The Structure of Matter:

A Survey of Modern Physics

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Preface

The Structure of Matter grew from lecture notes prepared for a one-year course on modern physics, taken by university juniors majoring in one of the science or engineering fields. My aim in designing the course was to achieve the following goals:

a) To expose the students to the new ideas ("new" to the students with their one- to two-year background in classical physics) of relativity, statistical physics, and quantum physics.

b) To keep the mathematical level of the technical treatment as low as possible so as to avoid putting a double burden on the student, who has to assimilate a large number of ideas in the course.

c) To stress the importance of quantitative aspects of physics, and to teach the students the importance of, and the possibility of, getting a roughly correct answer to simple questions without the full use of the machinery of quantum mechanics.

d) To discuss a large number of phenomena whose explanation lies in quantum physics, but whose manifestation looms very large in present research areas.

Let me elaborate: the students for whom this book is written are expected to have, as their physics background, a one-year course on calculus-based general physics, and, preferably, a course on wave phenomena. Their mathematical background should be a year of calculus, some acquaintance with simple differential equations, and a quarter course on algebra, that is, vectors, linear transformations, and matrices. In general, such students will not have had any exposure to the theory of relativity, and the first eight chapters deal with that subject.

The mathematics of the theory of relativity is very simple, and I have kept the development at a simple level. Much attention is paid to the basic ideas of relativity, and the main phenomena are discussed on three levels: first, using clocks and rods, second, using the formulas of the Lorentz transformation, and third, using Minkowski diagrams. Relativity finds a very important application in the kinematics of elementary particle physics scattering and production, and a chapter is devoted to that subject. The equivalence principle can also be discussed at the mathematical level reached by the students, and Chapter 8 deals with some aspects of Einstein's theory of gravitation. The chapter on Relativity and Electromagnetism was sometimes omitted from the course, the material being left to the student for reading on the "cultural" rather than the "essential" level. This separation is not in any way stressed in the text: only the *Notes and Comments* at the end of each chapter are distinctly cultural—they are a substitute for conversations with the interested student.

Statistical physics is included in the book because it is important to teach students something about probability theory and about statistical reasoning. The main accomplishment, using the ideas developed in Chapter 9, is the derivation of the ubiquitous Boltzmann factor $e^{-E/kT}$ whose importance in physics, chemistry, and biology cannot be overestimated. A brief discussion of kinetic theory is essential to an understanding of the structure of matter, and a large number of applications, ranging from the barometer formula to Brownian motion and equipartition, are discussed in the last chapter of Part II. There is usually no time to discuss all of the applications, and the division into essential and supplemental is left up to the individual instructor.

Quantum mechanics is the method that must be used for the discussion of the structure of matter. The theory grew out of radiation theory and out of classical electron theory. These roots of quantum physics are discussed in Part III on the Old Quantum Theory. A special feature of this section is a more detailed than usual discussion of the Lorentz electron theory, which still provides an excellent qualitative understanding of many optical phenomena.

Our introduction to quantum mechanics deals primarily with one-dimensional problems, so that the mathematical complications are not allowed to obscure the important new physical ideas. A great deal of attention is paid to the role of the uncertainty relations in the interpretation of quantum mechanical results. New phenomena, such as barrier penetration, are discussed in detail, with attention paid to physical applications and to numerical estimates. The harmonic-oscillator problem is solved in detail, providing instruction in the handling of ordinary differential equations. The discussion of the two-particle Schrödinger equation brings with it a totally new quantum phenomenon, the Pauli Exclusion Principle, which is discussed in great detail and illustrated by a discussion of the Fermi sea and neutron stars.

With the introduction of the quantum mechanical angular momentum, the mathematical level goes somewhat above what has been required heretofore: in my experience, angular momentum and spin are perhaps the hardest subjects to develop a feeling for at this level. Given the facts about angular momentum, the discussion of the hydrogen atom is relatively straightforward, in that it follows the steps that appeared in the solution of the harmonic oscillator. The section on the more advanced topics in quantum mechanics ends with a series of topics in which the manifestations of spin in a variety of physical systems is discussed. This material is treated more qualitatively, and falls into the cultural category, although some of the material appears later in the discussion of atoms and molecules.

In Part VI of the book a variety of physical systems is discussed semiquantitatively. Here I have tried to build on the exact results obtained for simple one-dimensional systems and to get a feeling for the magnitudes involved. The structure of atoms can be discussed with the help of what is known about the hydrogen atom and the exclusion principle, and that approach easily extends to a discussion of simple molecules. The new aspect of rotational and vibrational motion, and associated spectra, is discussed, as is the Raman effect. In Chapter 33 the mathematical treatment becomes a little more intense again, with a brief discussion of time-dependent perturbation theory. Much of this material can be omitted, but the notion of matrix element that appears here is important, since that is used to study selection rules. Radioactivity and applications are also discussed in this chapter. The development naturally leads to a discussion of stimulated emission, and a more thorough than usual discussion of lasers, and their uses.

The remaining chapters of the book deal with (i) solid state physics, (ii) nuclear physics, and (iii) elementary partical physics. It is obviously impossible to cover more than the basic phenomena in a survey course, and even then, the material is more than can be covered in a reasonable allotment of time. My own interests have dictated a light coverage of the first two topics, and a detailed discussion of the third, but other instructors may make different choices and assign the additional material as cultural reading. The solid state section deals with crystal lattices, specific heats of solids (the Einstein and Debye theories), and the Mössbauer effect; with metals and the Fermi distribution function, electronic specific heat; and with semiconductors and superconductors. The material discussed in the nuclear physics section includes the semi-empirical mass formula, the liquid drop model, and the shell model, with a brief discussion of collective effects. The section on elementary particle physics is, because of my own interests, somewhat more extensive. It covers antiparticles, neutrinos, and beta decay, the strong interactions and their symmetries, and the quark model, including the notions of "flavor" and "color."

The transition from lecture notes to book manuscript was much aided by the useful comments of Professors A. Goldman and R. N. Dexter; by some general suggestions of Professor Paul Stoler, and above all, by the very detailed criticism and the countless suggestions for improvements that were provided by Professor Herbert Kabat. I am very grateful to all of them, and to the individuals and institutions who have kindly consented to the reproduction of figures. My special thanks go to the students of the modern physics course that I taught for four years: their excitement about the new ideas of relativity and quantum physics, and their healthy skepticism strongly motivated me to write and rewrite the notes on which this book is based.

S.G.

Minneapolis, Minnesota
February 1979

Contents

PART I
Relativity

The bulk of this book deals with the structure of matter and the understanding that quantum mechanics provides of it. It is nevertheless impossible, even though we live in a world in which most atomic constituents move with velocities very much smaller than that of light, to ignore the theory of relativity, as the view of space-time ushered in by its discovery has become an integral part of every physicist's thinking about the world of matter. In particular, relativity plays a crucial part in our thinking about the fundamental particles. I have therefore chosen to begin this book with a section on relativity. In keeping with the assumed level of mathematical training of the potential reader, I have kept the mathematics as simple as I could. No tensor notation is used, and the four-dimensional treatment of electromagnetism is only briefly discussed. I have not discussed the energy-momentum tensor, and the Lorentz transformation laws for quantities more complicated than four-vectors. The same mathematical inhibitions, together with a lack of space (and time in lecturing) have kept me from a more thorough discussion of the elements of the Einstein theory of gravitation. Nevertheless, I feel that the topics covered do provide an adequate undergraduate education in this fascinating subject. The bibliography at the end of the book attempts to point the student and the instructor in directions of deeper coverage.

1
The Relativity Postulates

Galilean relativity Observers find it convenient to use reference frames to describe physical phenomena. An important question is how different observers describe the same phenomena, and the theory of relativity is concerned with this question. An *event*, such as a collision between two infinitesimally small bodies, the absorption of a flash of light by a small body, an explosion, and so on, can be described by its location, and the time at which it is observed to occur. The location may be described by the coordinates (x, y, z) relative to a set of coordinate axes, called the reference frame. Newton's first law of motion singles out a set of frames. In these frames, called *inertial frames*, the first law, according to which an absence of forces implies uniform motion, is true. There are frames in which the first law does not hold: a body at rest relative to a rotating frame, for example on a turntable, will experience an acceleration. It is conventional, to preserve the equation

$$\text{Force} = \text{mass} \times \text{acceleration}$$

to introduce fictitious centrifugal forces, but these forces are not due to any external agency; they reflect noninertial effects.

The laws of mechanics are described by an equation of the type

$$M_i \frac{d^2\mathbf{r}_i}{dt^2} = \mathbf{F}_i \tag{1.1}$$

with the force, if it is due to another particle, for example, of the form (Fig. 1.1)

$$\mathbf{F}_i = \mathbf{F}_i(\mathbf{r}_i - \mathbf{r}_j). \tag{1.2}$$

It should be stressed that all known forces, gravitational, electromagnetic, nuclear, and weak have the characteristic that they only depend on the *relative* location of the source of the force and the body acted on, and not on the location of the body relative to some "center of the universe."

An observer in a frame moving at a uniform velocity relative to the original frame in which the coordinates of the two particles in interaction are $\mathbf{r}_1$ and $\mathbf{r}_2$, say, will assign coordinates (Fig. 1.2)

$$\begin{aligned} \mathbf{r}_1' &= \mathbf{r}_1 - \mathbf{u}t \\ \mathbf{r}_2' &= \mathbf{r}_2 - \mathbf{u}t \end{aligned} \tag{1.3}$$

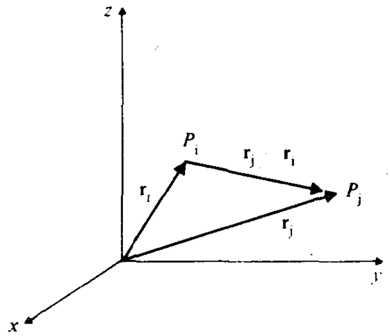

Figure 1.1. Vectors describing the positions and the separation between two interacting particles.

to the particles, provided the two frames coincided at $t = 0$ and the observer is moving with velocity **u** relative to the initial frame. The new observer will see different velocities for the particles

$$\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{u} \tag{1.4}$$

but the accelerations are the same for constant **u**, since

$$\frac{d^2\mathbf{r}'}{dt^2} = \frac{d^2\mathbf{r}}{dt^2}. \tag{1.5}$$

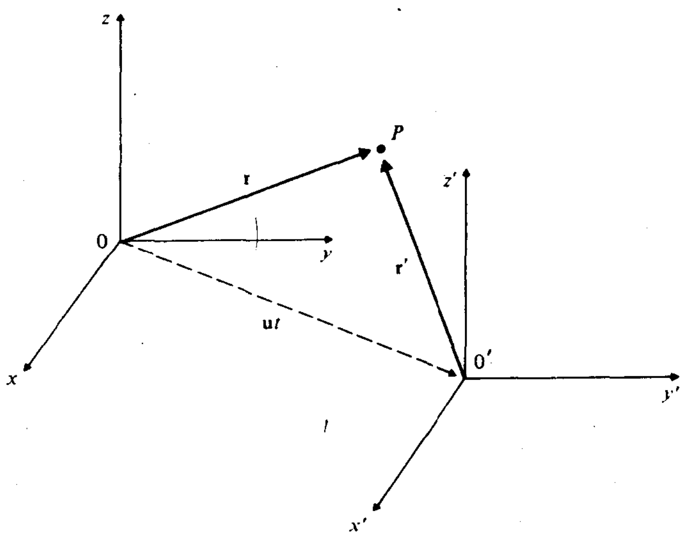

Figure 1.2. The primed frame moves with velocity **u** relative to the unprimed frame, and they coincide at $t = 0$. The point P is described differently relative to the two coordinate frames. The figure shows how $\mathbf{r}' = \mathbf{r} - \mathbf{u}t$.

Also, since

$$\mathbf{r}_1' - \mathbf{r}_2' = \mathbf{r}_1 - \mathbf{r}_2, \tag{1.6}$$

the force, which only depends on the difference, is unchanged under the transformation. Thus the observer would write down exactly the same equation of motion. What we have shown is that

the laws of mechanics are form-invariant
under the Galilean transformation

$$\mathbf{r} \rightarrow \mathbf{r}' = \mathbf{r} - \mathbf{u}t$$

or, equivalently,

the laws of mechanics are the same in
all inertial frames.

We do not assert that the solutions of the equations of motion are identical. To determine the motion we need initial conditions, and Eq.(1.4) shows that a given initial velocity in the unprimed frame differs from the initial velocity in the primed frame. What is important is that upon examination of the trajectories of the particles in interaction, an observer should not be able to distinguish which inertial frame is singled out. There is no singling out if the differences in trajectories can be ascribed to different initial conditions.

We do not know at this time what determines an inertial frame. Newton worried about this problem and satisfied himself with the assertion that there must exist an "absolute space," and inertial frames are those that are at rest or in a state of uniform motion relative to it. The Austrian physicist and philosopher Ernest Mach had another point of view, namely that inertial frames are somehow determined by the distribution of matter in the universe. There is a simple experiment that brings out this point: the surface of water in a bucket is flat when the bucket is at rest relative to the stars in the sky, and it is curved when the stars are rotating relative to the bucket. Is the acceleration of the distant stars the source of the noninertial forces? Mach believed that it is, and his point of view is one that is very attractive to many cosmologists. It is, however, difficult to incorporate Mach's Principle into a model of the universe.

The aether With the development of the laws of electrodynamics by James Clerk Maxwell, a crisis of sorts arose. The laws of electrodynamics were not invariant under the Galilean transformation

$$\mathbf{r} \rightarrow \mathbf{r}' = \mathbf{r} - \mathbf{u}t \tag{1.7}$$

according to which the velocity undergoes the transformation

$$\mathbf{v} \rightarrow \mathbf{v}' = \mathbf{v} - \mathbf{u}. \tag{1.8}$$

The most direct way to see this without actually examining Maxwell's equations is to recall that these equations predict that electromagnetic radiation propagates with the velocity $c = 3 \times 10^{10}$ cm/sec. We may ask: velocity relative to what? It is not correct to say that this is the velocity relative to the source of the radiation; that, at least, is *not* what Maxwell's equations say. An answer that had won a certain amount of acceptance in the latter part of the nineteenth century was that c was the velocity relative to the *aether*, a substance that filled the universe and made wave propagation possible. The aether was favored by old models of wave motion, according to which waves could only propagate in a medium, in which there was something that could be induced to oscillate by the action of the source. The medium was recognized to be very peculiar, since it only sustained transverse oscillations, but the assumption of its existence removed the possibility of invariance of the laws of electromagnetism under the galilean transformations. Since the aether provided a preferred frame, it removed the problem.

It seemed reasonable to assume that the aether was not tied in its motion to the earth, and it then became an interesting question of what the velocity of the earth relative to the aether was. The velocity of the earth relative to the sun is approximately 30 km/sec (the distance to the sun is approximately 1.5×10^8 km and it takes a year to get around a nearly circular orbit), and one might expect that an "aether wind" of such speed might be detectable. In 1887 A.A. Michelson and E.W. Morley carried out the first of a series of brilliant experiments designed to measure the speed of this aether wind, and they found the remarkable result that the velocity of light is the same for light traveling along the earth's orbital motion as for light moving transversely to that direction. This equality has now been established to within 1 km/sec.

Michelson-Morley experiment The idea of the experiment designed by Michelson and Morley is the following: consider light moving from a point P to a mirror M_1 (Fig. 1.3) along the east-west axis, and assume the aether wind is blowing from the east with speed v. The propagation of light against the wind proceeds with speed $c - v$; the speed is $c + v$ when the light goes with the wind. Thus the time of flight from P to M_1 and back is

$$t_1 = \frac{l_1}{c+v} + \frac{l_1}{c-v} = \frac{2l_1 c}{c^2 - v^2} \approx \frac{2l_1}{c}\left(1 + \frac{v^2}{c^2}\right) \tag{1.9}$$

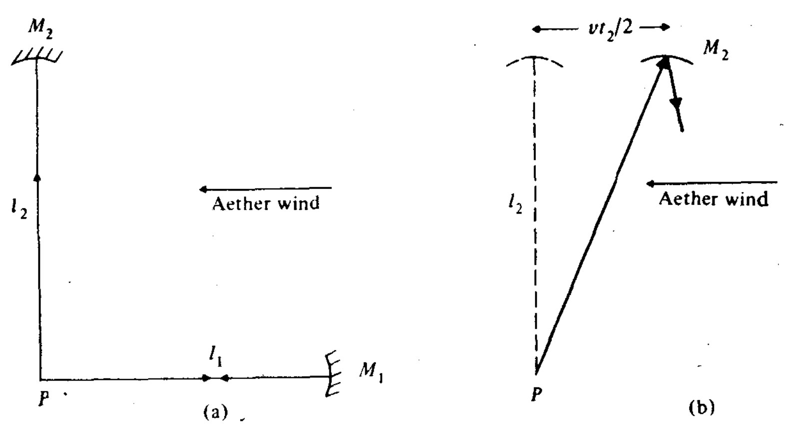

Figure 1.3. The optical paths in the Michelson Morley interferometer. (a) The location of the mirrors and the effective source P relative to the direction of the aether wind. (b) The actual direction that the light ray must travel in the aether wind to travel from P to M_2 and back.

where l_1 is the distance from P to M_1. When the light is moving transversely to the wind, a calculation of the time of flight from the point P to the mirror M_2 and back to P must take into account the delay due to the fact that the light is effectively being "blown off course." If the time of flight is t_2, that is, $t_2/2$ for the one-way trip, the distance covered is really the hypotenuse of a triangle whose sides are l_2 and $vt_2/2$, respectively. Thus

$$ct_2/2 = [l_2^2 + (vt_2/2)^2]^{1/2}. \tag{1.10}$$

Hence

$$c^2 t_2^2 = 4l_2^2 + v^2 t_2^2,$$

and therefore the time t_2 is given by the formula

$$t_2 = \frac{2l_2}{c} \frac{1}{(1 - v^2/c^2)^{1/2}} \approx \frac{2l_2}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right). \tag{1.11}$$

The time difference is given by

$$\Delta_{12} = t_1 - t_2 = \frac{2}{c}(l_1 - l_2) + \frac{v^2}{c^2}\left(\frac{2l_1}{c} - \frac{l_2}{c}\right). \tag{1.12}$$

Suppose we now rotate the apparatus through 90° so that PM_2 now points against the aether wind, and PM_1 points southward. We are now effectively interchanging l_1 and l_2, and, taking into account that we always want the time difference (parallel to wind) − (transverse to wind), we get

$$\Delta_{12}^{\text{rot}} = \frac{2}{c}(l_1 - l_2) + \frac{v^2}{c^2}\left(\frac{l_1}{c} - \frac{2l_2}{c}\right). \tag{1.13}$$

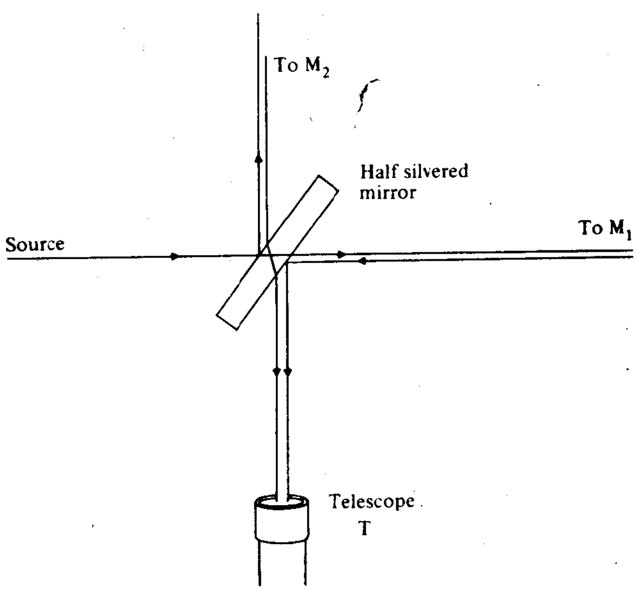

Figure 1.4. The schematics of the Michelson-Morley experiment.

The change in the time difference as a result of the rotation is

$$\Delta_{12} - \Delta_{12}^{\text{rot}} = \frac{v^2}{c^2}\frac{l_1 + l_2}{c} \equiv \Delta t \tag{1.14}$$

which is very tiny indeed, being proportional to $(v/c)^2$. If we make a reasonable guess that $v \approx 3 \times 10^6$ cm/sec, then $(v/c)^2 \approx 10^{-8}$. Such an accurate measurement became possible through the use of an optical device, the interferometer, designed by Michelson. A beam of light is split in two by a half-silvered mirror (Fig. 1.4); part of the beam is reflected by M_2 and part is reflected by M_1. The two parts of the beam recombine and are observed with the telescope T. Because the wavefronts are not exactly parallel (it is impossible to make the mirrors exactly perpendicular to each other, and so on) interference fringes will appear in the field of vision of the telescope. If care is taken that the mirrors and the source are well stabilized, the interference fringes will not change. Upon rotation of the apparatus, a change in the difference of the two path lengths of the beams will be introduced. That change is $c\Delta t$, and if the light has wavelength λ, there will be a shift in the pattern of interference fringes (this is equivalent to moving one mirror a little). The number of fringes shifted is

$$\Delta n = \frac{c\Delta t}{\lambda} = \frac{l_1 + l_2}{\lambda}\left(\frac{v}{c}\right)^2. \tag{1.15}$$

Michelson and Morley used light with wavelength 5.9×10^{-5} cm (= 5900 Å) and their apparatus was such that $l_1 \approx l_2 \approx 11$ meters. Thus the expected shift was

$$\Delta n \simeq \frac{2.2 \times 10^3}{5.9 \times 10^{-5}}(10^{-4})^2 \simeq 0.37.$$

A shift of as few as 0.04 fringes could have been detected, and *the absence of any observable shift* was a remarkable result.

Stellar aberration A possible way to evade the conclusion that there was no aether could be the assertion that perhaps the aether is locally "dragged" by the earth and thus more or less at rest relative to the terrestrial laboratory. This explanation is in conflict with the observation of *stellar aberration*. Consider the observation of a distant star with a telescope (Fig. 1.5). If the earth were at rest relative to the distant star, and the star were directly overhead, the telescope would have to be pointed straight up. With the earth moving with speed v, it is evident that the telescope must be tilted at a slight angle relative to the original vertical direction to allow the light from the star to pass through the telescope without hitting the sides. (An everyday analogy is the tilt of an umbrella carried by a running pedestrian in the rain on a windless day). The light, as seen by the observer, is emitted by a source moving with horizontal velocity v, so that the components of the light velocity are $c \sin\theta = v$ and $c\cos\theta$. For $v/c \ll 1$ the tilt angle is $\theta \simeq v/c$. One cannot, of course, be sure that a given star is exactly overhead, but, whatever its azimuth, the earth will be moving toward it at some time of the year, and away from it six months later. Thus a maximum change in the angle, given by

$$\Delta\theta \simeq 2\frac{v}{c} \simeq 2 \times 10^{-4} \text{ radians} = 44''$$

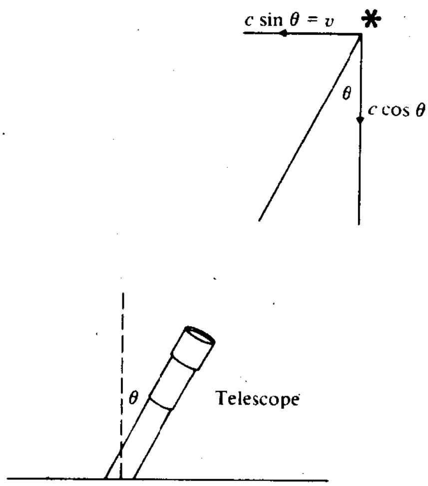

Figure 1.5. Aberration of light. The angle θ measures the tilt of the telescope necessary to avoid having the light strike the walls of the telescope.

is expected, and is in fact observed. These observations were first made by James Bradley in 1728, and provided clear evidence for the motion of the earth around the sun. Were the aether to be dragged by the earth out to some distance, the maximum angle would be different in some model-dependent way (in terms of our analogy, the angle of the umbrella would change if the running pedestrian were accompanied by a wind in his or her vicinity).

Lorentz-Fitzgerald contraction Faced with the need to find another explanation for the results of the Michelson-Morley experiment, G.F. Fitzgerald, and independently, H.A. Lorentz (perhaps the leading theoretical physicist of the latter part of the nineteenth century) proposed an ingenious, though *ad hoc* solution. This was, that as a result of some mechanism intrinsic to the molecular forces that bind the atomic constituents of matter, materials in motion with velocity v through the aether should be contracted in length in the direction of motion by a factor of $(1 - v^2/c^2)^{1/2}$. It is clear from our discussion of the experiment of Michelson and Morley that a replacement of l_1 by $l_1(1 - v^2/c^2)^{1/2}$ and a replacement of l_2 by $l_2(1 - v^2/c^2)^{1/2}$ in the two orientations, respectively, will lead to the observed null result. This explanation was not satisfying in that, without an explanation of the contraction of moving bodies, it merely replaced one mystery by another. We shall soon see that in a sense it was technically the correct explanation, but it did leave the aether untouched, though unobservable.

The Einstein postulates Apparently uninfluenced by the surprising outcome of the Michelson-Morley experiment, Albert Einstein in 1905 culminated his researches into the nature of time and motion by a most remarkable paper, in which he postulated that

I. *All laws of physics are the same in all inertial frames,* and

II. *The speed of light, taken to be the maximum speed of propagation of a signal, is the same in all inertial frames.*

The first postulate maintains the integrity of the belief in the relativity of motion; the second postulate, in effect, restates in generalized form the results of Michelson and Morley. This postulate, by contradicting the general validity of invariance under Galilean transformations, that is, invariance under

$$\mathbf{v} \to \mathbf{v}' = \mathbf{v} - \mathbf{u},$$

forces us to modify the laws of mechanics. The modification must, of course, be consistent with Newtonian mechanics in the domain where $(v/c)^2$ effects can be neglected. We shall soon see that this modification can indeed be carried out.

NOTES AND COMMENTS

1. In the interest of brevity only the Michelson-Morley experiment was described in this chapter. There were earlier attempts to detect motion through the aether, and these are discussed in most textbooks on Special Relativity. I found the discussion in A.P. French, *Special Relativity*, W.W. Norton, New York, 1968, very clear.
2. Einstein's paper which appeared in *Annalen der Physik*, *17*, p. 891 (1905) appears in translation in *The Principles of Relativity*, Dover Publications, 1951, together with some of the other classic papers on special as well as general relativity. It is a remarkably readable paper.
3. Some historical background on the theory of relativity, and in particular on the question of the significance of the Michelson-Morley experiment for the actual development of the theory may be found in an extremely stimulating series of essays in *Thematic Origins of Scientific Thought, Kepler to Einstein*, by Gerald Holton, Harvard University Press, Cambridge, Massachusetts, 1973. Biographical material may be found in many books. I particularly enjoyed *Albert Einstein, Creator and Rebel*, by Banesh Hoffmann, New American Library, New York, 1972 and *Einstein* by Jeremy Bernstein, Viking Press, New York, 1973.
4. One might be tempted to ask the question: "Why is the maximum speed of propagation also the speed of light?" I do not know whether I have a really good answer to that. When the notion of rest mass is discussed in Chapter 5 we shall see that it is objects with zero rest mass that move with the maximum speed. It is generally believed that both electromagnetic radiation and neutrinos move in this way, and there are some deep reasons for believing that this *must* be so for radiation. The question "why is the speed of light 3×10^{10} cm/sec?" has no deep meaning. The particular value happens to be a consequence of our choice of unit of length in terms of the circumference of the earth, and the unit of time in terms of how long it takes the earth to go around the sun. A sensible set of units would be one in which $c = 1$. Thus if we insist on the meter as the unit of length, a

sensible set of relativistic units would have the time unit, the "tick," say, be approximately 3.3×10^{-9} sec. Astronomers keep the year as a unit of time for certain matters, and define the unit of distance as the light-year, which is roughly 9.5×10^{15} meters.

5. A useful mnemonic is

$$1 \text{ year} \approx \pi \times 10^7 \text{ sec.}$$

6. A modern version of the Michelson-Morley experiment was performed with lasers. In this experiment one actually carried out a measurement of a "classical" Doppler shift (see Chapter 2) of light moving with the aether wind relative to light moving across the aether wind. The absence of a shift yields the same conclusions as the Michelson-Morley experiment, but the accuracy was such that the effect was less than 10^{-3} of the classically expected effect, compared with the less than 10% effect in the original experiment. The laser experiment was performed by T.S. Jaseja, A. Javan, J. Murray, and C.H. Townes, *Physical Review*, 133 A, 1221 (1964).
7. It is important to remember that it is only the speed of light in a vacuum that is to be identified with the maximum speed. Light in a medium travels with speed c/n, where n is the refractive index, and in such a medium an electron, for example, can travel faster than that. When its speed exceeds c/n, it emits radiation, called *Čerenkov* radiation after its discoverer. The radiation is emitted in a forward cone of half angle given by $\cos\theta = c/nv$, where v is the electron speed.
8. It is very difficult to test Mach's Principle. A possible consequence might be that different cosmological conditions aeons ago implied different values for physical constants such as the electric charge and the gravitational constant, G. Such differences would have observable effects on planetary history. The case for such changes in G is made in F. Hoyle *From Stonehenge to Modern Cosmology*, W.H. Freeman, San Francisco, 1969, but I believe that there is fairly universal scepticism about the conclusions among astronomers. A recent experiment on the measurement of two spectral lines from a quasar with a red shift of the order of 0.524 gives us a measure of how the two spectral lines were related thousands of millions of years ago. The upper limit on the change in the electric charge is of the order of 10^{-12} per year.
9. Recently observed pulses from neutron stars (pulsars) in double star formations emitting X-rays (binary x-ray sources) have been used to show that the speed of light is independent of the velocity of the source to an accuracy of better than one part in 10^9. The most recent summary of tests of Special Relativity may be found in D. Newman, G.W. Ford, A. Rich, and E. Sweetman, *Physical Review Letters*, **40**, 1355 (1978). This reference is probably more useful for instructors than for students.

PROBLEMS

1.1 Consider a collision between two objects, in which both energy and momentum are conserved. Show that an observer in a frame moving with velocity v relative to the original frame also sees energy and momentum conserved, provided there is conservation of mass in the process.