Proceedings

Institute of Acoustics

ACOUSTICS '89

1989 Spring Conference

Outdoor Sound Propagation
Aircraft Noise
Environmental Noise

Vol. 11, Part 5 pp189 - 436 (1989)

Proceedings

Institute of Acoustics

ACOUSTICS '89

~~1989 S~~pring Conference

Outdoor Sound Propagation
Aircraft Noise
Environmental Noise

Vol. 11, Part 5 pp189 - 436 (1989)

Published by

The Institute of Acoustics
25 Chambers Street
Edinburgh

Proceedings Editor

R. Lawrence

Printed by

Rayross Printers

CONTENTS

1989 R W B STEPHENS LECTURE

Review of Outdoor Sound Propagation - the Sound Field, Micrometeorology and Topography — 1
T F W Embleton
National Research Council, Canada

Warning Signals and Behavioural Response

Guidelines for the design of auditory warning sounds — 17
R D Patterson
University of Cambridge

The effects of warning format and keyboard layout on reaction times to auditory warnings — 25
S H James & M R James
Royal Aerospace Establishment

Sound levels for the British Rail inductive loop warning system — 43
M C Lower (1), R D Patterson (2), P Cosgrove (2) & R Milroy (2)
(1) University of Southampton
(2) University of Cambridge

Auditory warnings for the British Rail inductive loop warning system — 51
R D Paterson (1), P Cosgrove (1), R Milroy (1) & M C Lower (2)
(1) University of Cambridge
(2) University of Southampton

Auditory warnings for fixed and rotary wing aircraft — 59
G M Rood
Royal Aerospace Establishment

The perceived urgency of auditory warnings — 73
J Edworthy, S Loxley, E Geelhoed & I Dennis
Polytechnic South West

The specification of a toxic alarm system for a large chemical plant complex — 81
G Taylor, A Giles & B D Claybrook
ICI plc

Auditory icons — 91
W W Gaver
Rank Xerox Ltd

Vibration and Human Response

Groundborne vibration generated by HGVs - effects of vehicle, road and ground parameters 99
G R Watts
Transport and Road Research Laboratory

The wayside vibration from trains 111
T M Dawn
British Railways Board

Ground vibration and its effects on building habitability 121
R Randell
University of Southampton

Comparative annoyance from railway noise and building vibration 129
H J Woodroof
University of Southampton

Improved operator performance from reduced vibration 135
J A Lines & R M Stayner
A F R C Engineering

Active engine mounts for improving vehicle comfort 143
C F Ross, G P Eatwell, A J Langley & C M Dorling
Topexpress Ltd

Effects of low frequency whole-body sinusoidal vibration on speech 151
M R Taylor
Smiths Industries

Vibration dose values - notes for consultants 159
A W James
Sound Research Laboratories Ltd

Measurement and analysis techniques for the evaluation of human exposure to vibration 167
C M Nelson
ISVR, University of Southampton

Survey of exposure to hand-arm vibration in Great Britain 175
K Kyriakides
Health and Safety Executive

Outdoor Sound Propagation

Acoustic scattering by sub-surface inhomogeneities 189
D L Berry (1), S N Chandler-Wilde (2) &
K Attenborough (1)
(1) The Open University
(2) Coventry Polytechnic

Long distance sound propagation over grounds 197
J N B Harriott (1), S N Chandler-Wilde (2) &
D C Hothersall (1)
(1) Bradford University
(2) Coventry Polytechnic

Numerical modelling of noise propagation over barriers 207
D C Hothersall (1), S N Chandler-Wilde (2) &
N M Hajmirzae (1)
(1) Bradford University
(2) Coventry Polytechnic

Interface waves at air/air-filled poroelastic media boundaries 215
Y Chen & K Attenborough
The Open University

An acoustic propagation model allowing a multiply layered poro-elastic ground surface 223
S Tooms & K Attenborough
The Open University

Analysis of spherical-wave propagation over absorbing ground 231
F P Mechel
Fraunhofer Institut für Bauphysik

Calculation of sound propagation from a point source over an impedance boundary 257
C Howorth, K Attenborough & N W Heap
The Open University

The ground wave in outdoor sound propagation 265
J Wempen
University of Oldenburg

Ground characterisation from short range propagation measurements 273
H M Hess, K Attenborough & N W Heap
The Open University

The prediction of sound due to a panel radiating over an impedance plane 281
K M Li, K Attenborough & N W Heap
The Open University

Reverberation and attenuation by trees: measured and modelled 289
W H T Huisman
University of Nijmegen

Creeping wave theory applied to impulse propagation in the atmosphere 297
C G Don & A J Cramond
Chisholm Institute of Technology

Propagation of sound from a point source into the acoustic shadow produced by wind speed and temperature gradients close to the ground 305
M West, F Walkden & R A Sack
University of Salford

Measurement of acoustic propagation through the atmosphere 311
P J Soilleux
Royal Signals Research Establishment

The prediction of noise from construction sites 323
R Wentang & K Attenborough
The Open University

Aircraft Noise

A study of annoyance due to general and business aviation noise 331
J B Ollerhead (1), I D Diamond (2), J G Walker (2), J B Critchley (3) & S A Bradshaw (2)
(1) Loughborough University
(2) University of Southampton
(3) Civil Aviation Authority

Practical measurement of ground noise and its implications on prediction techniques 339
P Henson & J G Charles
Bickerdike Allen Partners

Insulation of mobile homes subjected to military aircraft noise 347
D M Bexon (1), S M Dryden (2), P Moore (2) & R J Weston (1)
(1) RAF ICOM
(2) WS Atkins Engineering Sciences

Experience in the measurement of noise from military aircraft 361
W Stubbs & R B Gillham
Wimpey Laboratories Ltd

In-flight measurement of boundary layer noise and its vibration effects 369
D Sims
British Aerospace (Dynamics) Ltd

Environmental Noise

Neighbourhood noise disturbance 377
W A Utley & E C Keighley
Building Research Establishment

The suitability of A-weighted sound power for labelling domestic appliances 385
J R Brookes & K Attenborough
The Open University

Some applications for the single number rating of hearing protectors 391
R J Weston
RAF ICOM

The Noise Council survey of clay pigeon shooting noise 397
H G Leventhall
Commins-BBM Partnership

Progressing motorcycle noise control - a Noise Council perspective 405
A J Gilbert (1) & J D Clegg (2)
(1) Warrington Borough Council
(2) Bolton Metropolitan Borough Council

Industrial claddings: Sound absorption and transmission 409
N J H Alexander, D E O'Connor & R J Orlowski
University of Salford

Prediction of community response to noise from urban light rail systems 419
B M Shield, D W Birden, J P Roberts & M L Vuillermoz
Southbank Polytechnic

Environmental noise aspects of a gas turbine power generating plant 429
B C Postlethwaite
Acoustic Technology Ltd

Physical Acoustics and Ultrasonics

Deconvolution by the maximum entropy method as applied to ultrasonic surface characterisation 437
P F Smith & M A Player
University of Aberdeen

Acoustic nonlinearity parameters and higher-order elastic constants of crystals 445
J H Cantrell
University of Cambridge

Piezoelectric detection of signals in scanning electron acoustic microscopy 453
M Qian, J H Cantrell & F J Rocca
University of Cambridge

The pulse enhancement of unstable cavitation by the mechanisms of bubble migration 461
T G Leighton (1), M J W Pickworth (2), A J Walton (1) & P P Dendy (2)
(1) University of Cambridge
(2) Addenbrookes Hospital

Reflection from media with continuous variation of density, sound speed and attenuation 471
R C Chivers
University of Surrey

Ultrasonic absorption measurements in thin films of adhesive polymers 477
R E Challis & T Alper
University of Keele

A wide bandwidth ultrasonic absorption spectrometer for liquid materials 487
R E Challis & A K Holmes
University of Keele

Instrumentation

Calibration at high power of pulsed acoustic transducers 495
R A Hazelwood
Aconics Partners

Measured transducer field distributions for improved measurement and imaging 503
J R Blakey, R C Chivers & R A Bacon
University of Surrey

Application of acoustic reciprocity principles 513
F J Fahy
University of Southampton

A two-microphone technique for measuring acoustic impedance in the free field: Effects of sample area and measurement errors 521
D C Waddington & R J Orlowski
University of Salford

Student Session

The influence of airflow resistance on the acoustic absorption properties of polyurethane foams 533
G Rodwell & C G Rogers
Sheffield City Polytechnic

Air absorption in acoustic scale models and the substitution of nitrogen 541
P Wright
University of Salford

The effect of loading on the impact sound insulation of concrete floors with a floating raft on a resilient layer 543
M A Stewart, D J Mackenzie & D G Falconer
Heriot-Watt University

Annoyance and impulsivity judgements of environmental noises 551
C G Swift, I H Flindell & C G Rice
University of Southampton

Loudness of low frequency impulses 559
C W Dilworth
University of Salford

Analysis of the amplitude statistics of random signals 561
A Shadbolt
University of Salford

A hemispherical cap array for noise source location 567
M H D Santer
University of Southampton

An expert systems approach to understanding signals and systems 569
A M Raper (1) & J K Hammond (2)
(1) Anderson Consulting
(2) ISVR, Southampton University
(1988 Simon Alport Prize Lecture)

SWCC for predicting the differences between shallow and deep water explosions 583
N Hogwood
University of Southampton

The effects of bubble curtains on underwater shockwave propagation 589
S P Boyle
University of Southampton

Auditory hazard to divers from underwater explosions 597
C Herbert
University of Southampton

The acoustic loading of enclosures on loudspeakers 599
D R Philip
University of Salford

The viewing of loudspeaker modes by laser speckle 607
M Brooke
University of Wales, Cardiff

Open Session

Computerised auto-testing of sound level meters 613
C P Stollery
Cirrus Research Ltd

The effect of flanged joints on noise and vibrational transmission 619
R S Ming, G Stimpson & N Lalor
University of Southampton

Transmission of noise through rubber-metal composite springs 627
A H Muhr
Malaysian Rubber Producers Research Association

The use of time-reversed decay measurements in speech studios 635
B Rasmussen (1) & J H Rindel (2)
(1) Bruel & Kjaer, Denmark
(2) Technical University of Denmark

Sound generation in a duct with a bulk-reacting liner 643
A Cummings
University of Hull

A study of stick-slip for selected materials 651
G J McNulty (1), G R Symmons (1) & Z Ming (2)
(1) Sheffield City Polytechnic
(2) Chengdu University, P R China

ACOUSTIC SCATTERING BY SUB-SURFACE INHOMOGENEITIES

D.L.Berry(1), S.N.Chandler-Wilde(2) and K.Attenborough(1)

(1) Faculty of Technology, Open University, Milton Keynes, MK7 6AA
(2) Department of Mathematics, Coventry Polytechnic, Coventry, CV1 5FB.

Introduction

The application of the boundary integral equation (BIE) method to problems of acoustic scattering is widely known ([1] - [3]). The method has been used, in the context of outdoor sound propagation, to predict the propagation of acoustic waves over a plane of homogeneous impedance, perturbed by the introduction of a region of inhomogeneous impedance ([4], [5]), or by the introduction of a noise barrier [6]. Here, the BIE method is applied to the problem of the prediction of the acoustic field due to a point source in a homogeneous quiescent atmosphere, above a homogeneous rigid porous half-space containing a rigid inhomogeneity.

Theory

The Boundary Value Problem

The geometry is shown in figure 1. An obstacle, labelled S, with smooth, rigid surface ∂S is embedded in a porous half-space, characterised by a complex impedance Z_2 and a complex wavenumber k_2. The upper half-space, denoted U_+, contains air, and is assumed to be characterised by real impedance and wavenumber, Z_1 and k_1, respectively. To define the other notation in figure 1, $U_- := \mathbf{R}^3 \backslash (\overline{S \cup U_+})$ denotes the porous medium, and $\Gamma = \{(x,y,z) \in \mathbf{R}^3 | z = 0\}$ the boundary between the two half spaces. It is intended to determine the value of the complex acoustic pressure $p(\mathbf{r}, \mathbf{r}_0)$, at points $\mathbf{r} \in \mathbf{R}^3$ given a source point $\mathbf{r}_0 \in U_+$, when the plane surface, Γ, is insonified by a monofrequency point source. The complex acoustic pressure is assumed to satisfy the following boundary value problem:
an inhomogeneous Helmholtz equation for $\mathbf{r} \in U_+$,

$$(\nabla^2 + k_1^2)p(\mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0); \qquad (1)$$

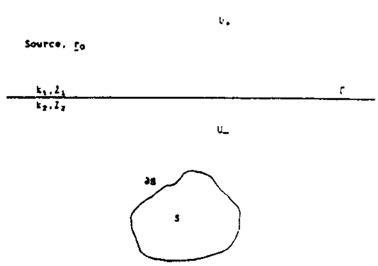

Figure 1: The physical situation

the Helmholtz equation for $\mathbf{r} \in U_-$,

$$(\nabla^2 + k_2^2)p(\mathbf{r}, \mathbf{r}_0) = 0; \tag{2}$$

the Neumann boundary condition for $\mathbf{r} \in \partial S$, for a rigid scatterer,

$$\frac{\partial p(\mathbf{r}, \mathbf{r}_0)}{\partial n(\mathbf{r})} = 0; \tag{3}$$

continuity of complex acoustic pressure, for $\mathbf{r} \in \Gamma$,

$$p_+(\mathbf{r}, \mathbf{r}_0) = p_-(\mathbf{r}, \mathbf{r}_0); \tag{4}$$

continuity of normal velocity, for $\mathbf{r} \in \Gamma$,

$$\alpha \frac{\partial p_+(\mathbf{r}, \mathbf{r}_0)}{\partial z} = \frac{\partial p_-(\mathbf{r}, \mathbf{r}_0)}{\partial z}; \tag{5}$$

and Sommerfeld's radiation conditions in U_+ and U_-. In the above, a time dependence $e^{-i\omega t}$ is assumed and suppressed; $p(\mathbf{r}, \mathbf{r}_0)$ denotes the complex pressure at $\mathbf{r} = (x, y, z)$ due to a source at $\mathbf{r}_0 = (x_0, y_0, z_0)$; the subscripts $+/-$ denote the limiting values of a function as Γ is approached from the U_+/U_- side; $n(\mathbf{r})$ denotes the normal to the surface ∂S directed into S at point $\mathbf{r}$; $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ denotes the Laplacian; and α is defined by, $\alpha = (Z_2 k_2)/(Z_1 k_1)$.

REFORMULATION AS AN INTEGRAL EQUATION

To formulate the integral equation, the solution to the following simpler problem is required. Let the Green's function $G(\mathbf{r}, \mathbf{r}_0)$, satisfy the following boundary value problem, for each $\mathbf{r}_0 \in \mathbb{R}^3 \backslash \Gamma$: an inhomogeneous Helmholtz equation for $\mathbf{r} \in U_+$,

$$(\nabla^2 + k_1^2)G(\mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0); \tag{6}$$

an inhomogeneous Helmholtz equation for $\mathbf{r} \in U_-$,

$$(\nabla^2 + k_2^2)G(\mathbf{r}, \mathbf{r}_0) = \alpha\delta(\mathbf{r} - \mathbf{r}_0); \tag{7}$$

jump conditions for $\mathbf{r} \in \Gamma$,

$$G_+(\mathbf{r}, \mathbf{r}_0) = G_-(\mathbf{r}, \mathbf{r}_0); \tag{8}$$

and

$$\alpha \frac{\partial G_+(\mathbf{r}, \mathbf{r}_0)}{\partial z} = \frac{\partial G_-(\mathbf{r}, \mathbf{r}_0)}{\partial z}; \tag{9}$$

and Sommerfeld's radiation conditions in U_+ and U_-. Note that in the case when no obstacle is present,

$$p(\mathbf{r}, \mathbf{r}_0) = G(\mathbf{r}, \mathbf{r}_0), \tag{10}$$

for $\mathbf{r} \in \mathbb{R}^3$ and $\mathbf{r}_0 \in U_+$; but $G(\mathbf{r}, \mathbf{r}_0)$ is defined also when $\mathbf{r}_0 \in U_-$. In physical terms, $G(\mathbf{r}, \mathbf{r}_0)$ is the complex acoustic pressure at point $\mathbf{r}$ in a medium consisting of two half spaces of different impedances and/or propagation constants due to a simple point source at point $\mathbf{r}_0$ of unit volume strength; the point $\mathbf{r}_0$ may lie in either half space. The factor α, included in equation (7), ensures that G satisfies reciprocity.

To obtain an integral equation, consider regions V_1 and V_2, V_1/V_2 consisting of that part of U_+/U_- contained within a large sphere of radius R, centred on the origin, and the boundary, Γ, but excluding small spheres, σ_r and σ_{r_0} of radii ε, centred on r and r_0. The interiors of the spheres σ_r and σ_{r_0} are excluded so that the conditions of Green's second theorem are safisfied by p and G in regions V_1 and V_2. Applying Green's second theorem to regions V_1 and V_2, the following two equations are obtained:

$$\int_{\partial V_i} (p(\mathbf{r}_s, \mathbf{r}_0)\frac{\partial G(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)} - G(\mathbf{r}_s, \mathbf{r})\frac{\partial p(\mathbf{r}_s, \mathbf{r}_0)}{\partial n(\mathbf{r}_s)})ds(\mathbf{r}_s) = 0, \tag{11}$$

for $i = 1, 2$, $\mathbf{r}_0 \in U_+$ and $\mathbf{r} \in \mathbf{R}^3$. Letting the radius of the small spheres, ϵ, tend to zero, and the radius of the large hemisphere, R, tend to infinity, and using the Sommerfeld radiation conditions satisfied by p and G, these two equations become

$$\kappa_1(\mathbf{r})p(\mathbf{r}, \mathbf{r}_0) = G(\mathbf{r}_0, \mathbf{r}) - \int_\Gamma p_+(\mathbf{r}_s, \mathbf{r}_0)\frac{\partial G_+(\mathbf{r}_s, \mathbf{r})}{\partial z_s} - G_+(\mathbf{r}_s, \mathbf{r})\frac{\partial p_+(\mathbf{r}_s, \mathbf{r}_0)}{\partial z_s}ds(\mathbf{r}_s), \tag{12}$$

where $\kappa_1(\mathbf{r}) := 1$ for $\mathbf{r} \in U_+$, $1/2$ for $\mathbf{r} \in \Gamma$, 0 for $\mathbf{r} \in U_- \bigcup \overline{S}$; and,

$$\alpha\kappa_2(\mathbf{r})p(\mathbf{r}, \mathbf{r}_0) = \int_\Gamma p_-(\mathbf{r}_s, \mathbf{r}_0)\frac{\partial G_-(\mathbf{r}_s, \mathbf{r})}{\partial z_s} - G_-(\mathbf{r}_s, \mathbf{r})\frac{\partial p_-(\mathbf{r}_s, \mathbf{r}_0)}{\partial z_s}ds(\mathbf{r}_s) + \int_{\partial S} p(\mathbf{r}_s, \mathbf{r}_0)\frac{\partial G(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)}ds(\mathbf{r}_s), \tag{13}$$

where $\kappa_2(\mathbf{r}) := 1$ for $\mathbf{r} \in U_-$, $1/2$ for $\mathbf{r} \in \Gamma \bigcup \partial S$, 0 for $\mathbf{r} \in U_+$. Multiplying equation (12) by α, and adding this to equation (13) and making use of the conditions on Γ satisfied by p and G (equations (4), (5), (8) and (9)), equations (12) and (13) can be combined to give the following equation, for $\mathbf{r}_0 \in U_+$ and $\mathbf{r} \in \mathbf{R}^3$:

$$\alpha\kappa(\mathbf{r})p(\mathbf{r}, \mathbf{r}_0) = \alpha G(\mathbf{r}_0, \mathbf{r}) + \int_{\partial S} p(\mathbf{r}_s, \mathbf{r}_0)\frac{\partial G(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)}ds(\mathbf{r}_s), \tag{14}$$

where $\kappa(\mathbf{r}) := 1$ for $\mathbf{r} \in \mathbf{R}^3\backslash\overline{S}$, $1/2$ for $\mathbf{r} \in \partial S$. Provided $Imk_2 > 0$, as argument similar to ([1], p.103) shows, this BIE has a unique solution, so that this formulation of the problem is equivalent to the boundary value problem, equations (1) to (5). The condition $Imk_2 > 0$ is satisfied since k_2 is the propagation constant for a porous medium.

SOLUTION OF THE INTEGRAL EQUATION

In general, a closed form solution of equation (14) is not possible. A numerical solution can be obtained by a simple boundary element method as follows. Firstly, p is determined on ∂S by forming and solving a set of linear equations. Once p on ∂S is found, the value of p at any other point, $\mathbf{r} \in \mathbf{R}^3\backslash S$ can be determined by performing the integral of equation (14).

Equation (14), restricted to $\mathbf{r} \in \partial S$, is a weakly singular Fredholm integral equation of the second kind. A difficulty in the numerical solution of this integral equation is that the kernel function tends to infinity as r approaches $\mathbf{r}_s$. This difficulty is resolved by applying the modification of Burton [7]. For $\mathbf{r} \in \partial S$, equation (14) is written:

$$\begin{aligned}\alpha\frac{1}{2}p(\mathbf{r}, \mathbf{r}_0) \;\; = \;\; & \alpha G(\mathbf{r}_0, \mathbf{r}) + \int_{\partial S} p(\mathbf{r}_s, \mathbf{r}_0)\frac{\partial G(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)} - p(\mathbf{r}, \mathbf{r}_0)\frac{\partial G_0(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)}ds(\mathbf{r}_s) \\ & + p(\mathbf{r}, \mathbf{r}_0)\int_{\partial S}\frac{\partial G_0(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)}ds(\mathbf{r}_s).\end{aligned} \tag{15}$$

where $G_0(\mathbf{r}_s, \mathbf{r}) = -\alpha/(4\pi|\mathbf{r}_s - \mathbf{r}|)$ is the principal singularity of $G(\mathbf{r}_s, \mathbf{r})$. From Gauss' theorem, the last integral can be integrated exactly, giving:

$$\int_{\partial S} \frac{\partial G_0(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)} ds(\mathbf{r}_s) = -\frac{\alpha}{2}, \tag{16}$$

for $\mathbf{r} \in \partial S$. Hence, equation (15) can be rewritten as:

$$\alpha p(\mathbf{r}, \mathbf{r}_0) = \alpha G(\mathbf{r}_0, \mathbf{r}) + \int_{\partial S} p(\mathbf{r}_s, \mathbf{r}_0) \frac{\partial G(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)} - p(\mathbf{r}, \mathbf{r}_0) \frac{\partial G_0(\mathbf{r}_s, \mathbf{r})}{\partial n(\mathbf{r}_s)} ds(\mathbf{r}_s). \tag{17}$$

According to Burton [7], the new integrand, taken as a whole, remains finite as $\mathbf{r}$ approaches $\mathbf{r}_s$.

Now, if the surface ∂S is split into N boundary elements, $\partial S_1, \partial S_2, \ldots, \partial S_N$, from equation (17) it follows that:

$$\alpha p(\mathbf{r}_j, \mathbf{r}_0) = \alpha G(\mathbf{r}_0, \mathbf{r}_j) + \sum_{l=1}^{N} I_{jl}, \tag{18}$$

for $j = 1(1)N$, where $\mathbf{r}_j$ is the midpoint of element ∂S_j, and:

$$I_{jl} = \int_{\partial S_l} p(\mathbf{r}_s, \mathbf{r}_0) \frac{\partial G(\mathbf{r}_s, \mathbf{r}_j)}{\partial n(\mathbf{r}_s)} - p(\mathbf{r}_j, \mathbf{r}_0) \frac{\partial G_0(\mathbf{r}_s, \mathbf{r}_j)}{\partial n(\mathbf{r}_s)} ds(\mathbf{r}_s). \tag{19}$$

For $j \neq l$, the approximation can be made that:

$$I_{jl} \approx A_l \left(p(\mathbf{r}_l, \mathbf{r}_0) \frac{\partial G(\mathbf{r}_l, \mathbf{r}_j)}{\partial n(\mathbf{r}_l)} - p(\mathbf{r}_j, \mathbf{r}_0) \frac{\partial G_0(\mathbf{r}_l, \mathbf{r}_j)}{\partial n(\mathbf{r}_l)} \right), \tag{20}$$

where A_l is the area of ∂S_l, and for $j = l$:

$$I_{jl} \approx 0. \tag{21}$$

Thus, the following linear equations are satisfied approximately by the unknown values $p(\mathbf{r}_j, \mathbf{r}_0)$:

$$\alpha p(\mathbf{r}_j, \mathbf{r}_0) = \alpha G(\mathbf{r}_0, \mathbf{r}_j) + \sum_{l=1 (l \neq j)}^{N} A_l \left(p(\mathbf{r}_l, \mathbf{r}_0) \frac{\partial G(\mathbf{r}_l, \mathbf{r}_j)}{\partial n(\mathbf{r}_l)} - p(\mathbf{r}_j, \mathbf{r}_0) \frac{\partial G_0(\mathbf{r}_l, \mathbf{r}_j)}{\partial n(\mathbf{r}_l)} \right), \tag{22}$$

for $j = 1(1)N$. These approximately satisfied set of N linear equations for the values of p at the midpoints $\mathbf{r}_l$ of ∂S_l can be written in the standard form:

$$\sum_{l=1}^{N} a_{jl} p(\mathbf{r}_l, \mathbf{r}_0) = \alpha G(\mathbf{r}_0, \mathbf{r}_j) \tag{23}$$

for $j = 1(1)N$, and where:

$$a_{jl} = \left[\alpha + \sum_{i=1 (i \neq j)}^{N} \frac{\partial G_0(\mathbf{r}_i, \mathbf{r}_j)}{\partial n(\mathbf{r}_i)} A_i \right] \delta_{jl} - (1 - \delta_{jl}) \frac{\partial G(\mathbf{r}_l, \mathbf{r}_j)}{\partial n(\mathbf{r}_l)} A_l \tag{24}$$

where δ_{jl} is the Kronecker delta.

If the obstacle is axisymmetric about an axis perpendicular to the surface Γ and if the elements are numbered as indicated in figure 2, the matrix $[a_{jl}]$ is *Block - Circulant* of order $N = m \times n$, with

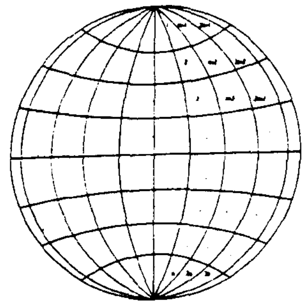

Figure 2: Element configuration for a sphere, including numbering system for the elements.

block entries of order m. A definition of this matrix type may be found in Davis [8]. The solution of equation (23) can be performed using the subroutine CGSLC of the Toeplitz package, from the Argonne National Laboratory [9]. The value of p at any point elsewhere in $r \in \mathbb{R}^3 \backslash S$, is then given approximately by substituting the calculated values of p on ∂S into:

$$\alpha\kappa(\mathbf{r})p(\mathbf{r},\mathbf{r}_0) = \alpha G(\mathbf{r}_0,\mathbf{r}) + \sum_{l=1}^{N} p(\mathbf{r}_l,\mathbf{r}_0)\frac{\partial G(\mathbf{r}_l,\mathbf{r})}{\partial n(\mathbf{r}_l)}A_l, \tag{25}$$

where $\kappa(\mathbf{r}) := 1$ for $\mathbf{r} \in \mathbb{R}^3\backslash\overline{S}$, $1/2$ for $\mathbf{r} \in \partial S$.

THE GREEN'S FUNCTIONS AND IMPEDANCE MODELS

The Green's function can be expressed exactly as an inverse Hankel transform ([10], equation (1)), which can, by various transformations, and by deformation of the path of integration, be expressed in a form more suitable for numerical integration. But, for the calculations performed in this paper, the following simple approximation was used:

$$G(\mathbf{r},\mathbf{r}_0) \approx -\frac{\epsilon}{4\pi}\left\{\frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|} + R_p\frac{e^{ik|\mathbf{r}-\mathbf{r}_0'|}}{|\mathbf{r}-\mathbf{r}_0'|}\right\} \tag{26}$$

when $\mathbf{r}$ and $\mathbf{r}_0$ are both on the same side of the interface, where $\mathbf{r}_0'$ denotes the geometrical image point of $\mathbf{r}_0$ in the plane Γ, $k = k_1(k_2)$ and $\epsilon = 1(\alpha)$ if $\mathbf{r}_0$ is in $U_+(U_-)$, and R_p denotes the plane wave reflection coefficient:

$$R_p = (\alpha cos\theta - (n^2 - sin^2\theta)^{\frac{1}{2}})/(\alpha cos\theta + (n^2 - sin^2\theta)^{\frac{1}{2}}) \tag{27}$$

($Re\{(n^2 - sin^2\theta)^{\frac{1}{2}}\} > 0$) if the source and receiver lie in U_+, where $\theta = cos^{-1}(|z+z_0|/|\mathbf{r}-\mathbf{r}_0'|)$ is the angle of incidence, and $n = k_2/k_1$ is the refractive index. If the source and receiver lie in U_- then R_p is given by (27) but with α and n replaced by α^{-1} and n^{-1} respectively.

To calculate $G(\mathbf{r},\mathbf{r}_0)$ when $\mathbf{r}_0 \in U_+$, $\mathbf{r} \in U_-$ the approximation of Richards *et al* ([10], equation (25)) has been used, i.e.:

$$G(\mathbf{r},\mathbf{r}_0) \approx exp\left[-ik_1 z(n^2 - sin^2\theta)^{\frac{1}{2}}\right] G(\mathbf{r}_\Gamma,\mathbf{r}_0) \tag{28}$$

Receiver position /m		Boundary Element Method $p_{scattered}/p_{direct}$			Classical Theory $p_{scattered}/p_{direct}$
x	y	m = 8 n = 16 h = 0.1963 m	m = 16 n = 32 h = 0.0982 m	m = 32 n = 64 h = 0.0491 m	
1	0	-0.0057 + 0.5853 i	-0.0051 + 0.5824 i	-0.0050 + 0.5817 i	-0.0045 + 0.5808 i
$1/\sqrt{2}$	$1/\sqrt{2}$	-0.2030 + 0.0804 i	-0.1919 - 0.0804 i	-0.1971 - 0.0805 i	-0.1965 - 0.0807 i
0	1	0.0693 + 0.1626 i	0.0645 + 0.1609 i	0.0635 + 0.1606 i	0.0628 + 0.1606 i
$-1/\sqrt{2}$	$1/\sqrt{2}$	-0.2486 - 0.0798 i	-0.2455 + 0.0762 i	-0.2449 - 0.0754 i	-0.2448 - 0.0755 i
-1	0	-0.0391 - 0.2797 i	-0.0426 - 0.2761 i	-0.0434 - 0.2754 i	-0.0437 - 0.2754 i

Table 1: Comparison of the values of the ratio of the scattered field to the direct field using the boundary element method and classical theory for a rigid sphere of radius 0.5m, in an infinite medium of propagation constant $k = (5.0 + 0.05i)/m$. For the boundary element method, the source is taken at $x = -1000.0m$.

where $\mathbf{r}_\Gamma = (x, y, 0)$ denotes the point on Γ directly above $\mathbf{r}$ and $\theta = cos^{-1}(z_0/|\mathbf{r}_\Gamma - \mathbf{r}_0|)$. When using the approximation (28), $G(\mathbf{r}_\Gamma, \mathbf{r}_0)$ is approximated by equation (26). The approximation (28) is stated [10] to be valid provided n has an appreciable imaginary component. $G(\mathbf{r}, \mathbf{r}_0)$, when $\mathbf{r}_0 \in U_-$, $\mathbf{r} \in U_+$ is calculated from the approximation (28) by using reciprocity.

In the absence of the obstacle, the lower medium is assumed to react like a semi - infinite rigid porous half space. A simple four parameter model of Attenborough [11] can be used to predict the values of normalised propagation constant, and normalised surface impedance for such media. The normalised surface impedance and normalised propagation constant both require characteristic values for the lower medium, i.e. pore shape factor ratio, grain shape factor, effective flow resistivity and porosity.

COMPARISON WITH ANALYTICAL RESULTS

If, in the above theory, $k_1 = k_2$ and $Z_1 = Z_2$ then equation (14) predicts the values of the pressure field in the presence of the rigid obstacle but in the absence of the plane boundary, i.e. the rigid obstacle is in an infinite homogeneous medium. This means that a direct comparison with the classical results for the scattering of sound from a rigid obstacle can be made. Morse and Ingard, [12], give expressions for the field scattered by a rigid sphere, where the incident plane wave is $exp(ikx)$, travelling in the positive x-direction. If, in the above theory, the source is positioned sufficiently far from the obstacle, such that the incident field at the obstacle is assumed approximately plane then such a direct comparison can be made. Table 1 shows the results for the ratio of the scattered field to the direct field, at several points close to a sphere of radius 0.5 m, within a porous medium of propagation constant $k = (5.0 + 0.05i)/m$. The source for the BIE method is positioned at $x = -1000.0m$ ($y = 0, z = 0$), the centre of the sphere being taken as the origin and the number of elements is varied. It is observed that there is convergence as the number of elements is increased.

COMPARISON WITH EXPERIMENTAL RESULTS

A set of simple laboratory measurements within a small anechoic chamber were carried out to compare with the theory. In these experiments, a porous half-space was modelled by a 0.5m thick layer of pea-gravel (maximum grain size 0.95cm). A wooden hemisphere of 0.125m diameter was used as the