



# Electronic devices and circuits

MILLMAN & HALKIAS

# **ELECTRONIC DEVICES AND CIRCUITS**

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ELECTRONIC DEVICES AND CIRCUITS

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# PREFACE

This book, intended as a text for a first course in electronics for electrical engineering or physics students, has two primary objectives: to present a clear, consistent picture of the internal physical behavior of many electronic devices, and to teach the reader how to analyze and design electronic circuits using these devices.

Only through a study of physical electronics, particularly solid-state science, can the usefulness of a device be appreciated and its limitations be understood. From such a physical study, it is possible to deduce the external characteristics of each device. This characterization allows us to exploit the device as a circuit element and to determine its large-signal (nonlinear) behavior. A small-signal (linear) model is also obtained for each device, and analyses of many circuits using these models are given. The approach is to consider a circuit first on a physical basis, in order to provide a clear understanding and intuitive feeling for its behavior. Only after obtaining such a qualitative insight into the circuit is mathematics (through simple differential equations) used to express quantitative relationships.

Methods of analysis and features which are common to many different devices and circuits are emphasized. For example, Kirchhoff's, Thévenin's, Norton's, and Miller's theorems are utilized throughout the text. The concepts of the load line and the bias curve are used to establish the quiescent operating conditions in many different circuits. Calculations of input and output impedances, as well as current and voltage gains, using small-signal models, are made for a wide variety of amplifiers.

A great deal of attention is paid to the effects of feedback on input and output resistance, nonlinear distortion, frequency response, and the stabilization of voltage or current gains of the various devices and circuits studied. In order that the student appreciate the different applications of these circuits, the basic building blocks (such as untuned amplifiers, power amplifiers, feedback amplifiers, oscillators, and power suppliers) are discussed in detail.

For the most part, real (commercially available) device characteristics are employed. In this way the reader may become familiar with the order of magnitude of device parameters, the variability of these parameters within a given type and with a change of temperature, the effect of the inevitable shunt capacitances in circuits, and the effect of input and output resistances and loading on circuit operation. These

considerations are of utmost importance to the student or the practicing engineer since the circuits to be designed must function properly and reliably in the physical world, rather than under hypothetical or ideal circumstances.

There are over 600 homework problems, which will test the student's grasp of the fundamental concepts enunciated in the book and will give him experience in the analysis and design of electronic circuits. In almost all numerical problems realistic parameter values and specifications have been chosen. An answer book is available for students, and a solutions manual may be obtained from the publisher by an instructor who has adopted the text.

This book was planned originally as a second edition of Millman's "Vacuum-tube and Semiconductor Electronics" (McGraw-Hill Book Company, New York, 1958). However, so much new material has been added and the revisions have been so extensive and thorough that a new title for the present text seems proper. The changes are major and have been made necessary by the rapid developments in electronics, and particularly by the continued shift in emphasis from vacuum tubes to transistors and other semiconductor devices. Less than 25 percent of the coverage relates to vacuum tubes; the remainder is on solid-state devices, particularly the bipolar transistor. In recognition of the growing importance of integrated circuits and the field-effect transistor, an entire chapter is devoted to each of these topics. But to avoid too unwieldy a book, it was decided not to consider gas tubes, silicon-controlled rectifiers, polyphase rectifiers, tuned amplifiers, modulation, or detection circuits. The companion volume to this book, Millman and Taub's "Pulse, Digital, and Switching Waveforms" (McGraw-Hill Book Company, New York, 1965), gives an extensive treatment of the generation and processing of nonsinusoidal waveforms.

Considerable thought was given to the pedagogy of presentation, to the explanation of circuit behavior, to the use of a consistent system of notation, to the care with which diagrams are drawn, and to the many illustrative examples worked out in detail in the text. It is hoped that these will facilitate the use of the book in self-study and that the practicing engineer will find the text useful in updating himself in this fast-moving field.

The authors are very grateful to P. T. Mauzey, Professor H. Taub, and N. Voulgaris, who read portions of the manuscript and offered constructive criticism. We thank Dr. Taub also because some of our material on the steady-state characteristics of semiconductor devices and on transistor amplifiers parallels that in Millman and Taub's "Pulse, Digital, and Switching Waveforms." We acknowledge with gratitude the influence of Dr. V. Johannes and of the book "Integrated Circuits" by Motorola, Inc. (McGraw-Hill Book Company, New York, 1965) in connection with Chapter 15. We express our particular appreciation to Miss S. Silverstein, administrative assistant of the Electrical Engineering Department of The City College, for her most skillful service in the preparation of the manuscript. We also thank J. T. Millman and S. Thanos for their assistance.

*Jacob Millman  
Christos C. Halkias*

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*Johnson* · Transmission Lines and Networks  
*Koenig and Blackwell* · Electromechanical System Theory  
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*LePage* · Analysis of Alternating-current Circuits  
*LePage* · Complex Variables and the Laplace Transform for Engineering  
*LePage and Seely* · General Network Analysis  
*Levi and Panzer* · Electromechanical Power Conversion  
*Ley, Lutz, and Rehberg* · Linear Circuit Analysis  
*Linvill and Gibbons* · Transistors and Active Circuits  
*Littauer* · Pulse Electronics  
*Lynch and Truxal* · Introductory System Analysis  
*Lynch and Truxal* · Principles of Electronic Instrumentation  
*Lynch and Truxal* · Signals and Systems in Electrical Engineering  
*McCluskey* · Introduction to the Theory of Switching Circuits  
*Manning* · Electrical Circuits  
*Meisel* · Principles of Electromechanical-energy Conversion  
*Millman* · Vacuum-tube and Semiconductor Electronics  
*Millman and Halkias* · Electronic Devices and Circuits  
*Millman and Seely* · Electronics  
*Millman and Taub* · Pulse and Digital Circuits  
*Millman and Taub* · Pulse, Digital, and Switching Waveforms  
*Mishkin and Braun* · Adaptive Control Systems  
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*Nanavati* · An Introduction to Semiconductor Electronics  
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*Ruston and Bordogna* · Electric Networks: Functions, Filters, Analysis  
*Ryder* · Engineering Electronics  
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*Schwarz and Friedland* · Linear Systems  
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*Seely* · Electronic Engineering  
*Seely* · Introduction to Electromagnetic Fields  
*Seely* · Radio Electronics  
*Seifert and Steeg* · Control Systems Engineering  
*Siskind* · Direct-current Machinery  
*Skilling* · Electric Transmission Lines  
*Skilling* · Transient Electric Currents  
*Spangenberg* · Fundamentals of Electron Devices  
*Spangenberg* · Vacuum Tubes  
*Stevenson* · Elements of Power System Analysis  
*Stewart* · Fundamentals of Signal Theory

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**Strauss** · Wave Generation and Shaping  
**Su** · Active Network Synthesis  
**Terman** · Electronic and Radio Engineering  
**Terman and Pettit** · Electronic Measurements  
**Thaler** · Elements of Servomechanism Theory  
**Thaler and Brown** · Analysis and Design of Feedback Control Systems  
**Thaler and Pastel** · Analysis and Design of Nonlinear Feedback Control Systems  
**Thompson** · Alternating-current and Transient Circuit Analysis  
**Tou** · Digital and Sampled-data Control Systems  
**Tou** · Modern Control Theory  
**Truxal** · Automatic Feedback Control System Synthesis  
**Tuttle** · Electric Networks: Analysis and Synthesis  
**Valdes** · The Physical Theory of Transistors  
**Van Bladel** · Electromagnetic Fields  
**Weinberg** · Network Analysis and Synthesis  
**Williams and Young** · Electrical Engineering Problems

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# 1 / ELECTRON BALLISTICS AND APPLICATIONS

In this chapter we present the fundamental physical and mathematical theory of the motion of charged particles in electric and magnetic fields of force. In addition, we discuss a number of the more important electronic devices that depend on this theory for their operation.

The motion of a charged particle in electric and magnetic fields is presented, starting with simple paths and proceeding to more complex motions. First a uniform electric field is considered, and then the analysis is given for motions in a uniform magnetic field. This discussion is followed, in turn, by the motion in parallel electric and magnetic fields and in perpendicular electric and magnetic fields.

## 1-1 CHARGED PARTICLES

The charge, or quantity, of negative electricity of the electron has been found by numerous experiments to be  $1.602 \times 10^{-19}$  C (coulomb). The values of many important physical constants are given in Appendix A. Some idea of the number of electrons per second that represents current of the usual order of magnitude is readily possible. For example, since the charge per electron is  $1.602 \times 10^{-19}$  C, the number of electrons per coulomb is the reciprocal of this number, or approximately,  $6 \times 10^{18}$ . Further, since a current of 1 A (ampere) is the flow of 1 C/sec, then a current of only 1 pA (1 picoampere, or  $10^{-12}$  A) represents the motion of approximately 6 million electrons per second. Yet a current of 1 pA is so small that considerable difficulty is experienced in attempting to measure it.

In addition to its charge, the electron possesses a definite mass. A direct measurement of the mass of an electron cannot be made, but the ratio  $e/m$  of the charge to the mass has been determined by a

number of experimenters using independent methods. The most probable value for this ratio is  $1.759 \times 10^{11}$  C/kg. From this value of  $e/m$  and the value of  $e$ , the charge on the electron, the mass of the electron is calculated to be  $9.109 \times 10^{-31}$  kg.

The charge of a positive ion is an integral multiple of the charge of the electron, although it is of opposite sign. For the case of singly ionized particles, the charge is equal to that of the electron. For the case of doubly ionized particles, the ionic charge is twice that of the electron.

The mass of an atom is expressed as a number that is based on the choice of the atomic weight of oxygen equal to 16. The mass of a hypothetical atom of atomic weight unity is, by this definition, one-sixteenth that of the mass of monatomic oxygen. This has been calculated to be  $1.660 \times 10^{-27}$  kg. Hence, in order to calculate the mass in kilograms of any atom, it is necessary only to multiply the atomic weight of the atom by  $1.660 \times 10^{-27}$  kg. A table of atomic weights is given in Appendix C.

The radius of the electron has been estimated as  $10^{-15}$  m, and that of an atom as  $10^{-10}$  m. These are so small that all charges are considered as mass points in the following sections.

**Classical and Wave-mechanical Models of the Electron** The foregoing description of the electron (or atom) as a tiny particle possessing a definite charge and mass is referred to as the *classical model*. If this particle is subjected to electric, magnetic, or gravitational fields, it experiences a force, and hence is accelerated. The trajectory can be determined precisely using Newton's laws, provided that the forces acting on the particle are known. In this chapter we make exclusive use of the classical model to study electron ballistics. The term *electron ballistics* is used because of the existing analogy between the motion of charged particles in a field of force and the motion of a falling body in the earth's gravitational field.

For large-scale phenomena, such as electronic trajectories in a vacuum tube, the classical model yields accurate results. For small-scale systems, however, such as an electron in an atom or in a crystal, the classical model treated by Newtonian mechanics gives results which do not agree with experiment. To describe such subatomic systems properly it is found necessary to attribute to the electron a wavelike property which imposes restrictions on the exactness with which the electronic motion can be predicted. This wave-mechanical model of the electron is considered in Chap. 2.

## 1-2 THE FORCE ON CHARGED PARTICLES IN AN ELECTRIC FIELD

The force on a unit positive charge at any point in an electric field is, by definition, the electric field intensity  $\mathcal{E}$  at that point. Consequently, the force on a positive charge  $q$  in an electric field of intensity  $\mathcal{E}$  is given by  $q\mathcal{E}$ , the resulting force



being in the direction of the electric field. Thus,

$$\underline{f_q = q\mathcal{E}} \quad (1-1)$$

where  $f_q$  is in newtons,  $q$  is in coulombs, and  $\mathcal{E}$  is in volts per meter. Boldface type is employed wherever vector quantities (those having both magnitude and direction) are encountered.

The mks (meter-kilogram-second) rationalized system of units is found most convenient for the subsequent studies. Therefore, unless otherwise stated, this system of units is employed.

In order to calculate the path of a charged particle in an electric field, the force, given by Eq. (1-1), must be related to the mass and the acceleration of the particle by Newton's second law of motion. Hence

$$f_q = q\mathcal{E} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt} \quad (1-2)$$

where  $m$  = mass, kg

$\mathbf{a}$  = acceleration, m/sec<sup>2</sup>

$\mathbf{v}$  = velocity, m/sec

The solution of this equation, subject to appropriate initial conditions, gives the path of the particle resulting from the action of the electric forces. If the magnitude of the charge on the electron is  $e$ , the force on an electron in the field is

$$\underline{f = -e\mathcal{E}} \quad (1-3)$$

The minus sign denotes that the force is in the direction opposite to the field.

In investigating the motion of charged particles moving in externally applied force fields of electric and magnetic origin, it is implicitly assumed that the number of particles is so small that their presence does not alter the field distribution.

### 1-3 CONSTANT ELECTRIC FIELD

Suppose that an electron is situated between the two plates of a parallel-plate capacitor which are contained in an evacuated envelope, as illustrated in Fig. 1-1. A difference of potential is applied between the two plates, the direction of the electric field in the region between the two plates being as shown. If the distance between the plates is small compared with the dimensions of the plates, the electric field may be considered to be uniform, the lines of force pointing along the negative  $X$  direction. That is, the only field that is present is  $\mathcal{E}$  along the  $-X$  axis. It is desired to investigate the characteristics of the motion, subject to the initial conditions

$$v_x = v_{0x} \quad x = x_0 \quad \text{when } t = 0 \quad (1-4)$$