15183

Electromagnetomechanical Interactions in Deformable Solids and Structures

edited by

Y. Yamamoto and K. Miya

ELECTROMAGNETOMECHANICA INTERACTIONS IN DEFORMABLE SOLIDS AND STRUCTURES

Proceedings of the IUTAM Symposium held in Tokyo, Japan, 12-17 October, 1986

Edited by

Yoshiyuki YAMAMOTO
Tokyo Denki University
Hatoyama, Saitama
Japan

and

Kenzo MIYA
University of Tokyo
Tokai, Ibaraki
Japan

1987

NORTH-HOLLAND
AMSTERDAM • NEW YORK • OXFORD • TOKYO

ISBN: 0 444 70231 8

Published by:
ELSEVIER SCIENCE PUBLISHERS B.V.
P.O. Box 1991
1000 BZ Amsterdam
The Netherlands

Sole distributors for the U.S.A. and Canada:
ELSEVIER SCIENCE PUBLISHING COMPANY, INC.
52 Vanderbilt Avenue
New York, N.Y. 10017
U.S.A.

Library of Congress Cataloging-in-Publication Data

IUTAM Symposium (1986 : Tokyo, Japan)
Electromagnetomechanical interactions in deformable solids and structures.

Includes bibliographies.
1. Deformations (Mechanics)--Congresses. 2. Electromagnetic interactions--Congresses. 3. Ferromagnetic materials--Congresses. 4. Eddy currents (Electric)--Congresses. I. Yamamoto, Yoshiyuki, 1924- II. Miya, Kenzo, 1940- . III. Title.
TA417.6.I98 1986 620.1'123 87-8871
ISBN 0-444-70231-8

PRINTED IN THE NETHERLANDS

THE IUTAM SYMPOSIUM IN TOKYO-OCT. 1986

PREFACE

The International Symposium on "Electromagnetomechanical Interactions in Deformable Solids and Structures" was held in the Sanjo Hall of the University of Tokyo in Tokyo, Japan, between 12-17 October, 1986, under the sponsorship of the International Union of Theoretical and Applied Mechanics and with the support of the National Committee for Theoretical and Applied Mechanics of the Science Council of Japan. The scientific committee of the symposium consisted of eight members. The local organizing committee and the local executive committee were formed to execute tasks efficiently.

The first IUTAM-IUPAP symposium relating to electromagnetomechanical behaviour of solid continua was held in Paris, France, in July 1983 and was composed of a new group of scientists and engineers. The 1986 symposium was proposed in response to the desire that a similar kind of IUTAM symposium should be held in Japan. It was authorized by IUTAM in 1984.

The basic concept of this symposium was placed on the stimulation and exchange of creative ideas and the promotion of advanced investigations into electromagnetomechanical interaction phenomena. The importance of research in this field has been well recognized due to the rapid developments which have, recently, been achieved. These developments are particularly evident in both nuclear fusion reactor technology and electroacoustic device technology in the light of electromagnetoelasticity.

The scope of the symposium was extended to:
(1) Magnetomechanics of structural components;
(2) Magnetomechanics of superconducting magnets;
(3) Wave propagation in magnetic fields;
(4) Acoustoelectricity and piezoelectricity;
(5) The application of eddy current techniques in flaw detection;
(6) New numerical schemes for eddy currents and magnetic field analysis.

Six distinguished scholars were invited to deliver general lectures which surveyed recent developments and explored new ideas and methods for dealing with problems in this recently developing field. A wide range of theoretical, experimental, and numerical research was reported and discussed on the above-mentioned subjects. A total of seventy-two papers were presented at the symposium. Forty-one participants from Japan attended the presentations together with more than fifty participants from overseas. All the presentations were made orally. The authors prepared their papers in "camera-ready" form so that they could be included in these proceedings.

Financial support was provided by the International Union of Theoretical and Applied Mechanics and the Japanese Society for the Promotion of Science. Donations from the Japanese World Exposition Commemorative Fund, the Mitsubishi Foundation, and the Kashima Foundation are cordially acknowledged.

All work concerned with the preparatory management of the symposium, related events, and the editorial duties involved in preparing these proceedings were conducted by the staff of Professor Miya's laboratory under the auspices of the local executive committee. We greatly appreciate their devoted efforts and con-

tributions to this symposium. We believe that its success might not have been achieved without their cooperation. We also believe that this symposium has, no doubt, motivated researchers in these areas of study which will lead to more advanced and fruitful results in the field of electromagnetomechanics.

October, 1986

Yoshiyuki YAMAMOTO
Kenzo MIYA

LIST OF PARTICIPANTS

U.S.A.

D.W. Weissenburger
F.C. Moon
L. Turner
W. Lord
H.T. Tiersten
P.J. Chen
Z.J. Cendes
K. Hara
H.T. Savage
E.S. Bobrov

U.S.S.R.

S.A. Ambartsumian

POLAND

J. Stefaniak
W. Kurnik
B. Maruszewski
J.P. Nowacki
K. Majorkowska-Knap
D. Rogula
J. Kapelewski
M. Zorawski
M. Sztyren
Cz. Rymarz

CANADA

S. Dost

CZECHOSLOVAKIA

J. Brilla

FRANCE

A. Bossavit
G.A. Maugin
B. Collet

WEST GERMANY

J. Lenz
K.P. Jüngst
U. Neumann

NETHERLANDS

A.A.F. Van De Ven
P.H. Van Lieshout
Y. Ersoy

UNITED KINGDOM

C.W. Trowbridge
C. Emson

SWEDEN

R.K.T. Hsieh
S.A. Zhou

SINGAPORE

C.-S. Lim

TURKEY

A.U. Erdem

IRELAND

F.M. McCarthy

JAPAN

M. Urata
A. Hatayama
Y. Fukai
Y. Nakasone
K. Kaiho
H. Ukikusa

K. Takeuchi
S. Imagawa
Y. Iwahashi
A. Kameari
F. Matsuoka
T. Yamanaka
Y. Shindo
S. Yamazaki
Y. Noguchi
E. Matsumoto
E. Fukushima
T. Hamajima
H. Maeda
K. Ishikawa
M. Uesaka
T. Takagi
Y. Wada
H. Mori
S. Motogi
T. Satow
K. Morimoto
Y. Nojima
M. Fujii

H. Kubo
Y. Arai
H. Hoshikawa
T. Honma
T. Shoji
H. Ohtsubo
T. Sugiura
O. Watanabe
J. Tani
K. Miya
N. Asami
T. Tone
Y. Shindo
N. Takatsu
G. Yagawa
T. Ohtani
H. Fujita
Y. Yamamoto
S. Minagawa
H. Ogiwara
H. Tohma
H. Yanagi

CONTENTS

*Invited paper.

Contents

A

ELECTROMAGNETIC BUCKLING
AND
STABILITY

Electromagnetomechanical Interactions in
Deformable Solids and Structures
Y. Yamamoto and K. Miya (Editors)
Elsevier Science Publishers B.V. (North-Holland)

MAGNETOELASTIC BUCKLING OF FERROMAGNETIC AND SUPERCONDUCTING STRUCTURES

A.A.F. van de Ven
Eindhoven University of Technology
Department of Mathematics and Computing Science
Eindhoven, The Netherlands

A general method for the determination of the buckling load for a slender body under electromagnetic influence is presented. The body may be either superconducting or soft ferromagnetic. Explicit results of this method are given for two examples, knowing the stability of a superconducting ring in its own field, and the buckling of a set of two ferromagnetic rods in an external magnetic field.

1. INTRODUCTION

One of the topics in electromagnetoelastic interaction theory is the study of the stability of a slender body loaded by forces of electromagnetic origin (see e.g. [1]). In general an electromagnetoelastic stability problem is governed by: the Maxwell equations, the equations of motion (or equilibrium equations), constitutive equations for the stresses and one or more electromagnetic field variables and, finally, the electromagnetic and mechanical boundary conditions. In these relations interaction terms occur; for instance the body force in the equation of motion is of electromagnetic origin. Moreover, and this is an even more essential effect, these relations all refer to the, a priori unknown, deformed state of the body (e.g. the boundary conditions must be applied on the deformed surface of the body). As a consequence, this set of relations is essentially non-linear.

In practice, electromagnetoelastic stability problems mainly occur in

a) superconductors,

b) ferromagnetic devices (with $\chi \approx \mu_r \gg 1$).

In the *Maxwell-Minkowski-model* (cf. [2]) which is used here for soft ferromagnetic media with large permeability (i.e. for $\mu_r^{-1} \approx 0$) the electromagnetic force consists only of a surface tension $\mathbf{T}^{(em)}$ (the body force is zero), and the same holds true for a superconductor. One has

$$a)\ \mathbf{T}^{(em)} = -\tfrac{1}{2}\mu_0(\mathbf{K},\mathbf{K})\mathbf{n} \quad , \quad b)\ \mathbf{T}^{(em)} = \tfrac{1}{2}\mu_0(\mathbf{M},\mathbf{n})^2\mathbf{n} \ . \tag{1.1}$$

($\mathbf{M}$: magnetization; $\mathbf{n}$: outward normal; $\mathbf{K}$: surface current).

Hence, from a mechanical point of view, a) and b) are identical. The differences, however, lie in the magnetic boundary conditions, which read ($^+$ outside, $^-$ inside the body)

$$\begin{array}{ll} a)\ (\mathbf{B}^- = \mathbf{0}) & b)\ (\mu_r^{-1} = 0) \\ (\mathbf{B}^+,\mathbf{n}) = 0 & (\mathbf{B}^+,\mathbf{n}) = (\mathbf{B}^-,\mathbf{n})\ , \\ (\mathbf{H}^+ \times \mathbf{n}) = -\mathbf{K} & (\mathbf{H}^+ \times \mathbf{n}) = \mathbf{0}\ . \end{array} \tag{1.2}$$

In spite of these differences, the stability analysis follows the same lines for both problems.

2. GENERAL DESCRIPTION OF THE PROCEDURE

We shall give here an outline of the main steps in the procedure leading to the determination of the buckling load for a slender body under electromagnetic influence. Our basic idea is: the final, or buckled, state is considered as a perturbation of some intermediate state, which here is approximated by the rigid-body state. The fields in the final state are decomposed into the rigid-body fields and the perturbations on this intermediate state. These perturbations are due to the deflections in buckling of the slender body and are assumed to be small. Therefore, the perturbed set may be linearized with respect to the perturbations. Thus, two sets of equations and boundary conditions are obtained

i) a set for the rigid-body fields;

ii) a linearized set for the perturbations.

Since this latter set is homogeneous, it always has the trivial- or zero-solution as a solution. It is only for special values of the electromagnetic load parameter (i.e. the basic external magnetic field or the total electric current) that this set has a non-trivial solution. The lowest of these values is called the *buckling value*.

For slender bodies (such as beams or rods, plates and rings) the displacement in buckling can be characterized by one or two global displacement parameters. For instance, the deflection of a slender *beam* is characterized by the transverse displacement of its central line, that of a thin *plate* by the normal displacement of its central plane, and that of a *ring* (in-plane) by the displacements in radial and tangential direction of its central line.

Since the perturbed fields arise from the deflections in buckling, it is logical to use for the solution of the perturbed system a separation of variables in which the perturbed fields are assumed to be proportional to the displacement parameters mentioned above. Let us explain this in somewhat more detail for the case, frequently met with, that the perturbed magnetic field quantities can be described by a magnetic potential $\phi=\phi(x_1,x_2,x_3)$, satisfying the 3-dimensional Laplace equation $\Delta\phi=0$. Denoting the displacement parameter by u, we propose the separation of variables (in a schematic notation)

$$\phi = \Phi u \quad .$$

Because this separation must be consisted with $\Delta\phi=0$, two equations for Φ and u follow.

Written out for a *beam*, the separation condition reads explicitely

$$\phi(x_1,x_2,x_3) = \Phi(x_1,x_2)u(x_3) \quad , \tag{2.1}$$

where x_3 is the coordinate along the axis of the beam. In order that this separation is consistent with $\Delta\phi=0$, there must exist a real parameter λ such that

$$\Delta\Phi(x_1,x_2) - \lambda^2\Phi(x_1,x_2) = 0 \quad , \quad u''(x_3) + \lambda^2 u(x_3) = 0 \quad . \tag{2.2}$$

The value of the parameter λ follows from the support conditions of the beam.

In a completely analogous way the plate and the ring can be treated, yielding

i) for the *plate* (x_1,x_2: coordinates in the plane of the plate)

$$\phi(x_1,x_2,x_3) = \Phi(x_3)w(x_1,x_2) \quad , \tag{2.3}$$

implies

$$\Phi''(x_3) - \lambda^2\Phi(x_3) = 0 \quad , \quad \Delta w(x_1,x_2) + \lambda^2 w(x_1,x_2) = 0 \quad ; \tag{2.4}$$

ii) for the ring (μ,η,θ are toroidal coordinates, of which θ is the pole angle; $w(\theta)$ is the radial displacement of the ring, which is taken inextensible)

$$\phi(x_1,x_2,x_3) = \Phi(\mu,\eta)w(\theta) \quad , \tag{2.5}$$

gives

$$\Delta\Phi(\mu,\eta) - m^2\Phi(\mu,\eta) = 0 \quad , \quad w''(\theta) + m^2 w(\theta) = 0 \quad , \tag{2.6}$$

with $m \in \mathbb{N}$, because w must be periodic in θ.

An equation for the displacement parameter u or w can be derived from the equilibrium equations or the equations of motion. For slender bodies these equations can be integrated to well-known one- or two-dimensional global equations, which are known as beam-, plate- or ring-equations. The load-parameters ocurring in these equations are always of electromagnetic origin (here, integrals of $\mathbf{T}^{(em)}$). Note that for an explicit determination of $\mathbf{T}^{(em)}$, and hence the load-parameter, the magnetic part of the perturbed system must be solved first.

Schematically, the whole procedure can now be recapitulated as follows: