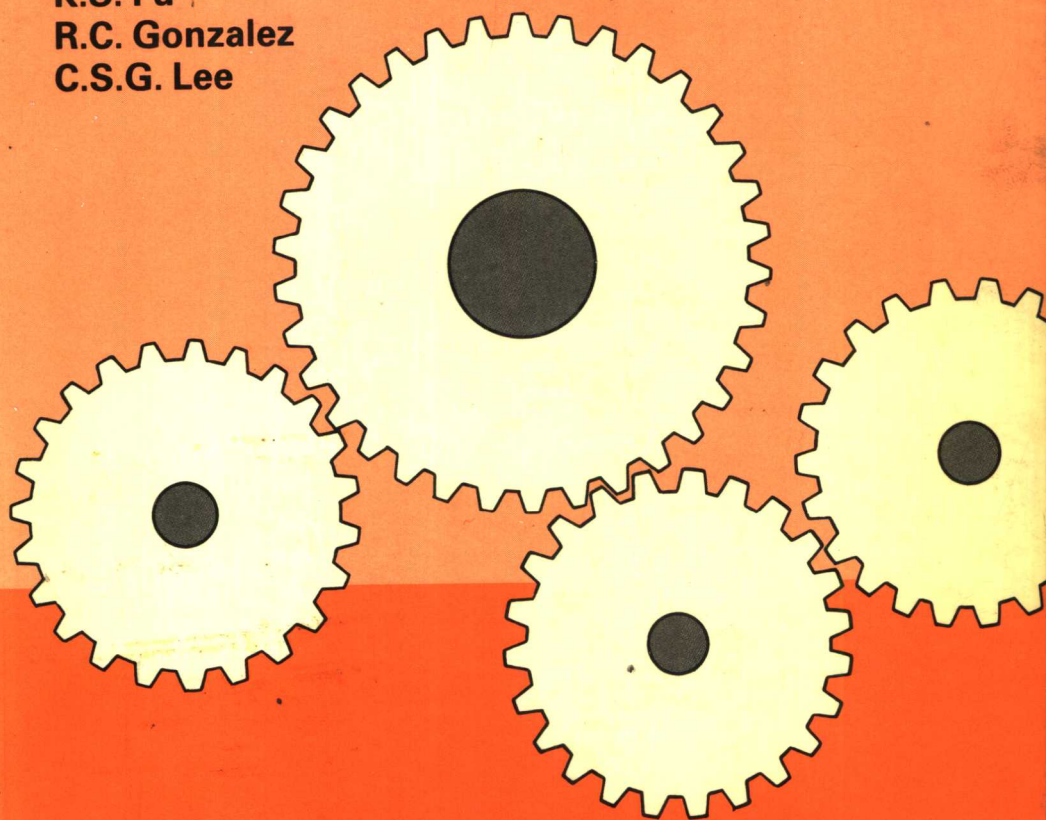


# ROBOTICS

Control, Sensing, Vision, and Intelligence

K.S. Fu  
R.C. Gonzalez  
C.S.G. Lee



McGRAW-HILL INTERNATIONAL EDITIONS

Industrial Engineering Series

---

# **ROBOTICS:**

## **Control, Sensing, Vision, and Intelligence**

---

**K. S. Fu**

*School of Electrical Engineering  
Purdue University*

**R. C. Gonzalez**

*Department of Electrical Engineering  
University of Tennessee  
and  
Perceptics Corporation  
Knoxville, Tennessee*

**C. S. G. Lee**

*School of Electrical Engineering  
Purdue University*

**McGraw-Hill Book Company**

New York St. Louis San Francisco Auckland Bogotá  
Hamburg London Madrid Mexico Milan Montreal New Delhi  
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

To  
Viola,  
Connie, and  
Pei-Ling

## **ROBOTICS: CONTROL, SENSING, VISION, AND INTELLIGENCE**

**INTERNATIONAL EDITION**

Copyright © 1987

Exclusive rights by McGraw-Hill Book Co., Singapore  
for manufacture and export. This book cannot be re-exported  
from the country to which it is consigned by McGraw-Hill.

1st Printing 1987

Copyright © 1987 by McGraw-Hill, Inc.

All rights reserved. No part of this publication may be  
reproduced or distributed in any form or by any means,  
or stored in a data base or a retrieval system, without  
the prior written permission of the publisher.

This book was set in Times Roman by House of Equations Inc.

The editor was Sanjeev Rao;

The production supervisor was Diane Renda;

Project supervision was done by Lynn Contrucci.

### **Library of Congress Cataloging-in-Publication Data**

Fu, K.S. (King Sun),

Robotics: control, sensing, vision and intelligence.

(McGraw-Hill series in CAD/CAM robotics and computer  
vision)

Bibliography; p.

Includes index.

1. Robotics I. Gonzalez, Rafael C. II. Lee,  
C. S. G. (C.S. George) III. Title.

TJ211.F82 1987 629.8'92 86-7156

ISBN 0-07-022625-3

ISBN 0-07-022626-1 (solutions manual)

**When ordering this title use ISBN 0-07-100421-1**

Printed and bound in Singapore by  
Chong Moh Offset Printing Pte Ltd

---

## ABOUT THE AUTHORS

---

K. S. Fu was the W. M. Goss Distinguished Professor of Electrical Engineering at Purdue University. He received his bachelor, master, and Ph.D. degrees from the National Taiwan University, the University of Toronto, and the University of Illinois, respectively. Professor Fu was internationally recognized in the engineering disciplines of pattern recognition, image processing, and artificial intelligence. He made milestone contributions in both basic and applied research. Often termed the "father of automatic pattern recognition," Dr. Fu authored four books and more than 400 scholarly papers. He taught and inspired 75 Ph.D.s. Among his many honors, he was elected a member of the National Academy of Engineering in 1976, received the Senior Research Award of the American Society for Engineering Education in 1981, and was awarded the IEEE's Education Medal in 1982. He was a Fellow of the IEEE, a 1971 Guggenheim Fellow, and a member of Sigma Xi, Eta Kappa Nu, and Tau Beta Pi honorary societies. He was the founding president of the International Association for Pattern Recognition, the founding editor in chief of the IEEE *Transactions of Pattern Analysis and Machine Intelligence*, and the editor in chief or editor for seven leading scholarly journals. Professor Fu died of a heart attack on April 29, 1985 in Washington, D.C.

R. C. Gonzalez is IBM Professor of Electrical Engineering at the University of Tennessee, Knoxville, and founder and president of Perceptics Corporation, a high-technology firm that specializes in image processing, pattern recognition, computer vision, and machine intelligence. He received his B.S. degree from the University of Miami, and his M.E. and Ph.D. degrees from the University of Florida, Gainesville, all in electrical engineering. Dr. Gonzalez is internationally known in his field, having authored or coauthored over 100 articles and 4 books dealing with image processing, pattern recognition, and computer vision. He received the 1978 UTK Chancellor's Research Scholar Award, the 1980 Magnavox Engineering Professor Award, and the 1980 M.E. Brooks Distinguished Professor Award for his work in these fields. In 1984 he was named Alumni Distinguished Service Professor

at the University of Tennessee. In 1985 he was named a distinguished alumnus by the University of Miami. Dr. Gonzalez is a frequent consultant to industry and government and is a member of numerous engineering professional and honorary societies, including Tau Beta Pi, Phi Kappa Phi, Eta Kappa Nu, and Sigma Xi. He is a Fellow of the IEEE.

C. S. G. Lee is an associate professor of Electrical Engineering at Purdue University. He received his B.S.E.E. and M.S.E.E. degrees from Washington State University, and a Ph.D. degree from Purdue in 1978. From 1978 to 1985, he was a faculty member at Purdue and the University of Michigan, Ann Arbor. Dr. Lee has authored or coauthored more than 40 technical papers and taught robotics short courses at various conferences. His current interests include robotics and automation, and computer-integrated manufacturing systems. Dr. Lee has been doing extensive consulting work for automotive and aerospace industries in robotics. He is a Distinguished Visitor of the IEEE Computer Society's Distinguished Visitor Program since 1983, a technical area editor of the IEEE *Journal of Robotics and Automation*, and a member of technical committees for various robotics conferences. He is a coeditor of *Tutorial on Robotics*, 2nd edition, published by the IEEE Computer Society Press and a member of Sigma Xi, Tau Beta Pi, the IEEE, and the SME/RI.

---

## PREFACE

---

This textbook was written to provide engineers, scientists, and students involved in robotics and automation with a comprehensive, well-organized, and up-to-date account of the basic principles underlying the design, analysis, and synthesis of robotic systems.

The study and development of robot mechanisms can be traced to the mid-1940s when master-slave manipulators were designed and fabricated at the Oak Ridge and Argonne National Laboratories for handling radioactive materials. The first commercial computer-controlled robot was introduced in the late 1950s by Unimation, Inc., and a number of industrial and experimental devices followed suit during the next 15 years. In spite of the availability of this technology, however, widespread interest in robotics as a formal discipline of study and research is rather recent, being motivated by a significant lag in productivity in most nations of the industrial world.

Robotics is an interdisciplinary field that ranges in scope from the design of mechanical and electrical components to sensor technology, computer systems, and artificial intelligence. The bulk of material dealing with robot theory, design, and applications has been widely scattered in numerous technical journals, conference proceedings, research monographs, and some textbooks that either focus attention on some specialized area of robotics or give a "broad-brush" look of this field. Consequently, it is a rather difficult task, particularly for a newcomer, to learn the range of principles underlying this subject matter. This text attempts to put between the covers of one book the basic analytical techniques and fundamental principles of robotics, and to organize them in a unified and coherent manner. Thus, the present volume is intended to be of use both as a textbook and as a reference work. To the student, it presents in a logical manner a discussion of basic theoretical concepts and important techniques. For the practicing engineer or scientist, it provides a ready source of reference in systematic form.

The mathematical level in all chapters is well within the grasp of seniors and first-year graduate students in a technical discipline such as engineering and computer science, which require introductory preparation in matrix theory, probability, computer programming, and mathematical analysis. In presenting the material, emphasis is placed on the development of fundamental results from basic concepts. Numerous examples are worked out in the text to illustrate the discussion, and exercises of various types and complexity are included at the end of each chapter. Some of these problems allow the reader to gain further insight into the points discussed in the text through practice in problem solution. Others serve as supplements and extensions of the material in the book. For the instructor, a complete solutions manual is available from the publisher.

This book is the outgrowth of lecture notes for courses taught by the authors at Purdue University, the University of Tennessee, and the University of Michigan. The material has been tested extensively in the classroom as well as through numerous short courses presented by all three authors over a 5-year period. The suggestions and criticisms of students in these courses had a significant influence in the way the material is presented in this book.

We are indebted to a number of individuals who, directly or indirectly, assisted in the preparation of the text. In particular, we wish to extend our appreciation to Professors W. L. Green, G. N. Saridis, R. B. Kelley, J. Y. S. Luh, N. K. Loh, W. T. Snyder, D. Brzakovic, E. G. Burdette, M. J. Chung, B. H. Lee, and to Dr. R. E. Woods, Dr. Spivey Douglass, Dr. A. K. Bejczy, Dr. C. Day, Dr. F. King, and Dr. L-W. Tsai. As is true with most projects carried out in a university environment, our students over the past few years have influenced not only our thinking, but also the topics covered in this book. The following individuals have worked with us in the course of their advanced undergraduate or graduate programs: J. A. Herrera, M. A. Abidi, R. O. Eason, R. Safabakhsh, A. P. Perez, C. H. Hayden, D. R. Cate, K. A. Rinehart, N. Alvertos, E. R. Meyer, P. R. Chang, C. L. Chen, S. H. Hou, G. H. Lee, R. Jungclas, Huarg, and D. Huang. Thanks are also due to Ms. Susan Merrell, Ms. Denise Smiddy, Ms. Mary Bearden, Ms. Frances Bourdas, and Ms. Mary Ann Pruder for typing numerous versions of the manuscript. In addition, we express our appreciation to the National Science Foundation, the Air Force Office of Scientific Research, the Office of Naval Research, the Army Research Office, Westinghouse, Martin Marietta Aerospace, Martin Marietta Energy Systems, Union Carbide, Lockheed Missiles and Space Co., The Oak Ridge National Laboratory, and the University of Tennessee Measurement and Control Center for their sponsorship of our research activities in robotics, computer vision, machine intelligence, and related areas.

*K. S. Fu  
R. C. Gonzalez  
C. S. Lee*

Professor King-Sun Fu died of a heart attack on April 29, 1985, in Washington, D.C., shortly after completing his contributions to this book. He will be missed by all those who were fortunate to know him and to work with him during a productive and distinguished career.

*R. C. G.*

*C. S. L.*



---

# CONTENTS

---

## Preface

|  |           |
|--|-----------|
| <b>1. Introduction</b>                                     | <b>1</b>  |
| 1.1. Background  | 1         |
| 1.2. Historical Development                                | 4         |
| 1.3. Robot Arm Kinematics and Dynamics                     | 6         |
| 1.4. Manipulator Trajectory Planning<br>and Motion Control | 7         |
| 1.5. Robot Sensing   | 8         |
| 1.6. Robot Programming Languages                           | 9         |
| 1.7. Machine Intelligence                                  | 10        |
| 1.8. References  | 10        |
| <b>2. Robot Arm Kinematics</b>                             | <b>12</b> |
| 2.1. Introduction  | 12        |
| 2.2. The Direct Kinematics Problem                         | 13        |
| 2.3. The Inverse Kinematics Solution                       | 52        |
| 2.4. Concluding Remarks                                    | 75        |
| References   | 76        |
| Problems   | 76        |
| <b>3. Robot Arm Dynamics</b>                               | <b>82</b> |
| 3.1. Introduction  | 82        |
| 3.2. Lagrange-Euler Formulation                            | 84        |
| 3.3. Newton-Euler Formulation                              | 103       |
| 3.4. Generalized D'Alembert<br>Equations of Motion         | 124       |
| 3.5. Concluding Remarks                                    | 142       |
| References   | 142       |
| Problems   | 144       |

|  |            |
|--|------------|
| <b>4. Planning of Manipulator Trajectories</b>           | <b>149</b> |
| 4.1. Introduction  | 149        |
| 4.2. General Considerations on Trajectory Planning       | 151        |
| 4.3. Joint-interpolated Trajectories                     | 154        |
| 4.4. Planning of Manipulator Cartesian Path Trajectories | 175        |
| 4.5. Concluding Remarks                                  | 196        |
| References   | 197        |
| Problems   | 198        |
| <b>5. Control of Robot Manipulators</b>                  | <b>201</b> |
| 5.1. Introduction  | 201        |
| 5.2. Control of the Puma Robot Arm                       | 203        |
| 5.3. Computed Torque Technique                           | 205        |
| 5.4. Near-Minimum-Time Control                           | 223        |
| 5.5. Variable Structure Control                          | 226        |
| 5.6. Nonlinear Decoupled Feedback Control                | 227        |
| 5.7. Resolved Motion Control                             | 232        |
| 5.8. Adaptive Control                                    | 244        |
| 5.9. Concluding Remarks                                  | 263        |
| References   | 265        |
| Problems   | 265        |
| <b>6. Sensing</b>  | <b>267</b> |
| 6.1. Introduction  | 267        |
| 6.2. Range Sensing                                       | 268        |
| 6.3. Proximity Sensing                                   | 276        |
| 6.4. Touch Sensors                                       | 284        |
| 6.5. Force and Torque Sensing                            | 289        |
| 6.6. Concluding Remarks                                  | 293        |
| References   | 293        |
| Problems   | 293        |
| <b>7. Low-Level Vision</b>                               | <b>296</b> |
| 7.1. Introduction  | 296        |
| 7.2. Image Acquisition                                   | 297        |
| 7.3. Illumination Techniques                             | 304        |
| 7.4. Imaging Geometry                                    | 307        |
| 7.5. Some Basic Relationships Between Pixels             | 328        |
| 7.6. Preprocessing                                       | 331        |
| 7.7. Concluding Remarks                                  | 359        |
| References   | 360        |
| Problems   | 360        |

|  |            |
|--|------------|
| <b>8. Higher-Level Vision</b>  | <b>362</b> |
| 8.1. Introduction  | 362        |
| 8.2. Segmentation  | 363        |
| 8.3. Description   | 395        |
| 8.4. Segmentation and Description of<br>Three-Dimensional Structures | 416        |
| 8.5. Recognition   | 424        |
| 8.6. Interpretation  | 439        |
| 8.7. Concluding Remarks  | 445        |
| References   | 445        |
| Problems   | 447        |
| <b>9. Robot Programming Languages</b>                                | <b>450</b> |
| 9.1. Introduction  | 450        |
| 9.2. Characteristics of Robot-<br>Level Languages                    | 451        |
| 9.3. Characteristics of Task-<br>Level Languages                     | 462        |
| 9.4. Concluding Remarks  | 470        |
| References   | 472        |
| Problems   | 473        |
| <b>10. Robot Intelligence and Task Planning</b>                      | <b>474</b> |
| 10.1. Introduction   | 474        |
| 10.2. State Space Search   | 474        |
| 10.3. Problem Reduction  | 484        |
| 10.4. Use of Predicate Logic   | 489        |
| 10.5. Means-Ends Analysis  | 493        |
| 10.6. Problem-Solving  | 497        |
| 10.7. Robot Learning   | 504        |
| 10.8. Robot Task Planning  | 506        |
| 10.9. Basic Problems in Task Planning                                | 509        |
| 10.10. Expert Systems and<br>Knowledge Engineering                   | 516        |
| 10.11. Concluding Remarks  | 519        |
| References   | 520        |
| <b>Appendix</b>  | <b>522</b> |
| A Vectors and Matrices   | 522        |
| B Manipulator Jacobian   | 544        |
| <br>Bibliography   | <br>556    |
| Index  | 571        |

## INTRODUCTION

One machine can do the work of a  
hundred ordinary men, but no machine  
can do the work of one extraordinary man.

*Elbert Hubbard*

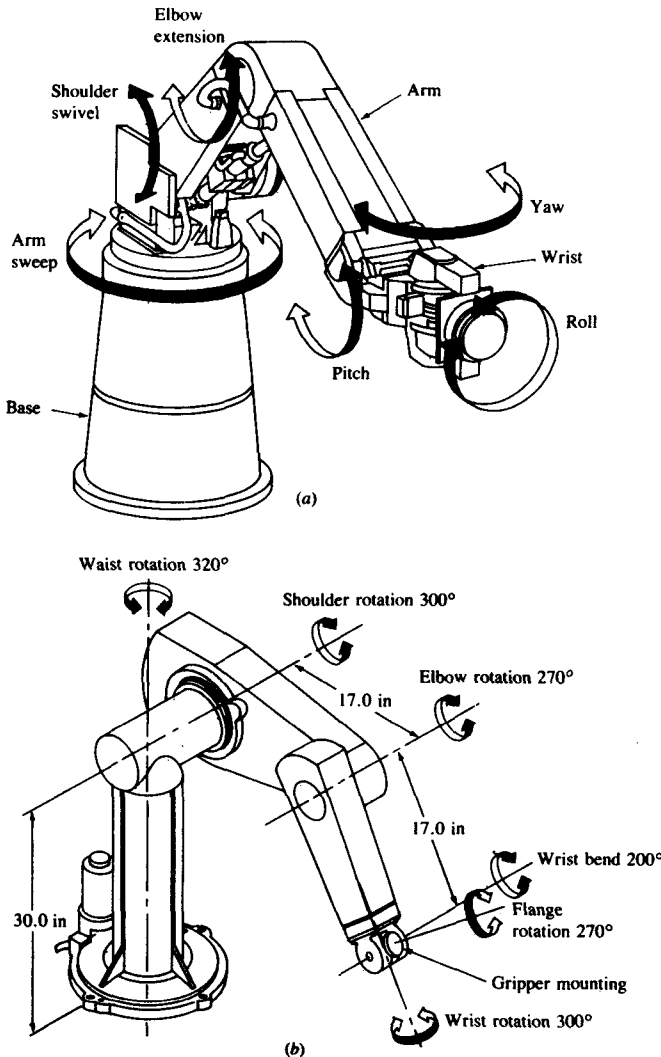
### 1.1 BACKGROUND

With a pressing need for increased productivity and the delivery of end products of uniform quality, industry is turning more and more toward computer-based automation. At the present time, most automated manufacturing tasks are carried out by special-purpose machines designed to perform predetermined functions in a manufacturing process. The inflexibility and generally high cost of these machines, often called *hard automation systems*, have led to a broad-based interest in the use of robots capable of performing a variety of manufacturing functions in a more flexible working environment and at lower production costs.

The word *robot* originated from the Czech word *robota*, meaning work. Webster's dictionary defines robot as "an automatic device that performs functions ordinarily ascribed to human beings." With this definition, washing machines may be considered robots. A definition used by the Robot Institute of America gives a more precise description of industrial robots: "A robot is a *reprogrammable multi-functional* manipulator designed to move materials, parts, tools, or specialized devices, through variable programmed motions for the performance of a variety of tasks." In short, a robot is a reprogrammable general-purpose manipulator with external sensors that can perform various assembly tasks. With this definition, a robot must possess *intelligence*, which is normally due to computer algorithms associated with its control and sensing systems.

An industrial robot is a general-purpose, computer-controlled manipulator consisting of several rigid links connected in series by revolute or prismatic joints. One end of the chain is attached to a supporting base, while the other end is free and equipped with a tool to manipulate objects or perform assembly tasks. The motion of the joints results in relative motion of the links. Mechanically, a robot is composed of an arm (or mainframe) and a wrist subassembly plus a tool. It is designed to reach a workpiece located within its work volume. The work volume is the sphere of influence of a robot whose arm can deliver the wrist subassembly unit to any point within the sphere. The arm subassembly generally can move with three degrees of freedom. The combination of the movements positions the

wrist unit at the workpiece. The wrist subassembly unit usually consists of three rotary motions. The combination of these motions orients the tool according to the configuration of the object for ease in pickup. These last three motions are often called *pitch*, *yaw*, and *roll*. Hence, for a six-jointed robot, the arm subassembly is the positioning mechanism, while the wrist subassembly is the orientation mechanism. These concepts are illustrated by the Cincinnati Milacron  $T^3$  robot and the Unimation PUMA robot arm shown in Fig. 1.1.



**Figure 1.1** (a) Cincinnati Milacron  $T^3$  robot arm. (b) PUMA 560 series robot arm.

Many commercially available industrial robots are widely used in manufacturing and assembly tasks, such as material handling, spot/arc welding, parts assembly, paint spraying, loading and unloading numerically controlled machines, space and undersea exploration, prosthetic arm research, and in handling hazardous materials. These robots fall into one of the four basic motion-defining categories (Fig. 1.2):

Cartesian coordinates (three linear axes) (e.g., IBM's RS-1 robot and the Sigma robot from Olivetti)

Cylindrical coordinates (two linear and one rotary axes) (e.g., Versatran 600 robot from Prab)

Spherical coordinates (one linear and two rotary axes) (e.g., Unimate 2000B from Unimation Inc.)

Revolute or articulated coordinates (three rotary axes) (e.g.,  $T^3$  from Cincinnati Milacron and PUMA from Unimation Inc.)

Most of today's industrial robots, though controlled by mini- and micro-computers, are basically simple positional machines. They execute a given task by

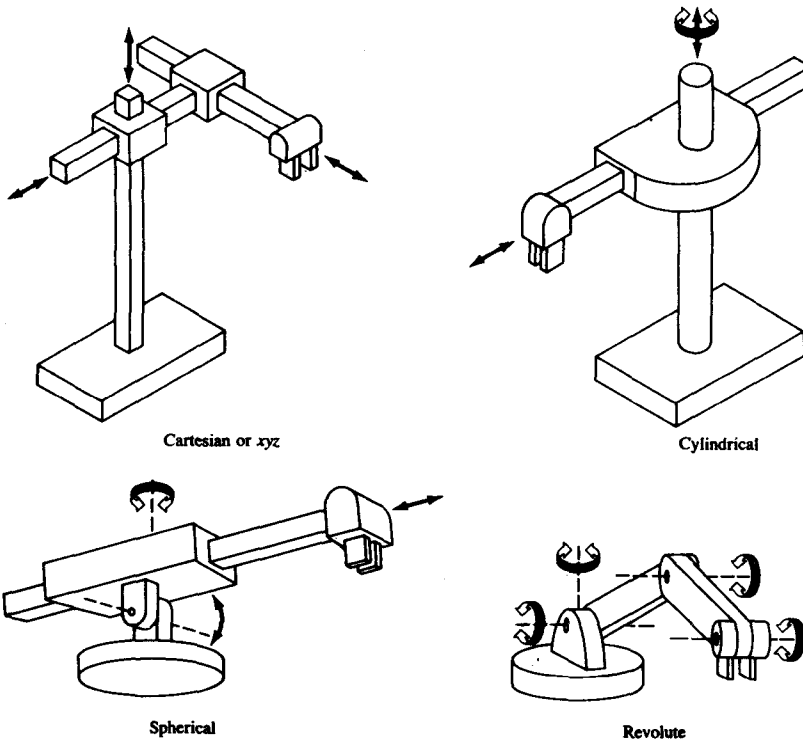


Figure 1.2 Various robot arm categories.

playing back prerecorded or preprogrammed sequences of motions that have been previously guided or taught by a user with a hand-held control-teach box. Moreover, these robots are equipped with little or no external sensors for obtaining the information vital to its working environment. As a result, robots are used mainly in relatively simple, repetitive tasks. More research effort is being directed toward improving the overall performance of the manipulator systems, and one way is through the study of the various important areas covered in this book.

## 1.2 HISTORICAL DEVELOPMENT

The word *robot* was introduced into the English language in 1921 by the playwright Karel Capek in his satirical drama, *R.U.R.* (Rossum's Universal Robots). In this work, robots are machines that resemble people, but work tirelessly. Initially, the robots were manufactured for profit to replace human workers but, toward the end, the robots turned against their creators, annihilating the entire human race. Capek's play is largely responsible for some of the views popularly held about robots to this day, including the perception of robots as humanlike machines endowed with intelligence and individual personalities. This image was reinforced by the 1926 German robot film *Metropolis*, by the walking robot Electro and his dog Sparko, displayed in 1939 at the New York World's Fair, and more recently by the robot C3PO featured in the 1977 film *Star Wars*. Modern industrial robots certainly appear primitive when compared with the expectations created by the communications media during the past six decades.

Early work leading to today's industrial robots can be traced to the period immediately following World War II. During the late 1940s research programs were started at the Oak Ridge and Argonne National Laboratories to develop remotely controlled mechanical manipulators for handling radioactive materials. These systems were of the "master-slave" type, designed to reproduce faithfully hand and arm motions made by a human operator. The master manipulator was guided by the user through a sequence of motions, while the slave manipulator duplicated the master unit as closely as possible. Later, force feedback was added by mechanically coupling the motion of the master and slave units so that the operator could feel the forces as they developed between the slave manipulator and its environment. In the mid-1950s the mechanical coupling was replaced by electric and hydraulic power in manipulators such as General Electric's Handyman and the Minotaur I built by General Mills.

The work on master-slave manipulators was quickly followed by more sophisticated systems capable of autonomous, repetitive operations. In the mid-1950s George C. Devol developed a device he called a "programmed articulated transfer device," a manipulator whose operation could be programmed (and thus changed) and which could follow a sequence of motion steps determined by the instructions in the program. Further development of this concept by Devol and Joseph F. Engelberger led to the first industrial robot, introduced by Unimation Inc. in 1959. The key to this device was the use of a computer in conjunction with a manipula-

tor to produce a machine that could be "taught" to carry out a variety of tasks automatically. Unlike hard automation machines, these robots could be reprogrammed and retooled at relative low cost to perform other jobs as manufacturing requirements changed.

While programmed robots offered a novel and powerful manufacturing tool, it became evident in the 1960s that the flexibility of these machines could be enhanced significantly by the use of sensory feedback. Early in that decade, H. A. Ernst [1962] reported the development of a computer-controlled mechanical hand with tactile sensors. This device, called the MH-1, could "feel" blocks and use this information to control the hand so that it stacked the blocks without operator assistance. This work is one of the first examples of a robot capable of adaptive behavior in a reasonably unstructured environment. The manipulative system consisted of an ANL Model-8 manipulator with 6 degrees of freedom controlled by a TX-O computer through an interfacing device. This research program later evolved as part of project MAC, and a television camera was added to the manipulator to begin machine perception research. During the same period, Tomovic and Boni [1962] developed a prototype hand equipped with a pressure sensor which sensed the object and supplied an input feedback signal to a motor to initiate one of two grasp patterns. Once the hand was in contact with the object, information proportional to object size and weight was sent to a computer by these pressure-sensitive elements. In 1963, the American Machine and Foundry Company (AMF) introduced the VERSATRAN commercial robot. Starting in this same year, various arm designs for manipulators were developed, such as the Roehampton arm and the Edinburgh arm.

In the late 1960s, McCarthy [1968] and his colleagues at the Stanford Artificial Intelligence Laboratory reported development of a computer with hands, eyes, and ears (i.e., manipulators, TV cameras, and microphones). They demonstrated a system that recognized spoken messages, "saw" blocks scattered on a table, and manipulated them in accordance with instructions. During this period, Pieper [1968] studied the kinematic problem of a computer-controlled manipulator while Kahn and Roth [1971] analyzed the dynamics and control of a restricted arm using bang-bang (near minimum time) control.

Meanwhile, other countries (Japan in particular) began to see the potential of industrial robots. As early as 1968, the Japanese company Kawasaki Heavy Industries negotiated a license with Unimation for its robots. One of the more unusual developments in robots occurred in 1969, when an experimental walking truck was developed by the General Electric Company for the U.S. Army. In the same year, the Boston arm was developed, and in the following year the Stanford arm was developed, which was equipped with a camera and computer controller. Some of the most serious work in robotics began as these arms were used as robot manipulators. One experiment with the Stanford arm consisted of automatically stacking blocks according to various strategies. This was very sophisticated work for an automated robot at that time. In 1974, Cincinnati Milacron introduced its first computer-controlled industrial robot. Called "The Tomorrow Tool," or  $T^3$ , it could lift over 100 lb as well as track moving objects on an assembly line.



During the 1970s a great deal of research work focused on the use of external sensors to facilitate manipulative operations. At Stanford, Bolles and Paul [1973], using both visual and force feedback, demonstrated a computer-controlled Stanford arm connected to a PDP-10 computer for assembling automotive water pumps. At about the same time, Will and Grossman [1975] at IBM developed a computer-controlled manipulator with touch and force sensors to perform mechanical assembly of a 20-part typewriter. Inoue [1974] at the MIT Artificial Intelligence Laboratory worked on the artificial intelligence aspects of force feedback. A landfall navigation search technique was used to perform initial positioning in a precise assembly task. At the Draper Laboratory Nevins et al. [1974] investigated sensing techniques based on compliance. This work developed into the instrumentation of a passive compliance device called *remote center compliance* (RCC) which was attached to the mounting plate of the last joint of the manipulator for close parts-mating assembly. Bejczy [1974], at the Jet Propulsion Laboratory, implemented a computer-based torque-control technique on his extended Stanford arm for space exploration projects. Since then, various control methods have been proposed for servoing mechanical manipulators.

Today, we view robotics as a much broader field of work than we did just a few years ago, dealing with research and development in a number of interdisciplinary areas, including kinematics, dynamics, planning systems, control, sensing, programming languages, and machine intelligence. These topics, introduced briefly in the following sections, constitute the core of the material in this book.

### 1.3 ROBOT ARM KINEMATICS AND DYNAMICS

Robot arm kinematics deals with the analytical study of the geometry of motion of a robot arm with respect to a fixed reference coordinate system without regard to the forces/moments that cause the motion. Thus, kinematics deals with the analytical description of the spatial displacement of the robot as a function of time, in particular the relations between the joint-variable space and the position and orientation of the end-effector of a robot arm.

There are two fundamental problems in robot arm kinematics. The first problem is usually referred to as the *direct* (or *forward*) *kinematics* problem, while the second problem is the *inverse kinematics* (or *arm solution*) problem. Since the independent variables in a robot arm are the joint variables, and a task is usually stated in terms of the reference coordinate frame, the inverse kinematics problem is used more frequently. Denavit and Hartenberg [1955] proposed a systematic and generalized approach of utilizing matrix algebra to describe and represent the spatial geometry of the links of a robot arm with respect to a fixed reference frame. This method uses a  $4 \times 4$  homogeneous transformation matrix to describe the spatial relationship between two adjacent rigid mechanical links and reduces the direct kinematics problem to finding an equivalent  $4 \times 4$  homogeneous transformation matrix that relates the spatial displacement of the hand coordinate frame to the reference coordinate frame. These homogeneous transformation matrices are also useful in deriving the dynamic equations of motion of a robot arm. In general, the