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Fundamentals  
of Digital  
Image Processing

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ANIL K. JAIN

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ANIL K. JAIN  
*University of California, Davis*



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# Preface

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Digital image processing is a rapidly evolving field with growing applications in science and engineering. Image processing holds the possibility of developing the ultimate machine that could perform the visual functions of all living beings. Many theoretical as well as technological breakthroughs are required before we could build such a machine. In the meantime, there is an abundance of image processing applications that can serve mankind with the available and anticipated technology in the near future.

This book addresses the fundamentals of the major topics of digital image processing: *representation*, *processing techniques*, and *communication*. Attention has been focused on mature topics with the hope that the level of discussion provided would enable an engineer or a scientist to design image processing systems or conduct research on advanced and newly emerging topics. Image representation includes tasks ranging from acquisition, digitization, and display to mathematical characterization of images for subsequent processing. Often, a proper representation is a prerequisite to an efficient processing technique such as enhancement, filtering and restoration, analysis, reconstruction from projections, and image communication. Image processing problems and techniques (Chapter 1) invoke concepts from diverse fields such as physical optics, digital signal processing, estimation theory, information theory, visual perception, stochastic processes, artificial intelligence, computer graphics, and so on. This book is intended to serve as a text for second and third quarter (or semester) graduate students in electrical engineering and computer science. It has evolved out of my class notes used for teaching introductory and advanced courses on image processing at the University of California at Davis.

The introductory course (Image Processing I) covers Chapter 1, Chapter 2 (Sections 2.1 to 2.8), much of Chapters 3 to 5, Chapter 7, and Sections 9.1 to 9.5. This material is supplemented by laboratory instruction that includes computer experiments. Students in this course are expected to have had prior exposure to one-dimensional digital signal processing topics such as sampling theorem, Fourier trans-

form, linear systems, and some experience with matrix algebra. Typically, an entry level graduate course in digital signal processing is sufficient. Chapter 2 of the text includes much of the mathematical background that is needed in the rest of the book. A student who masters Chapter 2 should be able to handle most of the image processing problems discussed in the text and elsewhere in the image processing literature.

The advanced course (Image Processing II) covers Sections 2.9, 2.13, and selected topics from Chapters 6, 8, 9, 10, and 11. Both the courses are taught using visual aids such as overhead transparencies and slides to maximize discussion time and to minimize in-class writing time while maintaining a reasonable pace. In the advanced course, the prerequisites include Image Processing I and entry level graduate coursework in linear systems and random signals.

Chapters 3 to 6 cover the topic of image representation. Chapter 3 is devoted to low-level representation of visual information such as luminance, color, and spatial and temporal properties of vision. Chapter 4 deals with image digitization, an essential step for digital processing. In Chapter 5, images are represented as series expansion of orthogonal arrays or *basis images*. In Chapter 6, images are considered as random signals.

Chapters 7 through 11 are devoted to image processing techniques based on representations developed in the earlier chapters. Chapter 7 is devoted to image enhancement techniques, a topic of considerable importance in the practice of image processing. This is followed by a chapter on image restoration that deals with the theory and algorithms for removing degradations in images. Chapter 9 is concerned with the end goal of image processing, that is, image analysis. A special image restoration problem is image reconstruction from projections—a problem of immense importance in medical imaging and nondestructive testing of objects. The theory and techniques of image reconstruction are covered in Chapter 10. Chapter 11 is devoted to image data compression—a topic of fundamental importance in image communication and storage.

Each chapter concludes with a set of problems and annotated bibliography. The problems either go into the details or provide the extensions of results presented in the text. The problems marked with an asterisk (\*) involve computer simulations. The problem sets give readers an opportunity to further their expertise on the relevant topics in image processing. The annotated bibliography provides a quick survey of the topics for the enthusiasts who wish to pursue the subject matter in greater depth.

### **Supplementary Course Materials**

Forthcoming with this text is an instructors manual that contains solutions to selected problems from the text, a list of experimental laboratory projects, and course syllabus design suggestions for various situations.

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Finally, I would like to dedicate this book to my favorite image, Mohini, my children Mukul, Malini, and Ankit, and to my parents, all, for their constant support and encouragement.

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