PRINCIPLES OF MANAGEMENT SCIENCE

Paul Newbold

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preface

This text is intended to introduce students to the principles of management science/operational research. The techniques discussed here are often described as "quantitative approaches to decision making in business," and constitute a powerful arsenal of weapons for attacking a wide range of management problems.

The book is aimed primarily at students of business: Its purpose is to provide a flavor of many management science techniques, and illustrate the potential for their application. Consequently, while effort is made to explain why procedures work, their theoretical development is not discussed in any depth. A college algebra course should provide an adequate mathematical preparation for the first nine chapters of the text. Much of the material in Chapters 11 through 17 requires also some introductory knowledge of probability and statistics. With this in mind, I have included in Chapter 10 a review of the pertinent material in these areas.

In introducing management science techniques throughout the text, I have chosen to proceed by immediately working through illustrative examples. These have been designed to permit the reader to develop a more concrete grasp of solution alogrithms, and of their potential for applications. Although, of necessity, these examples are simplified idealizations of real-world problems, it is hoped that they will convey insight into what can be achieved in practice with management science models. In addition, appended to many chapters are "Management Science in Action" sections, which provide discussions of more substantial applications.

There is more material in this text than can comfortably be covered in one semester, and, since the majority of the chapters are self-contained, a great deal of flexibility in designing a course is available. One possibility is

Chapter 1:	Introduction
Chapter 2–5:	Linear Programming
Chapter 8:	Integer Programming
Chapter 13:	Network Models
Chapter 14:	Inventory Models
Chapter 15:	Queueing
Chapter 16:	Simulation

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introducing management science

CHAPTER ONE

1.1. QUANTITATIVE MODELS FOR DECISION MAKING

Managers are paid to make decisions. In any organization, problems regularly arise for which a choice among alternative actions is possible, and different choices will impact differentially on the well-being of the organization. Consider, for example the following types of business problems:

1. In a multiproduct firm, what mix of products should be produced in the months ahead?
2. When, and in what quantities, should materials and parts required in the production process be ordered? What is the right balance between the costs associated with the exhaustion of stocks and those of holding large inventories?
3. Which research and development projects should be pursued, and what sums should be budgeted for those purposes?
4. How should a new product be marketed? What is the appropriate price, production level, and promotion strategy?
5. What is the most efficient system for the distribution of goods from factories to customer outlets?
6. How should members of a sales force be assigned to different regional territories?
7. At a customer service center, how many servers should be available at different times of the day?
8. How should an investment portfolio be constituted?

This partial listing is sufficient to display the range of problem types met by corporate management. Some, such as production scheduling and ordering are faced regularly, though that is not to imply that their appropriate solutions will remain fixed. Rather, as the environment in which a corporation operates evolves over time, the essential problem specifications will change, and hence so will the appropriate solutions. Others of our problems, such as new product introduction are of a one-off nature, and must often be attacked from the ground up, rather than through the modification of existing practice.

In spite of this diversity of problems, it is useful to broadly categorize management approaches to decision making as either **informal** or **formal.** The informal approach is based on managers' intuition, insight, and experience. There is no doubt that these qualities, which to some extent will be both sharpened and deepened through his or her career development, are invaluable to a manager. However, in this text our attention will be concentrated on more formal approaches to management decision-making. The methodology that we will discuss is generally referred to as **management science,** as it involves the application of the **scientific method** to problem solving by managers. This methodology involves the careful formulation of a problem, the collection of relevant data, the analysis of that data, and the derivation of conclusions on the basis of this analysis. One important ingredient of the methodology of the laboratory scientist is, however, more difficult to translate to the sphere of corporate mangement. It is often prohibitively expensive to **experiment** with real business systems. However, a good understanding of the workings of a system can often be achieved by building a **model** of that system, and model building constitutes an essential element of management science. In Chapter 16, we will see how **simulation** can sometimes be employed to allow at modest cost experimentation with a model of a real-world system.

The rational approach to management decision-making that has come to be called management science is also referred to on occasion as **operations research** or **decision science.** The three terms are virtually synonymous, and are used interchangeably in the literature. In essence, a formal analysis is to be made of a business decision-making problem. Although we will concentrate in this book on the formal analysis of problems, it should not be inferred that informal subjective judgment of managers is not also of value. On the contrary, the methods of management science should be viewed as a valuable aid to decision-making, often most profitably employed in conjunction with less formal considerations.

The management scientist constructs **quantitative models** to formulate and analyze business problems. This approach is gaining increased acceptance as managers increasingly find themselves confronted by complex problems, involving many factors, which are not well understood, and for which informally developed solutions have little appeal. Quantitative model building is also attractive in the solution of repetitive problems, such as reordering decisions, where much time and money can be saved through the repeated use of a formal solution algorithm. The growing popularity of management science methods can be put down to two factors. First, as successful implementations of these methods are reported, managers have grown increasingly willing to consider their application to their own problems. Second, the rapid development of the capabilities of the electronic computer has allowed the efficient storage and retrieval of vast quantities of information, and the carrying out of complex computations that would otherwise have been prohibitively tedious. This development has allowed the possibility of the formal analysis of very large models in the study of problems that a few years ago, would of necessity have had to be attacked informally.

In the next section we will discuss and illustrate the nature of management model building.

1.2. MODEL BUILDING

A model is a representation of a real entity, and may be constructed in order to gain some understanding of, or insight into, that entity. Model building is at least as much an art as a science, requiring a balance between realism and simplicity. A model should be sufficiently realistic to incorporate the important characteristics of the real-world system it depicts, but not so complex as to obscure those characteristics.

Three types of models are generally distinguished:

(i) *Iconic models* are physical representations of real objects, designed to resemble in appearance those objects. For example, when a tall building is planned, engineers may construct a small scale model of that building, and of its surrounding area and environment, in order to conduct stress tests in a wind tunnel.

(ii) *Analog models* are also physical models, but represent the entities under study by analogy rather than by replica. An example is a graph showing the movements over time of stock market prices; this provides a pictorial representation of numerical data. A further example is a barometer which represents changes in atmospheric pressure through movements of a needle.

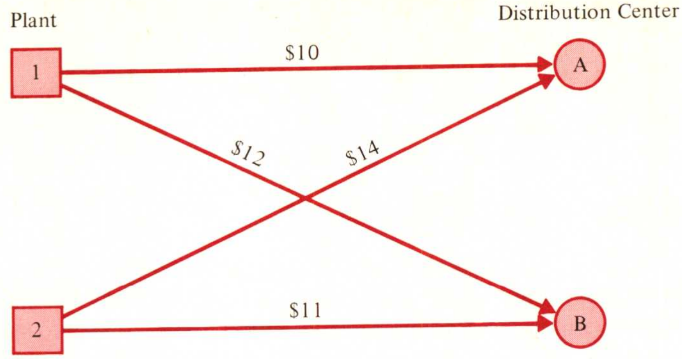
(iii) *Mathematical models,* or *symbolic models,* are more abstract representations than iconic or analog models. They attempt to provide, for example through an equation or system of equations, a description of a physical system.

Mathematical modeling is of central importance in management science, providing a means of describing and solving real-world problems. To illustrate, let us see how a problem can be expressed as a mathematical model.

A manufacturer of television sets has two plants and two distribution centers. For the coming week, distribution center A requires 300 sets, and center B, 250 sets. The maximum production capacities are 275 sets at plant 1, and 325 at plant 2. Television sets can be shipped from either plant to either distribution center, but shipping costs differ. Figure 1.1 shows the cost per set for shipments along each of the four possible routes. The manufacturer wants to supply the requirements at both distribution centers at the lowest possible total cost.

We now express this problem in mathematical form. The manufacturer must decide on four quantities—the numbers of sets to be shipped along each route. To develop a mathematical model of this problem, we must introduce symbols for each of these quantities. Let us set x_{1A} equal to the number of sets shipped from plant 1 to center A, x_{1B} the number from plant 1 to center B, x_{2A} the number from plant 2 to center A, and x_{2B} the number from plant 2 to center B.

FIGURE 1.1 Costs per unit for shipments of television sets.



Management is concerned about the total cost of the shipments. Since it costs $10 per set for shipments from plant 1 to distribution center A, if x_{1A} sets are sent along this route, the total cost will be $10 x_{1A}. Similarly, total shipment costs along the other three routes will be $12 x_{1B}, $14 x_{2A}, and $11 x_{2B}. Therefore, the total cost, in dollars, of all shipments will be

$$C = 10\,x_{1A} + 12\,x_{1B} + 14\,x_{2A} + 11\,x_{2B} \qquad (1.1)$$

The aim of this corporation is to choose x_{1A}, x_{1B}, x_{2A}, and x_{2B} so that the total cost given in equation (1.1) is as small as possible.

However, we do not as yet have a complete specification of the problem as limitations on the values that the variables x_{1A}, x_{1B}, x_{2A}, x_{2B} can take are imposed both by requirements at the two distribution centers and capacities of the two plants. Since 300 sets are needed at center A, this must be the total ($x_{1A} + x_{2A}$) of sets sent to that center from the two plants. We can therefore write

$$x_{1A} + x_{2A} = 300 \qquad (1.2)$$

Similarly, the requirement that 250 sets must be sent to distribution center B implies that

$$x_{1B} + x_{2B} = 250 \qquad (1.3)$$

Next, we must account for the limited production capacities at the two plants. The total number of sets shipped from plant 1 is $(x_{1A} + x_{1B})$. However, given the capacity of this plant, this number cannot exceed 275. This constraint is written

$$x_{1A} + x_{1B} \leq 275 \tag{1.4}$$

Similarly, as a result of the fact that plant 2 can produce at most 325 sets, we have

$$x_{2A} + x_{2B} \leq 325 \tag{1.5}$$

There is one further constraint on the values that can be taken by the variables x_{1A}, x_{1B}, x_{2A}, and x_{2B}; they cannot be negative. Thus we write

$$x_{1A}, x_{1B}, x_{2A}, x_{2B} \geq 0 \tag{1.6}$$

The objective of this corporation, then, is to find those values of x_{1A}, x_{1B}, x_{2A}, x_{2B} for which total shipment cost (1.1) is smallest, subject to the requirements that these values satisfy the constraints (1.2)–(1.6). Thus, the mathematical model of our problem is

$$\begin{aligned} \text{Minimize} \quad & 10\,x_{1A} + 12\,x_{1B} + 14\,x_{2A} + 11\,x_{2B} \\ \text{subject to} \quad & x_{1A} + x_{2A} = 300 \\ & x_{1B} + x_{2B} = 250 \\ & x_{1A} + x_{1B} \leq 275 \\ & x_{2A} + x_{2B} \leq 325 \\ & x_{1A}, x_{1B}, x_{2A}, x_{2B} \geq 0 \end{aligned}$$

The advantage of expressing our problem in symbolic form in this manner is that we can appeal to mathematical methods for its solution. This is a member of a broad class of problems, called **linear programming** problems, whose solution will be discussed in considerable detail in Chapters 2, 3, 4, and 5. (In fact, our problem belongs to a special subclass of linear programming problems called **transportation** problems. Solution algorithms for such problems will be considered in Chapter 6.)