Concrete Structures:
Stresses and Deformations

A. Ghali

R. Favre

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A. Ghali
Professor, The University of Calgary, Canada

R. Favre
Professor, Swiss Federal Institute of Technology, Lausanne, Switzerland

LONDON NEW YORK
Chapman and Hall

First published in 1986 by
Chapman and Hall Ltd
11 New Fetter Lane, London EC4P 4EE
Published in the USA by
Chapman and Hall
29 West 35th Street, New York NY 10001

Printed in Great Britain by
J.W. Arrowsmith Ltd., Bristol

ISBN 0 412 25620 7

British Library Cataloguing in Publication Data

Ghali, A.
Concrete structures: stresses and deformations.
1. Concrete construction
I. Title II. Favre, R.
624.1'834 TA439

ISBN 0-412-25620-7

Library of Congress Cataloging in Publication Data

Ghali, A. (Amin)
Concrete structures.

Bibliography: p.
Includes index.
1. Concrete construction. 2. Stresses and strains.
3. Concrete—Cracking. I. Favre, R. (Renaud),
1934- . II. Title.
TA681.G39 1986 624.1'834 85-15126
ISBN 0-412-25620-7

Acknowledgements

This book was produced through the collaboration of A. Ghali with R. Favre and his research group, mainly during sabbatical leaves spent at the Swiss Federal Institute of Technology, Lausanne. For completion of the work, A. Ghali was granted a Killam Resident Fellowship at The University of Calgary for which he is very grateful.

The authors would like to thank those who helped in the preparation of the book. In Lausanne, Dr M. Koprna, Research Associate, reviewed parts of the text and collaborated in writing Chapter 8 and Appendix A; Mr J. Trevino, Research Assistant, made a considerable contribution by providing solutions or checking the numerical examples and preparing the manuscript for the publisher; Mr B.-F. Gardel prepared the figures. In Calgary, Mr M. Elbadry and Mr A. Mokhtar, Graduate Students, checked parts of the text, Mr B. Unterberger prepared by computer the graphs of Appendix F; Miss C. Larkin produced an excellent typescript.

The authors deeply appreciate the work of Dr S. El-Gabalawy of the Department of English at The University of Calgary, who revised the manuscript.

Preface

In the last two or three decades, structural engineers have been concerned in design with the safety against failure, more than with the quality of structures in service conditions. It is not sufficient that a reinforced or prestressed concrete structure must have an adequate safety factor against failure, but the structure must also exhibit a satisfactory performance during its use. This can be achieved by checks on the stresses and the deformations under service conditions.

The following are qualities which are essential for a satisfactory performance:

(1) No excessive deflection should occur under the combined effect of prestressing, the self-weight of the structure and the superimposed dead load.
(2) Crack width should not be excessive under the effects of the above-mentioned loads combined with live and other transitory loads, settlement of supports and temperature variations. This makes it necessary to control steel stress, which is one of the main parameters affecting width of cracks.

Because of creep and shrinkage of concrete as well as relaxation of prestressed steel, the stress in concrete and steel in cross-section of a statically determinate structure varies with time. In addition, when the structure is statically indeterminate, the internal forces and the reactions are also time-dependent. The strains and consequently the displacements change considerably with time due to the same effects and also due to cracking when concrete stress exceeds its tensile strength. The purpose of the present text is to introduce effective methods for prediction of the true stresses and deformations during the life of the structure.

Unlike the design for safety against failure, serviceability checks should not ignore the effects of creep, shrinkage, relaxation, cracking, imposed deformations (e.g. support movements), temperature variations and changes in the statical system during construction.

Mechanical properties of concrete and steel differ from project to project and from one country to another. Specifically, the modulus of elasticity, creep and shrinkage of concrete can vary within a wide range. The methods of analysis presented in the text allow the designer to account for the effects of variance in these parameters. Appendix A, based on European and American codes, gives guidance on the choice of values of these parameters for use in design; codes of other countries may also be employed. The methods of analysis presented are mainly independent of codes.

The book is written for use by practising engineers, students and researchers. Discussions are limited to methods of analysis which are considered most suitable when a small desk calculator is employed or when a personal or large computer is used. Reference is made to available computer programs based on the methods of analysis discussed in this book. Design aids in the form of tables and graphs are presented in the text and in the appendices.

A. Ghali
R. Favre
Calgary, Canada
Lausanne, Switzerland
January, 1985

The SI system of units and British equivalents

Length	
metre (m)	1 m = 39.37 in
	1 m = 3.281 ft
Area	
square metre (m^2)	1 m^2 = 1550 in^2
	1 m^2 = 10.76 ft^2
Volume	
cubic metre (m^3)	1 m^3 = 35.32 ft^3
Moment of inertia	
metre to the power four (m^4)	1 m^4 = 2403×10^3 in^4
Force	
newton (N)	1 N = 0.2248 lb
Load intensity	
newton per metre (N/m)	1 N/m = 0.06852 lb/ft
newton per square metre (N/m^2)	1 N/m^2 = 20.88×10^{-3} lb/ft^2
Moment	
newton metre (N-m)	1 N-m = 8.851 lb-in
	1 N-m = 0.7376×10^{-3} kip-ft
	1 kN-m = 8.851 kip-in
Stress	
newton per square metre (pascal)	1 Pa = 145.0×10^{-6} lb/in^2
	1 MPa = 0.1450 ksi
Curvature	
$(metre)^{-1}$	1 m^{-1} = 0.0254 in^{-1}
Temperature change	
degree Celsius (°C)	1 °C = (5/9) °Fahrenheit

Energy and power

joule (J) = 1 N-m	1 J = 0.7376 lb-ft
watt (W) = 1 J/s	1 W = 0.7376 lb-ft/s
	1 W = 3.416 Btu/h

Nomenclature for decimal multiples in the SI System

10^9	giga (G)
10^6	mega (M)
10^3	kilo (k)
10^{-3}	milli (m)

Notation

The following is a list of symbols which are common in various chapters of the book. All symbols are defined in the text when they first appear and again when they are used in equations which are expected to be frequently applied. The sign convention adopted throughout the text is also indicated where applicable.

A	Cross-section area
$\{A\}$	Vector of actions (internal forces or reactions)
$\bar{A}$, $\bar{B}$ and $\bar{I}$	Area, first moment and moment of inertia of the age-adjusted transformed section, composed of area of concrete plus $\bar{\alpha}$ times area of reinforcement
B	First moment of area. For $\bar{B}$, see $\bar{A}$
b	Breadth of a rectangular section, or width of the flange of a T-section
c	Depth of compression zone in a fully-cracked section
D	Displacement
d	Distance between extreme compressive fibre to the bottom reinforcement layer
E	Modulus of elasticity
$\bar{E}_c$	$= E_c(t_0)/[1 + \chi\varphi(t, t_0)]$ = age-adjusted elasticity modulus of concrete
e	Eccentricity
F	Force
f	Stress related to strength of concrete or steel
$[f]$	Flexibility matrix
f_{ct}	Tensile strength of concrete
h	Height of a cross section
I	Moment of inertia. For $\bar{I}$, see $\bar{A}$
i, j, m, n	Integers
l	Length of a member
M	Bending moment. In a horizontal beam, a positive moment produces tension at the bottom fibre
M_r and/or N_r	Values of the bending moment and/or the axial force which are just sufficient to produce cracking

N	Normal force, positive when tensile
P	Force
r	Radius of gyration
$r(t, t_0)$	Relaxation function = concrete stress at time t due to a unit strain imposed at time t_0 and sustained to time t
$[S]$	Stiffness matrix
s_r	Spacing between cracks
T	Temperature
t	Time or age (generally in days)
W	Section modulus (length^3)
y	Coordinate defining location of a fibre or a reinforcement layer; y is measured in the downward direction from a specified reference point
α	$= E_s/E_c(t_0)$ = ratio of elasticity modulus of steel to elasticity modulus of concrete at age t_0
$\bar{\alpha}$	$= \alpha[1 + \chi\varphi(t, t_0)] = E_s/\bar{E}_c$ = ratio of elasticity modulus of steel to the age-adjusted elasticity modulus of concrete
α_t	Coefficient of thermal expansion (degree^{-1})
ε	Normal strain, positive for elongation
ζ	Coefficient of interpolation between strain, curvature and deflection values for cracked and fully-cracked conditions (states 1 and 2, respectively)
η	Dimensionless multiplier for calculation of time-dependent change in axial strain
κ	Dimensionless multiplier for calculation of curvature
ν	Poisson's ratio
ξ	Dimensionless shape function
ρ, ρ'	Ratio of tension and of compression reinforcement to the area (bd); $\rho = A_s/bd$; $\rho' = A_s'/bd$
σ	Normal stress, positive when tensile
τ	Instant of time
$\varphi(t, t_0)$	Creep coefficient of concrete = ratio of creep to the instantaneous strain due to a stress applied at time t_0 and sustained to time t
$\chi(t, t_0)$	Aging coefficient of concrete (generally between 0.6 and 0.9; see Section 1.7 and Figs A.1–12)
$\chi\varphi(t, t_0)$	$= \chi(t, t_0)\ \varphi(t, t_0)$ = aging coefficient × creep coefficient
χ_r	Relaxation reduction coefficient for prestressed steel
ψ	Curvature (length^{-1}). Positive curvature corresponds to positive bending moment
{ }	Braces indicate a vector; i.e. a matrix of one column
[ ]	A rectangular or a square matrix

→, ↷	Single-headed arrows indicate a displacement (translation or rotation) or a force (a concentrated load or a couple)
→→	Double-headed arrow indicates a couple or a rotation; its direction is that of the rotation of a right-hand screw progressing in the direction of the arrow

Subscripts

c	Concrete
cs	Shrinkage
m	Mean
ns	Non-prestressed steel
O	Reference point
0	Initial or instantaneous
pr	Relaxation in prestressed steel
ps	Prestressed steel
s	Steel
st	Total steel, prestressed and non-prestressed
u	Unit force effect, unit displacement effect
φ	Creep effect
1,2	Cracked or uncracked state

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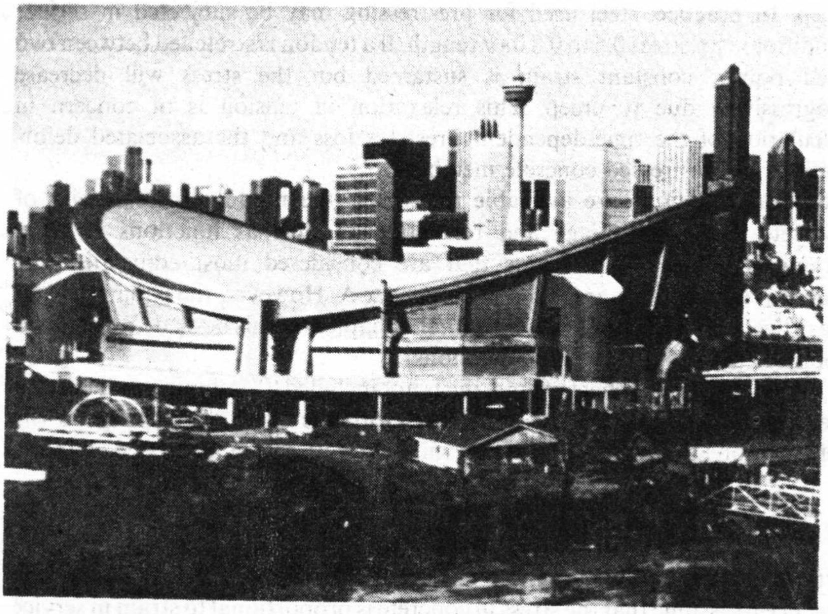
CHAPTER ONE

Creep and shrinkage of concrete and relaxation of steel



The 'Saddledome', Olympic Ice Stadium, Calgary, Canada. (Courtesy Genestar Structures Ltd. and J. Bobrowski and Partners Ltd.)

1.1 INTRODUCTION

The stress and strain in a reinforced or prestressed concrete structure are subject to change for a long period of time, during which creep and shrinkage of concrete and relaxation of the steel used for prestressing develop gradually. For analysis of the time-dependent stresses and deformations, it is necessary to employ time functions for strain or stress in the materials involved. In this chapter the basic equations necessary for the analysis are presented. The important parameters that affect the stresses or the strains are included in the equations, but it is beyond the scope of this book to examine how these parameters vary with the variations of the material properties.