Robert I. Soare

Recursively Enumerable Sets and Degrees

A Study of Computable Functions and Computably Generated Sets

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Preface to the Series

Perspectives in Mathematical Logic

(Edited by the Ω-group for "Mathematische Logik" of the Heidelberger Akademie der Wissenschaften)

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps of guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory, some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R. O. Gandy, A. Levy, G. H. Müller, G. Sacks, D. S. Scott) discussed the project in earnest and decided to go ahead with it.*

Professor F. K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the overall plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.

Oberwolfach, September 1975

Acknowledgements. *In starting our enterprise we essentially were relying on the personal confidence and understanding of Professor Martin Barner of the Mathematisches Forschungsinstitut Oberwolfach, Dr. Klaus Peters of Springer-Verlag and Dipl.-Ing. Penschuck of the Stiftung Volkswagenwerk. Through the Stiftung Volkswagenwerk we received a generous grant (1970–1973) as an initial help which made our existence as a working group possible.*

Since 1974 the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) has incorporated our enterprise into its general scientific program. The initiative for this step was taken by the late Professor F. K. Schmidt, and the former President of the Academy, Professor W. Doerr.

Through all the years, the Academy has supported our research project, especially our meetings and the continuous work on the Logic Bibliography, in an outstandingly generous way. We could always rely on their readiness to provide help wherever it was needed.

Assistance in many various respects was provided by Drs. U. Felgner and K. Gloede (till 1975) and Drs. D. Schmidt and H. Zeitler (till 1979). Last but not least; our indefatigable secretary Elfriede Ihrig was and is essential in running our enterprise.

We thank all those concerned.

Heidelberg, September 1982

R. O. Gandy *H. Hermes*
A. Levy *G. H. Müller*
G. Sacks *D. S. Scott*

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Author's Preface

One of the fundamental contributions of mathematical logic has been the precise formulation and study of computable functions. This program received an enormous impetus in 1931 with Gödel's Incompleteness Theorem which used the notion of a primitive recursive function and led during the mid-1930's to a variety of definitions of a computable (i.e., *recursive*) function by Church, Gödel, Kleene, Post, and Turing. It was soon proved that these various definitions each gave rise to exactly the same class of mathematical functions, the class now generally accepted (according to Church's Thesis) as containing precisely those functions intuitively regarded as "effectively calculable." Informally, these are the functions which could be calculated by a modern computer if one ignores restrictions on the amount of computing time and storage capacity.

Closely associated is the notion of a computably listable (so-called *recursively enumerable* (*r.e.*)) set of numbers, namely one which can be generated by a computable procedure. Indeed the notions are, in a sense, interchangeable because one can begin the study of computable functions either: (1) with the notion of a recursive function, and can then define an r.e. set as the range of such a function on the integers; or (2) with the notion of an r.e. set, and can then define a recursive function as one whose graph is r.e. (The latter approach is sometimes preferable in generalized recursion theory.)

Thus although this book is ostensibly about r.e. sets and their degrees, it is intended more generally as an introduction to the theory of computable functions, and indeed it is intended as a replacement for the well-known book by Rogers [1967], which is now both out of date and out of print. This book will serve as an introduction for both mathematicians and computer scientists. The first four chapters cover the basic theory of computable functions and r.e. sets including the Kleene Recursion Theorem and the arithmetical hierarchy. Basic finite injury priority arguments appear in Chapter 7. Well grounded in the fundamentals, the computer scientist can then turn to computational complexity.

In his epochal address to the American Mathematical Society E. L. Post [1944] stripped away the formalism associated with the development of recursive functions in the 1930's and revealed in a clear informal style the essential properties of r.e. sets and their role in Gödel's incompleteness theorem. Recursively enumerable sets have later played an essential role in many other famous undecidability results (such as the Davis-Matijasevič-Putnam-Robinson resolution of Hilbert's tenth problem on the solution of certain Diophantine equations, or the Boone-Novikov theorem on the unsolvability of the word problem for finitely presented groups). This essential role of r.e. sets is because of: (1) the widespread occurrence of r.e. sets in many branches of mathematics; and (2) the fact that there exist r.e. sets which are not computable (i.e., not *recursive*). The first such set (constructed by Church, Rosser, and Kleene jointly) was called by Post *creative* because its existence together with the representability of all r.e. sets even in such a small fragment of mathematics as elementary number theory implied the impossibility of mechanically listing all statements true in such a fragment. Post remarked: "The conclusion is inescapable that even for such a fixed, well-defined body of mathematical propositions, *mathematical thinking is, and must remain, essentially creative.* To the writer's mind, this conclusion must inevitably result in at least a partial reversal of the entire axiomatic trend of the late nineteenth and early twentieth centuries, with a return to meaning and truth as being of the essence of mathematics."

This book represents a kind of progress report over the last forty years on the programs, ideas, and hopes expressed in Post's paper. It is intended to follow the same informal style as Post, but with full mathematical rigor. In doing so, this book is in the style of its principal predecessors on the subject: Kleene [1952a]; Rogers [1967]; and Shoenfield [1971]; to whom the author acknowledges a great debt. It differs from these predecessors in: (1) its emphasis on intuition and pictures of complicated constructions (often accompanied by suggestive terminology intended to create a diagram or image in the reader's mind); and (2) its modular approach of first describing the strategy for meeting each single requirement, and later describing the process by which these various and often conflicting strategies may be fitted together. In this way the book attempts to unveil some of the secrets of classical recursion theory whose seemingly formidable technical obstacles have tended to frighten away the novice from appreciating its considerable intrinsic beauty and elegance.

Classical recursion theory (CRT) is the study of computable functions on ω (the nonnegative integers) as opposed to generalized recursion theory (GRT) which deals with computation in certain ordinals or higher type objects. The beauty of CRT lies in the simplicity of its fundamental notions, just as a classical painting of the Renaissance is characterized by simplicity of line and composition. For example, the notion of an r.e. set as one which can be effectively listed is one of the few fundamental notions of higher mathematics which can easily be explained to the common man. The art and architecture of the Renaissance are characterized by balance, harmony, and a world on a human scale, where the human figure is not dwarfed by his surroundings. In CRT the universe is merely the natural numbers ω

which the human mind can readily grasp and not some more abstract object. Further analogies between CRT and the classical art of the Renaissance may be found in the lectures in Bressanone, Italy (Soare [1981, §7]).

The ideas and methods of CRT (such as the priority method) have been useful not only in GRT but also in many other fields such as recursion theory on sets (so-called E-recursion), recursive model theory, the effective content of mathematics (particularly effective algebra and analysis), theoretical computer science and computational complexity, effective combinatorics (such as the extent to which classical combinatorial results like Ramsey's theorem can be effectivized), and models of formal systems such as Peano arithmetic. It is hoped that workers in these fields may find this book useful (particularly the first twelve chapters). The remainder of the book is written for the genuine devotee of recursion theory who wishes to be initiated into some of its inner mysteries.

Much of the material of this book has been presented in courses and seminars at the University of Chicago, and in short courses at various international mathematical meetings, for instance: the C.I.M.E. conference in Bressanone, Italy during June, 1979 on *Recursive Theory and Computational Complexity;* the British Logic Colloquium in Leeds during August, 1979 on *Recursion Theory: Its Generalizations and Applications;* and the conference in Bielefeld, Germany during July, 1981 on the *Priority Method in Recursion Theory.* I am indebted to my Ph.D. students at the University of Chicago: Victor Bennison, Peter Fejer, Steffen Lempp, David Miller, Steven Schwarz, and Michael Stob; and to the other graduate students Kathy Edwards, Michael Mýtilinaios, Nick Reingold, Francesco Ruggeri, Craig Smorynski, and Mitchell Stokes, all of whom have made the course stimulating and exciting to teach, and have contributed substantially to its present form. Special thanks are due for extensive contributions from L. Harrington, C. G. Jockusch, Jr., A. H. Lachlan, S. Lempp, M. Lerman, W. Maass, R. A. Shore, and T. Slaman. Many other mathematicians have supplied suggestions, corrections, stimulating conversations, or correspondence on the subject including among others F. Abramson, S. Ahmad, D. Alton, K. Ambos-Spies, K. Appel, M. Arslanov, M. Blum, T. Carlson, C. T. Chong, B. Cooper, J. Crossley, M. Davis, J. C. E. Dekker, A. Degtev, R. Downey, B. Dreben, R. Epstein, Y. Ershov, S. Friedman, R. O. Gandy, V. Harizanov, J. Hartmanis, L. Hay, P. Hinman, E. Herrmann, H. Hodes, S. Homer, Huang Wen Qi, I. Kalantari, E. B. Kinber, P. Kolaitis, G. Kreisel, S. Kurtz, R. Ladner, S. MacLane, A. Manaster, D. A. Martin, A. R. D. Mathias, T. McLaughlin, A. Meyer, T. Millar, A. Nerode, P. Odifreddi, J. Owings, D. Pokras, D. Posner, M. B. Pour-El, M. Ramachandran, J. Remmel, R. W. Robinson, J. Rosenstein, J. Royer, G. E. Sacks, L. Sasso, J. Shoenfield, S. Simpson, R. Smith, C. Smorynski, R. Solovay, S. Thomason, S. Wainer, Dong Ping Yang, C. E. M. Yates, and P. Young. Preliminary versions of this book were

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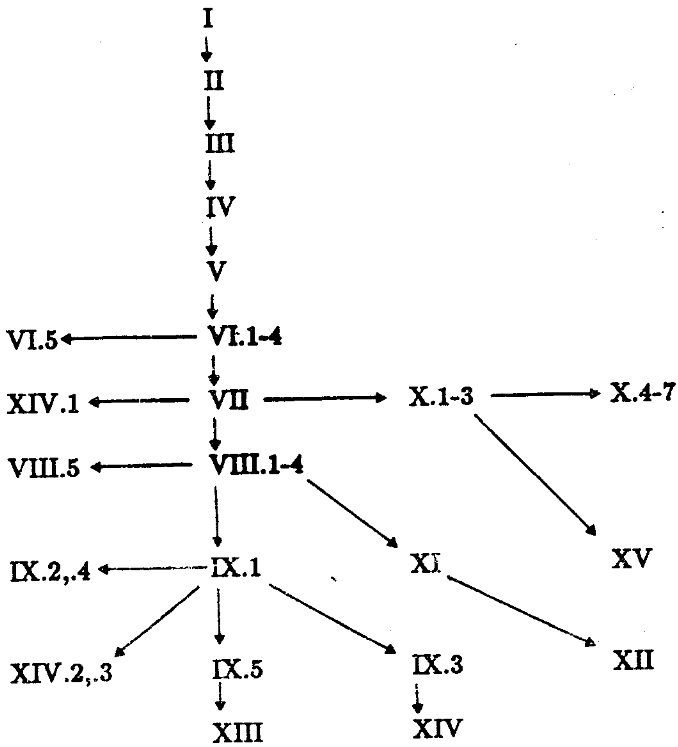
The subject matter of this book includes the contributions of many fine mathematicians, but in particular the unusually innovative discoveries (in historical order) of Stephen C. Kleene, Emil Post, Clifford Spector, Richard Friedberg, A. A. Muchnik, Gerald E. Sacks, and Alistair Lachlan. The book itself reflects the enormous debt which the author owes to his mathematical forbears: Anil Nerode, his thesis advisor, who taught him not only recursion theory but also the enthusiasm and confidence so essential to mathematical success, and to his "mathematical grandfather," Saunders MacLane, whose mathematical vigor, commitment to excellence, and strength of character have deeply influenced the author since his arrival at the University of Chicago in 1975. The author is very grateful to the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) for its travel support to attend meetings of the Ω-group from 1974 to 1983 to discuss outlines and preliminary versions of the book. These meetings with the editors and other members of the Ω-group were very helpful as the author's view emerged and changed over that period. The Academy also provided support for Steffen Lempp to proofread the entire typescript. A debt of gratitude goes to the author's wife Pegeen for her patience and understanding during the preparation of the book, and for her proofreading parts of the manuscript. The author is indebted to Fred Flowers for typing the first draft of Chapters I–X, to Richard Carnes for typesetting the entire manuscript in TeX, and to Terry Brown for drawing the diagrams and for typing the bibliography.

Chicago — Robert I. Soare
June 18, 1986

Major Dependencies Diagram



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Introduction

An initial segment of this book is intended as an introduction to the theory of recursive functions and r.e. sets. For this purpose, selected portions of Parts A, B, and C could be used in a one-semester or two-quarter course, the exact selection depending upon the lecturer's interests and the length of the course. In most parts of the book no knowledge of logic is necessary, but the reader will find it helpful. The minimum background is the mathematical sophistication normally acquired in an undergraduate course in modern algebra. The more advanced sections and exercises in Parts C and D include an exposition of some of the most important recent results and methods concerning r.e. sets, and are intended to bring the reader to the frontiers of current research.

The first three parts are grouped in three main historical periods, according to the dates of discovery of the main results and techniques. This is only a rough classification because, of course, not all the results in a given part were necessarily proved during the corresponding period. In general, however, they *could* have been proved at that time because the methods were available.

Part A corresponds to the period 1931–1943, beginning with the primitive recursive functions used in Gödel's incompleteness theorem in 1931. It includes various definitions of recursive functions and r.e. sets, their fundamental properties, the Kleene Recursion Theorem and its applications, relative computability and degrees of unsolvability, and the arithmetical hierarchy.

Part B covers the period 1944–1960 beginning with Post's 1944 address to the American Mathematical Society on r.e. sets and their decision problems. It includes Post's problem and his attempts to solve it using simple sets. It continues with the non-r.e. "oracle constructions" of Kleene and Post [1954] and Spector [1956] where a complicated condition such as "A is not computable from B" is decomposed into an infinite sequence $\{R_n : n \in \omega\}$ of simpler conditions called "requirements," say $A \neq \{n\}^B$, each one satisfied once and for all at some stage of the construction. Post's problem was solved using the finite injury priority method invented by Friedberg [1957] and independently by Muchnik [1957]. It combines the Kleene-Post type requirements with the effective constructions of Post thereby producing r.e. sets (rather than sets recursive in some oracle). The key innovation is to allow action taken at some stage for a given requirement R_m to be later "injured" by the action of some requirement R_n, $n < m$, of higher priority, so that R_m must be satisfied once again at a still later stage.

Part C covers the period from 1961 to the present, and stresses those constructions where the requirements may be infinitary in nature, for example positive requirements which cause infinitely many elements to enter an r.e. set A being constructed, or negative requirements which tend to restrain infinitely many elements from entering A. This includes the well-known infinite injury priority method and the minimal pair method for studying the r.e. degrees, **R**, as well as the Friedberg maximal set construction and its extensions by Lachlan and others for studying the lattice $\mathcal{E}$ of r.e. sets under inclusion. After considering **R** and $\mathcal{E}$ each separately, special attention is given to certain elegant results relating the algebraic structure of a set $A \in \mathcal{E}$ to its degree in **R** (namely to the degree of information which it encodes). Part D is devoted to more complicated results concerning $\mathcal{E}$ and **R**.

Definitions and Notation. Sections or theorems marked by * are not intended to be studied on a first reading of the book but contain either more difficult or supplementary material. The exercises are divided into three categories. Those unmarked are usually straightforward (at least with the copious hints). More difficult exercises are marked by $\diamond$ and the most difficult by $\diamond\diamond$.

We deal with sets and functions over the nonnegative integers $\omega = \{0, 1, 2, \dots\}$, and occasionally for technical convenience we include -1. Lower-case Latin letters $a, b, c, d, e, i, j, k, \dots, x, y, z$ denote integers; f, g, h (and occasionally other lower-case Latin letters) denote *total* functions from ω^n to ω, for $n \geq 1$; certain upper- and lower-case Greek letters $\Phi, \Psi, \Theta, \varphi, \psi, \theta$ denote (possibly) *partial* functions on ω (functions whose domain is some subset of ω); and upper-case Latin letters $A, B, C, \dots, X, Y, Z$ denote subsets of ω. The composition of two functions f and g is denoted by $f \circ g$ or simply by fg; $f^n(x)$ denotes the function $f(x)$ composed with itself n times; $\varphi(x)\downarrow$ denotes that $\varphi(x)$ is defined; $\varphi(x)\downarrow = y$ denotes moreover that $\varphi(x)$ has value y; $\varphi(x)\uparrow$ denotes that $\varphi(x)$ is undefined; $\varphi = \psi$ denotes equality of partial functions φ and ψ (which in other books and papers is often written $\varphi \simeq \psi$), namely that for all x, $\varphi(x)\downarrow$ iff $\psi(x)\downarrow$, and if $\varphi(x)\downarrow$ then $\varphi(x) = \psi(x)$; dom φ and ran φ denote the domain and range respectively of φ; χ_A denotes the characteristic function of A which is often identified with A and written simply as $A(x)$; $f \restriction x$ denotes the restriction of f to arguments $y < x$, and $A \restriction x$ denotes $\chi_A \restriction x$; f *majorizes* g if $g(x) \leq f(x)$ for all x, and f *dominates* g if $g(x) \leq f(x)$ for all but finitely many x. We write "1:1" for a function f which is one to one (injective), and "1:1 and onto" if f is bijective.

In addition to the usual set theoretic notation we use $A \subseteq^* B$ to denote that $A \subseteq B$ except for finitely many elements (namely that $A - B$ is finite);

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