

The book cover features a vibrant background with diagonal stripes in shades of red, purple, and blue. Overlaid on this is a black grid of thin lines that create a diamond-shaped pattern across the entire surface. The title is printed in a bold, white, sans-serif font, and the author's name is in a similar but slightly smaller font, both in italics.

PRINCIPLES OF MANAGEMENT SCIENCE

Paul Newbold

PRINCIPLES OF MANAGEMENT SCIENCE



Paul Newbold

University of Illinois, Urbana-Champaign

PRENTICE-HALL, Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging-in-Publication Data

Newbold, Paul.

Principles of management science.

Includes index.

1. Management science. I. Title.

HD30.23.N48 1986 658.4'001'51 85-28093

ISBN 0-13-701756-1

Editorial/production supervision by Margaret Rizzi
Cover and interior design by Suzanne Behnke
Manufacturing buyer: Ed O'Dougherty

© 1986 by Prentice-Hall
A Division of Simon & Schuster, Inc.
Englewood Cliffs, New Jersey 07632

All rights reserved. No part of this book may be
reproduced, in any form or by any means,
without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-701756-1 01

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Prentice-Hall of Southeast Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*
Whitehall Books Limited, *Wellington, New Zealand*

preface



This text is intended to introduce students to the principles of management science/operational research. The techniques discussed here are often described as “quantitative approaches to decision making in business,” and constitute a powerful arsenal of weapons for attacking a wide range of management problems.

The book is aimed primarily at students of business: Its purpose is to provide a flavor of many management science techniques, and illustrate the potential for their application. Consequently, while effort is made to explain why procedures work, their theoretical development is not discussed in any depth. A college algebra course should provide an adequate mathematical preparation for the first nine chapters of the text. Much of the material in Chapters 11 through 17 requires also some introductory knowledge of probability and statistics. With this in mind, I have included in Chapter 10 a review of the pertinent material in these areas.

In introducing management science techniques throughout the text, I have chosen to proceed by immediately working through illustrative examples. These have been designed to permit the reader to develop a more concrete grasp of solution algorithms, and of their potential for applications. Although, of necessity, these examples are simplified idealizations of real-world problems, it is hoped that they will convey insight into what can be achieved in practice with management science models. In addition, appended to many chapters are “Management Science in Action” sections, which provide discussions of more substantial applications.

There is more material in this text than can comfortably be covered in one semester, and, since the majority of the chapters are self-contained, a great deal of flexibility in designing a course is available. One possibility is

Chapter 1:	Introduction
Chapter 2–5:	Linear Programming
Chapter 8:	Integer Programming
Chapter 13:	Network Models
Chapter 14:	Inventory Models
Chapter 15:	Queueing
Chapter 16:	Simulation

The book has benefited greatly from the advice of many people who have read some or all of the chapters in draft form. In particular, I would like to thank S. Christian Albright, James A. Bartos, Richard J. Coppins, Lori S. Franz, George Heitmann, Michael R. Middleton, Laurence D. Richards, Robert Schellenberger, Daniel G. Shimshak, and William A. Verdini.

Finally, I must thank Steve Hotopp for computational assistance, Dixie Trinkle for expert typing, and my Prentice-Hall editors, Dennis Hogan and Margaret Rizzi, for much-needed encouragement and support throughout the development of this project.

contents



PREFACE xiii

1 INTRODUCING MANAGEMENT SCIENCE 1

- 1.1. Quantitative Models for Decision Making 1
- 1.2. Model Building 3
- 1.3. Deterministic and Stochastic Models 6
- 1.4. Some Management Science Problems: A Preview of Things to Come 7

2 LINEAR PROGRAMMING: GRAPHICAL SOLUTION 10

- 2.1. Areas of Application 10
- 2.2. A Maximization Problem 11
- 2.3. Setting Up the Problem 12
- 2.4. Graphic Description of the Problem 15
- 2.5. The Graphical Solution 22
- 2.6. Characterization of the Optimal Solution 23
- 2.7. Multiple Optimal Solutions, Infeasibility, and Unboundedness 29
- 2.8. Slack Variables 32
- 2.9. Redundant Constraints 34
- 2.10. A Minimization Problem and Its Solution 36
- 2.11. Surplus Variables 40
- Exercises 42

3 LINEAR PROGRAMMING: THE SIMPLEX METHOD 53

- 3.1. Introduction to the Simplex Method 53
- 3.2. A Constrained Maximization Problem: Initial Basic Feasible Solution 54
- 3.3. The Simplex Tableau 58
- 3.4. Improving the Initial Solution 61
- 3.5. Iterating to the Optimal Solution 66
- 3.6. A Further Example 68
- 3.7. Greater Than or Equal to and Equality Constraints in Maximization Problems:
Artificial Variables 72
- 3.8. Application of the Simplex Method to a Constrained Maximization Problem 78
- 3.9. Some Special Situations 81
- Exercises 91

MANAGEMENT SCIENCE IN ACTION

- Assigning Telephone Operators to Shifts: Betting by Phone in Australia 100

4 SENSITIVITY ANALYSIS AND DUALITY 102

- 4.1. Introduction to Sensitivity Analysis 102
- 4.2. Sensitivity to Changes in Objective Function Coefficients 103
- 4.3. Sensitivity to Changes in Right-hand Side Values of Constraints 108
- 4.4. Adding a New Decision Variable 115
- 4.5. Sensitivity Analysis for a Constrained Minimization Problem 116
- 4.6. A Further Example 123
- 4.7. Duality 128
- 4.8. Optimal Solutions of Primal and Dual Problems 131
- Exercises 136

5 COMPUTER SOLUTION OF LINEAR PROGRAMMING PROBLEMS, FORMULATION, AND APPLICATIONS 141

- 5.1. Using the Computer to Solve Linear Programming Problems 141
- 5.2. Application to Production Planning 147
- 5.3. Labor Scheduling 156
- 5.4. Financial Planning Application 161
- 5.5. Portfolio Selection Application 166
- Exercises 172

MANAGEMENT SCIENCE IN ACTION

- Planning Life Insurance Purchases: An Application of Linear Programming 190
- Planning Hospital Admissions: A Constrained Maximization Problem 191

6 SPECIAL LINEAR PROGRAMMING ALGORITHMS: TRANSPORTATION AND ASSIGNMENT PROBLEMS 192

- 6.1. Introduction 192
- 6.2. The Transportation Problem 193
- 6.3. Initial Feasible Solution for Transportation Problems: Vogel's Approximation Method (VAM) 197
- 6.4. Finding the Optimal Solution for Transportation Problems: The Modified Distribution (MODI) Method 201
- 6.5. Unbalanced Demand and Supply in Transportation Problems 208
- 6.6. Maximization Problems and the Transportation Algorithm 211
- 6.7. Production Planning and the Transportation Algorithm 216
- 6.8. The Assignment Problem 219
- 6.9. Finding the Optimal Solution for Assignment Problems: The Hungarian Method 222
- 6.10. Unbalanced Assignment Problems 225
- 6.11. Solving Maximization Assignment Problems 226
- Exercises 229

MANAGEMENT SCIENCE IN ACTION

Assigning Students to Company Interviews: Student Placement at the University of Minnesota 241

7 GOAL PROGRAMMING 243

- 7.1. Concepts of Goal Programming 243
- 7.2. Goal Programming without Priorities 247
- 7.3. Goal Programming with Priorities 250
- Exercises 255

MANAGEMENT SCIENCE IN ACTION

Public Health Resource Allocation: The Special Supplemental Food Program for Women, Infants, and Children 259

Balanced Nutrients: Diet Planning in the Third World 260

8 INTEGER LINEAR PROGRAMMING 261

- 8.1. Some Integer Linear Programming Problem Formulations 261
- 8.2. Solving All-integer Linear Programming Problems: The Branch-and-Bound Method 272
- 8.3. Using the Branch-and-Bound Method to Solve Mixed-Integer Linear Programming Problems 279
- 8.4. Using the Branch-and-Bound Method to Solve Zero-One Integer Linear Programming Problems 281

8.5. Integer Linear Programming Problems: Formulation and Applications	284
Exercises	294

MANAGEMENT SCIENCE IN ACTION

Design of an Automated Assembly System: An Application of 0–1 Integer Programming	303
Scheduling Radioisotope Production at General Electric: An Application of Mixed Integer Programming	304

9 DYNAMIC PROGRAMMING 305

9.1. Concepts of Dynamic Programming	305
9.2. A Shortest Route Problem	306
9.3. A Job Scheduling Problem	310
9.4. An Inventory Control Problem	315
Exercises	324

MANAGEMENT SCIENCE IN ACTION

Oil Stockpiles: Planning for Disruptions in Supply	330
--	-----

10 PROBABILITY AND STATISTICAL DISTRIBUTIONS 331

10.1. Uncertainty in Management Science	331
10.2. Random Experiment, Outcomes, Events	331
10.3. What Is Probability?	337
10.4. Probability Postulates and Their Consequences	339
10.5. Probability Rules	342
10.6. Bayes' Theorem	346
10.7. Random Variables and Their Expectations	348
10.8. Discrete Random Variables	351
10.9. The Binomial Distribution	353
10.10. The Poisson Distribution	356
10.11. Continuous Random Variables	358
10.12. The Normal Distribution	359
10.13. The Exponential Distribution	366
Exercises	367

11 DECISION THEORY 375

11.1. Decision Making in an Uncertain Environment	375
11.2. Solutions Not Using State-of-Nature Probabilities	378
11.3. The Expected Monetary Value Criterion	382
11.4. Used Sample Information: Bayesian Analysis	387

11.5. The Value of Sample Information	391
11.6. A Further Example	397
11.7. Risk and Utility	402
Exercises	409

MANAGEMENT SCIENCE IN ACTION

Choosing an Auxiliary Device for Icebreakers: Decision Making by the U.S. Coast Guard	421
Newspaper Advertising Rates: When to Raise Prices	421

12 **FORECASTING 423**

12.1. Introduction	423
12.2. Simple Exponential Smoothing	426
12.3. The Holt-Winters Exponential Smoothing Procedure	430
12.4. Forecasting from Regression Models	435
12.5. Forecasting from ARIMA Models: Box-Jenkins Methods	441
Exercises	443

MANAGEMENT SCIENCE IN ACTION

Extrapolation of Past Experience: Forecasting Emergency Work Load for the British Gas Corporation	447
---	-----

13 **NETWORK MODELS 448**

13.1. Introduction	448
13.2. PERT/CPM Networks	449
13.3. Project Scheduling with Uncertain Activity Times: Stochastic PERT	452
13.4. Trade-offs between Costs and Time: CPM	464
13.5. Control of Project Costs: PERT/Cost	471
13.6. The Shortest Route Problem	477
13.7. The Minimal Spanning Tree Problem	485
13.8 The Maximal Flow Problem	489
Exercises	494

MANAGEMENT SCIENCE IN ACTION

A Network for Product Distribution: The Problem of a Chemical Firm	509
--	-----

14 **INVENTORY MODELS 510**

14.1. Inventory Holding: Benefits and Costs	510
14.2. The Economic Order Quantity (EOQ) Model	512
14.3. The Economic Production Lot Size Model	521
14.4. Allowing for Stock-outs	526

14.5. Allowing for Quantity Discounts	530
14.6. The Material Requirements Planning (MRP) Approach	532
14.7. Reorder Point Models: Allowing for Stochastic Demand	535
14.8. Allowing for Declining Value of Items in Inventory	543
Exercises	549
MANAGEMENT SCIENCE IN ACTION	
Management of Spare Parts: The U.S. Army Wholesale Supply System	558

15 **QUEUEING 559**

15.1. The Waiting Line Problem	559
15.2. A Single-Channel Queueing Model	561
15.3. A Multiple-Channel Queueing Model	571
15.4. Some Other Queueing Models	576
15.5. The Economic Analysis of Waiting Line Problems	584
Exercises	587
Appendix A15.1	595
MANAGEMENT SCIENCE IN ACTION	
Police Patrol Operations: A Complex Queueing System	600

16 **SIMULATION 601**

16.1. Introduction	601
16.2. Simulation of a Queueing System	605
16.3. Validation of a Simulation Model	615
16.4. Generation of Random Numbers for Computer Simulation	619
16.5. Simulation of an Inventory System	622
Exercises	626
MANAGEMENT SCIENCE IN ACTION	
Simulation of a Queueing System: The Reproduction Unit of an Aerospace Manufacturer	632

17 **MARKOV PROCESSES AND THEIR APPLICATIONS 634**

17.1. Analysis of a Two-State Model	634
17.2. Application to Brand Shares	642
17.3. Markov Chains with Absorbing States	647
Exercises	651
Appendix A17.1	657

18 **MANAGEMENT INFORMATION AND DECISION SUPPORT SYSTEMS 667**

- 18.1. Introduction 667
- 18.2. Transaction Processing Systems 669
- 18.3. Information Systems 670
- 18.4. Decision Support Systems 671

MANAGEMENT SCIENCE IN ACTION

- Academic Resource Planning: A Decision Support System Based on Goal
Programming 673

APPENDIX: TABLES 675

- 1. Probability Function of the Binomial Distribution 675
- 2. Values of $e^{-\lambda}$ 680
- 3. Cumulative Distribution Function of the Standard Normal Distribution 681
- 4. Probability That a Multiserver Queueing System Will Be Idle 683
- 5. Some Uniformly Distributed Random Numbers 385
- 6. Cutoff Points of the Chi-square Distribution 686

ANSWERS TO SELECTED EVEN-NUMBERED EXERCISES 688

INDEX 699

introducing management science

CHAPTER ONE

1.1. QUANTITATIVE MODELS FOR DECISION MAKING

Managers are paid to make decisions. In any organization, problems regularly arise for which a choice among alternative actions is possible, and different choices will impact differentially on the well-being of the organization. Consider, for example the following types of business problems:

1. In a multiproduct firm, what mix of products should be produced in the months ahead?
2. When, and in what quantities, should materials and parts required in the production process be ordered? What is the right balance between the costs associated with the exhaustion of stocks and those of holding large inventories?
3. Which research and development projects should be pursued, and what sums should be budgeted for those purposes?
4. How should a new product be marketed? What is the appropriate price, production level, and promotion strategy?
5. What is the most efficient system for the distribution of goods from factories to customer outlets?
6. How should members of a sales force be assigned to different regional territories?
7. At a customer service center, how many servers should be available at different times of the day?
8. How should an investment portfolio be constituted?

This partial listing is sufficient to display the range of problem types met by corporate management. Some, such as production scheduling and ordering are faced regularly, though that is not to imply that their appropriate solutions

will remain fixed. Rather, as the environment in which a corporation operates evolves over time, the essential problem specifications will change, and hence so will the appropriate solutions. Others of our problems, such as new product introduction are of a one-off nature, and must often be attacked from the ground up, rather than through the modification of existing practice.

In spite of this diversity of problems, it is useful to broadly categorize management approaches to decision making as either **informal** or **formal**. The informal approach is based on managers' intuition, insight, and experience. There is no doubt that these qualities, which to some extent will be both sharpened and deepened through his or her career development, are invaluable to a manager. However, in this text our attention will be concentrated on more formal approaches to management decision-making. The methodology that we will discuss is generally referred to as **management science**, as it involves the application of the **scientific method** to problem solving by managers. This methodology involves the careful formulation of a problem, the collection of relevant data, the analysis of that data, and the derivation of conclusions on the basis of this analysis. One important ingredient of the methodology of the laboratory scientist is, however, more difficult to translate to the sphere of corporate management. It is often prohibitively expensive to **experiment** with real business systems. However, a good understanding of the workings of a system can often be achieved by building a **model** of that system, and model building constitutes an essential element of management science. In Chapter 16, we will see how **simulation** can sometimes be employed to allow at modest cost experimentation with a model of a real-world system.

The rational approach to management decision-making that has come to be called management science is also referred to on occasion as **operations research** or **decision science**. The three terms are virtually synonymous, and are used interchangeably in the literature. In essence, a formal analysis is to be made of a business decision-making problem. Although we will concentrate in this book on the formal analysis of problems, it should not be inferred that informal subjective judgment of managers is not also of value. On the contrary, the methods of management science should be viewed as a valuable aid to decision-making, often most profitably employed in conjunction with less formal considerations.

The management scientist constructs **quantitative models** to formulate and analyze business problems. This approach is gaining increased acceptance as managers increasingly find themselves confronted by complex problems, involving many factors, which are not well understood, and for which informally developed solutions have little appeal. Quantitative model building is also attractive in the solution of repetitive problems, such as reordering decisions, where much time and money can be saved through the repeated use of a formal solution algorithm. The growing popularity of management science methods can be put down to two factors. First, as successful implementations of these methods are reported, managers have grown increasingly willing to consider their application to their own problems. Second, the rapid development of the capabilities of the electronic computer has allowed the efficient storage and retrieval of vast quantities of information, and the carrying out of complex computations that

would otherwise have been prohibitively tedious. This development has allowed the possibility of the formal analysis of very large models in the study of problems that a few years ago, would of necessity have had to be attacked informally.

In the next section we will discuss and illustrate the nature of management model building.

1.2. MODEL BUILDING

A model is a representation of a real entity, and may be constructed in order to gain some understanding of, or insight into, that entity. Model building is at least as much an art as a science, requiring a balance between realism and simplicity. A model should be sufficiently realistic to incorporate the important characteristics of the real-world system it depicts, but not so complex as to obscure those characteristics.

Three types of models are generally distinguished:

(i) *Iconic models* are physical representations of real objects, designed to resemble in appearance those objects. For example, when a tall building is planned, engineers may construct a small scale model of that building, and of its surrounding area and environment, in order to conduct stress tests in a wind tunnel.

(ii) *Analog models* are also physical models, but represent the entities under study by analogy rather than by replica. An example is a graph showing the movements over time of stock market prices; this provides a pictorial representation of numerical data. A further example is a barometer which represents changes in atmospheric pressure through movements of a needle.

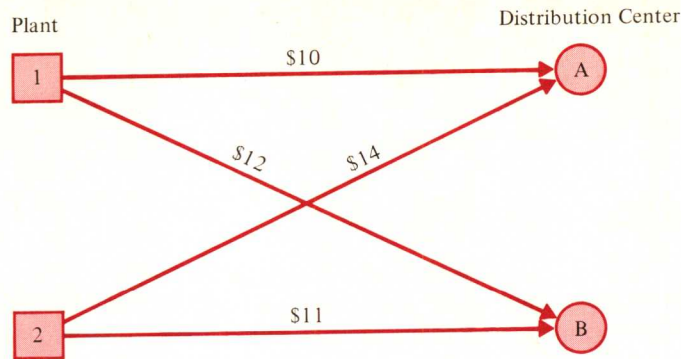
(iii) *Mathematical models*, or *symbolic models*, are more abstract representations than iconic or analog models. They attempt to provide, for example through an equation or system of equations, a description of a physical system.

Mathematical modeling is of central importance in management science, providing a means of describing and solving real-world problems. To illustrate, let us see how a problem can be expressed as a mathematical model.

A manufacturer of television sets has two plants and two distribution centers. For the coming week, distribution center A requires 300 sets, and center B, 250 sets. The maximum production capacities are 275 sets at plant 1, and 325 at plant 2. Television sets can be shipped from either plant to either distribution center, but shipping costs differ. Figure 1.1 shows the cost per set for shipments along each of the four possible routes. The manufacturer wants to supply the requirements at both distribution centers at the lowest possible total cost.

We now express this problem in mathematical form. The manufacturer must decide on four quantities—the numbers of sets to be shipped along each route. To develop a mathematical model of this problem, we must introduce symbols for each of these quantities. Let us set x_{1A} equal to the number of sets shipped from plant 1 to center A, x_{1B} the number from plant 1 to center B, x_{2A} the number from plant 2 to center A, and x_{2B} the number from plant 2 to center B.

FIGURE 1.1 Costs per unit for shipments of television sets.



Management is concerned about the total cost of the shipments. Since it costs \$10 per set for shipments from plant 1 to distribution center A, if x_{1A} sets are sent along this route, the total cost will be $\$10 x_{1A}$. Similarly, total shipment costs along the other three routes will be $\$12 x_{1B}$, $\$14 x_{2A}$, and $\$11 x_{2B}$. Therefore, the total cost, in dollars, of all shipments will be

$$C = 10 x_{1A} + 12 x_{1B} + 14 x_{2A} + 11 x_{2B} \quad (1.1)$$

The aim of this corporation is to choose x_{1A} , x_{1B} , x_{2A} , and x_{2B} so that the total cost given in equation (1.1) is as small as possible.

However, we do not as yet have a complete specification of the problem as limitations on the values that the variables x_{1A} , x_{1B} , x_{2A} , x_{2B} can take are imposed both by requirements at the two distribution centers and capacities of the two plants. Since 300 sets are needed at center A, this must be the total ($x_{1A} + x_{2A}$) of sets sent to that center from the two plants. We can therefore write

$$x_{1A} + x_{2A} = 300 \quad (1.2)$$

Similarly, the requirement that 250 sets must be sent to distribution center B implies that

$$x_{1B} + x_{2B} = 250 \quad (1.3)$$

Next, we must account for the limited production capacities at the two plants. The total number of sets shipped from plant 1 is $(x_{1A} + x_{1B})$. However, given the capacity of this plant, this number cannot exceed 275. This constraint is written

$$x_{1A} + x_{1B} \leq 275 \quad (1.4)$$

Similarly, as a result of the fact that plant 2 can produce at most 325 sets, we have

$$x_{2A} + x_{2B} \leq 325 \quad (1.5)$$

There is one further constraint on the values that can be taken by the variables x_{1A} , x_{1B} , x_{2A} , and x_{2B} ; they cannot be negative. Thus we write

$$x_{1A}, x_{1B}, x_{2A}, x_{2B} \geq 0 \quad (1.6)$$

The objective of this corporation, then, is to find those values of x_{1A} , x_{1B} , x_{2A} , x_{2B} for which total shipment cost (1.1) is smallest, subject to the requirements that these values satisfy the constraints (1.2)–(1.6). Thus, the mathematical model of our problem is

$$\begin{aligned} &\text{Minimize} && 10 x_{1A} + 12 x_{1B} + 14 x_{2A} + 11 x_{2B} \\ &\text{subject to} && x_{1A} + x_{2A} = 300 \\ & && x_{1B} + x_{2B} = 250 \\ & && x_{1A} + x_{1B} \leq 275 \\ & && x_{2A} + x_{2B} \leq 325 \\ & && x_{1A}, x_{1B}, x_{2A}, x_{2B} \geq 0 \end{aligned}$$

The advantage of expressing our problem in symbolic form in this manner is that we can appeal to mathematical methods for its solution. This is a member of a broad class of problems, called **linear programming** problems, whose solution will be discussed in considerable detail in Chapters 2, 3, 4, and 5. (In fact, our problem belongs to a special subclass of linear programming problems called **transportation** problems. Solution algorithms for such problems will be considered in Chapter 6.)