THE THEORY OF SOUND

J. W. S. RAYLEIGH

UNABRIDGED SECOND REVISED EDITION WITH AN
HISTORICAL INTRODUCTION BY ROBERT B. LINDSAY
VOLUME ONE

THE

THEORY OF SOUND

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WITH A HISTORICAL INTRODUCTION BY

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IN TWO VOLUMES

VOLUME I

SECOND EDITION REVISED AND ENLARGED

NEW YORK
DOVER PUBLICATIONS

This Dover edition, first published in 1945, is an
unabridged republication of the second revised and
enlarged edition of 1894. It is reprinted through
special arrangement with The Macmillan Company,
publishers of the original edition, and contains a
new Historical Introduction by Robert Bruce
Lindsay.

HISTORICAL INTRODUCTION
BY ROBERT BRUCE LINDSAY

THE current reprinting of Lord Rayleigh's "The Theory of Sound," first published in 1877, stimulates an inquiry into the reason why such a treatise still retains a position of importance in the literature of its field, when most scientific treatises of sixty-five years ago now possess for the most part historical interest only, and have long since been superseded by twentieth century standard works. It has seemed appropriate on this occasion to review briefly the historical development of the subject of acoustics in which this situation has occurred, and to pay some tribute to the character and contributions of the author of a book which continues to show such vitality. It is hoped that the following introductory comments will enhance the pleasure of those who continue to turn to Rayleigh's treatise for enlightenment and guidance in acoustics.

I. BIOGRAPHICAL SKETCH OF JOHN WILLIAM STRUTT, THIRD BARON RAYLEIGH (1842-1919)

The author of "The Theory of Sound" occupies an unusual position in the history of British physics if only because, while there are numerous examples of men raised to the peerage as a reward for outstanding scientific work, it is rare to find a peer by inheritance devoting himself to science. Lord Rayleigh was born John William Strutt, the eldest son of the second Baron Rayleigh of Terling Place, Witham in the county of Essex. His immediate ancestors were country gentlefolk with little or no interest in scientific pursuits, though one of his grandmothers was descended from a brother of Robert Boyle. In his boyhood Rayleigh exhibited no unusual precocity but apparently displayed the average boy's interest in the world about him. His schooling was rather scattered, short stays at Eton and Harrow being terminated by ill-health. He finally spent the four years preceding college at a small boarding school kept by a Rev. Mr. Warner in Highstead, Torquay, where he showed no interest in classics but began to develop decided competence in mathematics.

In 1861 at the age of nearly 20, young Rayleigh went up to Cambridge and entered Trinity College. Here he became a pupil of

E. J. Routh, the famous "coach" in applied mathematics. It was
under the guidance of Routh that he acquired the grasp of mathe-
matics which stood him in such good stead in his later research. The
system has often been criticized, but it ground the methods of ad-
vanced mathematical analysis essential for the physical scientist so
thoroughly into the candidate that they became a natural part of his
very being. It was not rigorous mathematics in the pure sense, but it
was vigorous mathematics, which served to cultivate a keen appre-
ciation of the particular method best suited for the solution of any
particular problem. Rayleigh also stated in after life that he had
profited greatly from the Cambridge lectures of Sir George G. Stokes,
who though Lucasian professor of mathematics was greatly interested
in experimental physics and performed many experimental demon-
strations for his classes. In the Mathematical Tripos of 1865, Ray-
leigh came out as Senior Wrangler and also became first Smith's
Prizeman. By this time he had clearly decided on a scientific career,
though the propriety of this was considered by some members of his
family rather doubtful in view of the social obligations inherent in
his ultimate succession to his father's title and position. Rayleigh
seems to have felt that such obligations should not be allowed to
interfere with his scientific work. In 1866 he was elected Fellow of
Trinity College, thus further emphasizing his scholarly leanings.
Curiously enough he replaced the usual grand tour of the continent
with a trip to the United States, then in the throes of reconstruction
after the Civil War.

In 1868 immediately after his return from America Rayleigh pur-
chased an outfit of experimental equipment. There was at that time
no university physical laboratory, though certain professors pos-
sessed apparatus for their own experimental purposes and for dem-
onstrations. Students received little or no direct encouragement to
embark on experimental investigations for themselves. This may
seem strange when one recalls that Cambridge had been for long the
home of Newton. Moreover, long before Rayleigh's undergraduate
days the immortal experiments of Young, Davy and Faraday, to men-
tion only a few, had already shed undying lustre on British science.
But this research had been carried on, by and large, outside the uni-
versities, which thus remained quite out of the current of real scien-
tific progress in physics well past the first half of the nineteenth
century. It was not until 1871 that Cambridge University established
a professorship of experimental physics; in 1873 the Cavendish
Laboratory was erected through the munificence of the Chancellor of
the University, the eighth Duke of Devonshire. James Clerk Maxwell

was elected the first Cavendish professor and served from 1871 to his untimely death in 1879. For the first time practical instruction from a distinguished physicist was provided at Cambridge.

To return to Rayleigh: it is interesting to observe that his first experimental investigations were on electricity and concerned the action of alternating currents on a galvanometer. The results were presented in a paper (his first) to the Norwich meeting of the British Association for the Advancement of Science in 1868. But he was soon thereafter deeply immersed in other things, including color vision and the pitch of resonators. The latter was his first work in acoustics and was apparently stimulated by his reading Helmholtz's famous work "On the Sensations of Tone" (1863). There was much correspondence about this and kindred matters with Maxwell, who was always eager to help along a youthful colleague. Rayleigh's experimental work was carried out at Terling in a rather crudely improvised laboratory. Later when the estate became his home by inheritance, more elaborate arrangements were made.

In 1871 Rayleigh married Evelyn Balfour, the sister of Arthur James Balfour, who was destined to gain much celebrity as a scholar, philosopher and statesman. He had become acquainted with Balfour as a fellow student at Cambridge. Shortly after his marriage a serious attack of rheumatic fever threatened for a time to cut short his career and left him much weakened in health. An excursion to Egypt was undertaken as a recuperative measure, and it was on a house boat trip up the Nile late in 1872 that the "Theory of Sound" had its genesis, the first part having been written with no access to a large library. The preparation of the treatise eventually extended over many years, and the two well-known volumes did not make their appearance from the press until 1877. In the meantime Rayleigh had succeeded to his father's title and had settled down at Terling. Changes were made to enable him to embark on more elaborate laboratory work, including experiments in acoustics and optics. It was during the period from 1871 to 1879 that he gave much attention to the diffraction of light and made copies of diffraction gratings. These investigations led to the introduction of the present standard definition of resolving power, a quantity of the utmost importance in specifying the performance of any optical instrument.

The premature death of Clerk Maxwell in 1879 left the Cavendish professorship vacant. Pressed by many scientific friends to stand for the post, Rayleigh finally consented, being partly influenced in his decision by the loss of income from his estate due to the agricultural depression of the late 70's. It does not appear that he ever contem-

plated retaining the professorship for an indefinite period, and indeed
he ultimately limited his tenure to five years. The pedagogical duties
of the Cavendish professor were not onerous: he was required to be
in residence for eighteen weeks during the academic year and to
deliver at least forty lectures in the course of this period. Rayleigh,
however, had no desire to interpret the job as a sinecure. He em-
barked vigorously on a program of developing elementary laboratory
instruction in a really elaborate way. It is difficult to appreciate today
what a task such a program involved sixty years ago. Collegiate in-
struction in practical physics was almost a new thing, and there was
little to go on save the teacher's imagination. Under Rayleigh's direc-
tion his demonstrators Glazebrook and Shaw, both of whom later
became men of note, the former in applied physics and the latter in
meteorology, developed laboratory courses for large classes in heat,
electricity and magnetism, properties of matter, optics and acoustics.
This was pioneer work of high order and had a beneficial influence
on the teaching of physics throughout England and ultimately else-
where.

Rayleigh was impressed at this time with the desirability of co-
operative research on a problem of importance and selected for this
purpose the redetermination of the standard electrical units. In
particular he wished to undertake a new evaluation of the relation
between the ohm, the practical unit of electrical resistance, and the
electromagnetic unit of resistance. The first precision work on this
problem had been carried out in 1863-64 under the auspices of the
British Association with Maxwell in charge. Later work by others had
disclosed considerable discrepancies. Rayleigh and his collaborators
devoted three years of labor to a repetition of the original experi-
ments with greater attention to sources of error. It is a tribute to
Rayleigh's great experimental care that his final results have not
been appreciably altered by more modern work. He appeared to
possess the uncanny power to make the simplest of equipment pro-
duce the utmost in precision.

In December 1884 Rayleigh returned to Terling, which he made
his scientific headquarters for the remainder of his life. It was close
enough to London to permit frequent visits to the metropolis for the
performance of official duties in connection with government or the
various professional societies in which he played a prominent role.
But he clearly enjoyed having his laboratory in his own home. Prob-
ably many contemporaries in the peerage, as well as the tenants on
the estate, thought him a trifle queer, but he went his way with typical
British imperturbability. The laboratory could hardly be considered

elaborate even when judged by contemporary standards. Rayleigh had a hatred of superfluous elegance and always stressed the desirability of simplicity in all research apparatus. Some of this feeling was undoubtedly inspired by his constitutional aversion to unnecessary expenditure; there was also a profound philosophical implication in the method which may be of value to the present day investigator, even when surrounded by highly intricate and sophisticated apparatus.

The life of a scientist working at his desk or in his laboratory has little to offer in the way of the dramatic, at least to the man in the street. It is inevitable that mankind in the large should find more emotional satisfaction in the contemplation of man's relations with his fellow creatures than in his relations with the physical environment. For the most part, too, scientific investigations involve a train of reasoning unfamiliar and intricate to the general run of people. Occasionally, however, a scientific discovery will be made which involves a relatively simple and clear cut situation, while at the same time it solves a puzzle originally as baffling as any detective story mystery. This was the case with the most dramatic popular episode in Rayleigh's career, namely the discovery of the rare gas argon in the atmosphere.

Already in his address to the Mathematics and Physics Section of the British Association at the Southampton meeting in 1882, Rayleigh had called attention to the desirability of a more precise determination of the densities of the so-called permanent gases, oxygen, hydrogen and nitrogen. The importance of this lay in its bearing on the problem of the atomic weights of the elements and hence the whole foundation of chemistry. This job Rayleigh now set for himself and devoted to it a good part of his own time and that of a skilled assistant for the better part of ten years, culminating in the famous joint announcement with Sir William Ramsay of the isolation of argon in 1895. The story is too well known for detailed repetition here. It furnishes a classic example of the importance of following up a small experimental discrepancy lying outside the limit of reasonable experimental error, in this case the difference between the density of nitrogen prepared from nitrogen compounds and nitrogen obtained by removing the oxygen of the air. It seems easy to say now that the larger value of the latter points directly to the existence in the air of a small amount of a gas heavier than nitrogen. But in 1895 this was not so simple and neither was the task of isolating the new gas. It is not too much to say that the subsequent discovery of all the other rare gases of the atmosphere was directly due to Rayleigh's patient,

ingenious and methodical investigation.

From 1887 to 1905, Lord Rayleigh served as Professor of Natural Philosophy at the Royal Institution of Great Britain as successor to John Tyndall, who in turn had succeeded Faraday. Unlike his predecessors Rayleigh spent comparatively little time in the laboratory of the Institution, confining his activity to the annual course of public lectures. These continued the tradition established by Faraday and Tyndall in covering the whole gamut of topics of physical interest with a profusion of experimental demonstrations. Sir Arthur Schuster says of Rayleigh in this connection: "Though not by nature a ready speaker, his lectures were effective." At any rate the auditor could always be confident that the speaker thoroughly understood what he was talking about.

In 1896 Rayleigh was appointed Scientific Adviser to Trinity House, a very ancient organization, dating back to Henry VIII, and having as its duties the erection and maintenance of such coastal installations as lighthouses, buoys and the like. For the next fifteen years he served faithfully and made numerous inspection trips. Some of his later work in optics and acoustics was suggested by problems arising in connection with the tests of lights and fog-signals. In spite of his devotion to his laboratory research, Rayleigh gave willingly of his time and energy to the deliberations of scientific committees of government and the various societies to which he belonged. Thus he was one of the leaders of the movement which led to the establishment of the National Physical Laboratory (the British counterpart of the National Bureau of Standards in Washington), and presided over the Executive Committee of the Laboratory until shortly before his death. He also served as President of the Advisory committee on Aeronautics from its inception in 1909 (at the instance of Prime Minister Asquith) until the time of his death. The activities of this committee were particularly important during the first world war from 1914-1918.

Among Lord Rayleigh's other public positions there is space only to mention his presidency of the Royal Society from 1905-1908 and his service as Chancellor of Cambridge University from 1908 until his death. Honors came to him in heaping measure, notable among them the Order of Merit, of which he was one of the first recipients in 1902, and the Nobel Prize in Physics in 1904.

Unlike most scientific men, Rayleigh was able to continue his work until his death, though he survived to the ripe old age of 76. He died on June 30, 1919, with three papers still unpublished. It is interesting that the last of these was one on acoustics: he never got

over his interest in sound.

The opinion of his contemporaries and successors places Rayleigh in that great group of nineteenth century physicists that have made British science famous all over the world, the group whose other members were Kelvin, Maxwell and Stokes. His position in the history of science is a great one. It is good to recall that he was above all a modest man and it is impossible to accept as otherwise than sincere the remarks he made when he received the Order of Merit: "the only merit of which he personally was conscious was that of having pleased himself by his studies, and any results that may have been due to his researches were owing to the fact that it had been a pleasure to him to become a physicist."

II. HISTORICAL DEVELOPMENT OF ACOUSTICS TO THE TIME OF RAYLEIGH

Introduction. Sound plays in our daily lives a part scarcely less important than motion and light, and the sense of hearing though by no means esteemed so precious as the sense of sight and the ability to locomote is yet so prized that the production of efficient hearing aids for the deaf is fast becoming a major industry. Life is full of sounds and we want to hear the pleasant and vital ones, while shunning the unpleasant and dangerous variety. All told we are becoming steadily more sound conscious, as the relatively enormous growth of the telephone, radio, phonographic recording and talking motion picture industries sufficiently attests.

In view of its importance, it might be supposed that the science of sound, technically known as acoustics, would loom as a substantial item in the history of the development of physical ideas. Strangely enough, in the standard histories this is by no means the case: the history of acoustics has been largely a neglected subject. A possible reason for this has been advanced by Whewell in his "History of the Inductive Sciences." The basic theory of the origin, propagation and reception of sound was proposed at a very early stage in the development of human thought in substantially the form which we accept today: the ancient Greeks, according to the most reasonable interpretation of the records, evidently were aware that sound somehow arises from the motion of the parts of bodies, that it is transmitted by the air through some undefined motion of the latter and in this way ultimately striking the ear produces the sensation of hearing. Vague as these ideas were they were yet clarity itself compared with the ancient views on the motion of solid bodies as well as on light and heat. The latter branches of physics had to go through a long course

of development in which theory succeeded .heory until the present stage was reached. As Whewell emphasizes, in acoustics the basic theory was laid down early and all that was needed was its implementation by the necessary analysis and its application to new problems as they arose. On the theoretical side the history of acoustics thus tends to be merged in the larger development of mechanics as a whole.

It has seemed eminently worth while, however, in connection with a re-issue of the greatest single work ever published in acoustics to take advantage of the occasion to review the history of those parts of mechanics and other branches of physics which have had a definite bearing on acoustical theory. In a small measure this may serve to supplement D. C. Miller's interesting "Anecdotal History of the Science of Sound" (1935), which is devoted mainly to a resumé of the experimental phenomena.

The problems of acoustics as already indicated are most conveniently divided into three main groups, viz: 1) the production of sound, 2) the propagation of sound, and 3) the reception of sound. We shall find it advantageous to organize the following historical outline accordingly.

The Production of Sound. The fact that when a solid body is struck a sound is produced must have been observed from the very earliest times. The additional fact that under certain circumstances the sounds produced are particularly agreeable to the ear furnished the basis for the creation of music, which also originated long before the beginning of recorded history. But music was an art for centuries before its nature began to be examined in a scientific manner. It is usually assumed that the first Greek philosopher to study the origin of musical sounds was Pythagoras in the 6th century B.C. He is supposed to have discovered that of two stretched strings fastened at the ends the higher note is emitted by the shorter one, and that indeed if one has twice the length of the other, the shorter will emit a note an octave above the other. By this time the notion of pitch had, of course, been developed, but its association with the frequency of the vibrations of the sounding body was probably not understood, and it does not appear that this concept emerged until the time of Galileo Galilei (1564-1642), the founder of modern physics. At the very end of the "First Day" of Galileo's "Discourses Concerning Two New Sciences," first published in 1638, the reader will find a remarkable discussion of the vibrations of bodies. Beginning with the well known *observations* on the isochronism of the simple pendulum and the dependence of the frequency of vibration on the length of the suspension, Galileo

goes on to describe the phenomenon of sympathetic vibrations or resonance by which the vibrations of one body can produce similar vibrations in another distant body. He reviews the common notions about the relation of the pitch of a vibrating string to its length and then expresses the opinion that the physical meaning of the relation is to be found in the number of vibrations per unit time. He says he was led to this point of view by an experiment in which he scraped a brass plate with an iron chisel and found that when a pure note of definite pitch was emitted the chisel cut the plate in a number of fine lines. When the pitch was high the lines were close together, while when the pitch was lower they were farther apart. Galileo was actually able to tune two spinet strings with two of these scraping tones; when the musical interval between the string notes was judged by the ear to be a fifth, the number of lines produced in the corresponding scrapings in the same total time interval bore precisely the ratio 3:2. The presumption is that if the octave had been tuned the ratio would have been 2:1, etc. It seems plain from a careful reading of Galileo's writings that he had a clear understanding of the dependence of the frequency of a stretched string on the length, tension and density. There was, of course, no question then of a dynamical discussion of the actual motion of the string: the theory of mechanics had not advanced far enough for that. But Galileo did make an interesting comparison between the vibrations of strings and pendulums in the endeavor to understand the reason why sounds of certain frequencies, i.e., those whose frequencies are in the ratio of two small integers, appear to the ear to combine pleasantly whereas others not possessing this property sound discordant. He observed that a set of pendulums of different lengths, set oscillating about a common axis and viewed in the original plane of their equilibrium positions present to the eye a pleasing pattern if the frequencies are simply commensurable, whereas they form a complicated jumble otherwise. This is a kinematic observation of great ingenuity and illustrates the fondness of the great Italian genius for analogy in physical description.

Credit is usually given to the Franciscan friar, Marin Mersenne (1588-1648) for the first correct published account of the vibrations of strings. This occurred in his "Harmonicorum Liber" published in Paris in 1636, two years before the appearance of Galileo's famous treatise on mechanics. However, it is now clear that Galileo's actual discovery antedated that of Mersenne. The latter did add one very important point: he actually measured the frequency of vibration of a long string and from this inferred the frequency of a shorter one of the same density and tension which gave a musical note. This was

apparently the first direct determination of the frequency of a musical sound.

Though later experimenters like Robert Hooke (1635-1703) tried to connect frequency of vibration with pitch by allowing a cog wheel to run against a piece of cardboard, the most thorough-going pioneer studies of this matter were made by Joseph Sauveur (1653-1716), who incidentally first suggested the name *acoustics* for the science of sound. He employed an ingenious use of the beats between the sounds from two organ pipes which were adjudged by the ear to be a semi tone apart, i.e., having frequencies in the ratio 15/16. By experiment he found that when sounded together the pipes gave 6 beats a second. By treating this number as the difference between the frequencies of the pipes the conclusion was that these latter numbers were 90 and 96 respectively. Sauveur also worked with strings and calculated (1700) by a somewhat dubious method the frequency of a given stretched string from the measured sag of the central point. It was reserved to the English methematician Brook Taylor (1685-1731), the celebrated author of Taylor's Theorem on infinite series, to be the first to work out a strictly dynamical solution of the problem of the vibrating string. This was published in 1713 and was based on an assumed curve for the shape of the string of such a character that every point would reach the rectilinear position in the same time. From the equation of this curve and the Newtonian equation of motion he was able to derive a formula for the frequency of vibration agreeing with the experimental law of Galileo and Mersenne. Though only a special case, Taylor's treatment paved the way for the more elaborate mathematical techniques of Daniel Bernoulli (1700-1782), D'Alembert (1717-1783) and Euler (1707-1783), involving the introduction of partial derivatives and the representation of the equation of motion in the modern fashion.

In the meantime it had already been observed, notably by Wallis (1616-1703) in England as well as by Sauveur in France, that a stretched string can vibrate in parts with certain points, which Sauveur called *nodes*, at which no motion ever takes place, whereas very violent motion takes place at intermediate points called *loops*. It was soon recognized that such vibrations correspond to higher frequencies than that associated with the simple vibration of the string as a whole without nodes, and indeed that the frequencies are integral multiples of the frequency of the simple vibration. The corresponding emitted sounds were called by Sauveur the *harmonic* tones, while the sound associated with the simple vibration was named the *fundamental*. The notation thus introduced (about 1700) has survived to the present

day. Sauveur noted the additional important fact that a vibrating string could produce the sounds corresponding to several of its harmonics at the same time. The dynamical explanation of this vibration was provided by Daniel Bernoulli in a celebrated memoir published by the Berlin Academy in 1755. Here he showed that it is possible for a string to vibrate in such a way that a multitude of simple harmonic oscillations are present at the same time and that each contributes independently to the resultant vibration, the displacement at any point of the string at any instant being the algebraic sum of the displacements for each simple harmonic node. This is the famous principle of the *coexistence of small oscillations*, also referred to as the *superposition principle*. It has proved of the utmost importance in the development of the theory of oscillations, though curiously enough its validity was at first strenuously doubted by D'Alembert and Euler, who saw at once that it led to the possibility of expressing any arbitrary function, e.g., the intial shape of a vibrating string, in terms of an infinite series of sines and cosines. The state of mathematics in the middle of the 18th century hardly permitted so bold a result. However, in 1822 Fourier (1768-1830) in his "Analytical Theory of Heat" did not hesitate to develop his celebrated theorem on this type of expansion with consequences of the greatest value for the advancement of acoustics.

The problem of the vibrating string was fully solved in elegant analytical fashion by J. L. Lagrange (1736-1813) in an extensive memoir of the Turin Academy in 1759. Here he supposed the string made up of a finite number of equally spaced identical mass particles and studied the motion of this system, establishing the existence of a number of independent frequencies equal to the number of particles. When he passed to the limit and allowed the number of particles to become infinitely great and the mass of each correspondingly small, these frequencies were found to be precisely the harmonic frequencies of the stretched string. The method of Lagrange was adopted by Rayleigh in his "Theory of Sound" and is indeed standard practise to-day, though most elementary books now develop the differential equation of motion of the string treated as a continuous medium by the method first set forth by D'Alembert in a memoir of the Berlin Academy of 1750. This differential equation we now call the wave equation, though the savants of the middle 18th century did not stress this interpretation.

In the memoir of Lagrange just referred to there is also a treatment of the sounds produced by organ pipes and musical wind instruments in general. The basic experimental facts were already known and

Lagrange was able to predict theoretically the approximate harmonic frequencies of closed and open pipes. The boundary conditions gave some trouble, as indeed they do to this day; in any case the problem impinges rather closely on the propagation of sound and as such is better treated in the next section.

The extension of the methods described in the preceding paragraphs to the vibrations of extended solid bodies like bars and plates naturally demanded a knowledge of the relation between the deformability of a solid body and the deforming force. Fortunately this problem had already been solved by Hooke, who in 1660 discovered and in 1676 announced in the form of the anagram *C E I I I N O S S S T T U V* the law "ut tensio sic vis" connecting the stress and strain for bodies undergoing *elastic* deformation. This law of course forms the basis for the whole mathematical theory of elasticity including elastic vibrations giving rise to sound. Its application to the vibrations of bars supported and clamped in various ways appears to have been made first by Euler in 1744 and Daniel Bernoulli in 1751, though it must be emphasized that dates of publication of memoirs do not always reflect accurately the time of discovery. The method used involved the variation of the expression for the work done in bending the bar. It is essentially that employed by Rayleigh in his treatise and leads of course to the well known equation of the fourth order in the space derivatives.

The corresponding analytical solution of the vibrations of a solid elastic plate came much later, though much experimental information was obtained in the latter part of the 18th century by the German E. F. F. Chladni (1756-1824), one of the greatest experimental acousticians. In 1787 he published his celebrated treatise "Entdeckungen über die Theorie des Klanges" in which he described his method of using sand sprinkled on vibrating plates to show the nodal lines. His figures were very beautiful and in a general way could be accounted for by considerations similar to those relating to vibrating strings. The exact forms, however, defied analysis for many years, even after the publication of Chladni's classic work "Die Akustik" in 1802. Napoleon provided for the Institute of France a prize of 3000 francs to be awarded for a satisfactory mathematical theory of the vibrations of plates. The prize was awarded in 1815 to Mlle. Sophie Germain, who gave the correct fourth order differential equation. Her choice of boundary conditions proved, however, to be incorrect. It was not until 1850 that Kirchhoff (1824-1887) gave a more accurate theory. The problem still provides considerable interest for workers even at the present time, both along theoretical

and experimental lines.

In the meantime the analogous problem of the vibrations of a flexible membrane, important for the understanding of the sounds emitted by drum heads, was solved first by S. D. Poisson (1781-1840), though he did not complete the case of the circular membrane. This was done by Clebsch (1833-1872) in 1862. It is significant that most of the theoretical work on vibration problems during the 19th century was done by persons who called themselves mathematicians. This was natural though perhaps somewhat unfortunate, since the choice of conditions did not always reflect actual experimentally attainable situations. Rayleigh's own work did much to rectify this condition, and nowadays the experimental and theoretical acousticians work hand in hand. The importance of this is evident in the design of such modern instruments as loud speakers and quartz crystal vibrators.

A more complete description of the historical development of sound producers would, of course, necessarily pay much attention to musical instruments. Unfortunately this development lay rather aside from the scientific progress in acoustics, a situation which has persisted in large measure even to recent times. There are signs, however, that the designers of new musical instruments are paying more attention to acoustical principles than previously was the case, and that the theory of acoustics will have a greater influence on music in the future than it has had in the past.

We have now brought our brief sketch of the production of sound up to the time of Rayleigh. We shall therefore proceed with the equally important problem of the propagation of sound.

The Propagation of Sound. From the earliest recorded observations there has been rather general agreement that sound is conveyed from one point in space to another through some activity of the air. Aristotle, indeed, emphasizes that there is actual motion of the air involved, but as was often the case with his notions on physics his expressions are rather vague. Since in the transmission of sound the air certainly does not appear to move, it is not surprising that other philosophers denied Aristotle's view. Thus even during the Galilean period the French philosopher Gassendi (1592-1655), in his revival of the atomic theory, attributed the propagation of sound to the emission of a stream of fine, invisible particles from the sounding body which, after moving through the air, are able to affect the ear. Otto von Guericke (1602-1686) expressed great doubt that sound is conveyed by a motion of the air, observing that sound is transmitted better when the air is still than when there is a wind. Moreover he had tried around the middle of the 17th century the experiment of

ringing a bell in a jar which was evacuated by means of his air pump, and claimed that he could still hear the sound. As a matter of fact, the first to try the bell-in-vacuum experiment was apparently the Jesuit Athanasius Kircher (1602-1680). He described it in his book "Musurgia Universalis", published in 1650, and concluded that air is not necessary for the transmission of sound. Undoubtedly there was not sufficient care to avoid transmission through the walls of the vessel. In 1660 Robert Boyle (1627-1691) in England repeated the experiment with a much improved air pump and more careful arrangements, and finally observed the now well known decrease in the intensity of the sound as the air is pumped out. He definitely concluded that the air is a medium for acoustic transmission, though presumably not the only one.

If air is the principal medium for the transmission of sound, the next question is: how rapidly does the propagation take place? As early as 1635 Gassendi while in Paris made measurements of the velocity of sound in air, using fire arms and assuming the passage of light as effectively infinite. His value was 1473 Paris feet per second. (The Paris foot is equivalent approximately to 32.48 cm.) Later by more careful measurements Mersenne showed this figure to be too high, obtaining 1380 Paris feet per second or about 450 meters/sec. Gassendi did note one matter of importance, namely that the velocity is independent of the pitch of the sound, thus discrediting the view of Aristotle, who had taught that high notes are transmitted faster than low notes. On the other hand Gassendi made the mistake of believing that the wind has no effect on the measured velocity of sound. In 1656 the Italian Borelli (1608-1679) and his colleague Viviani (1622-1703) made a more careful measurement and obtained 1077 Paris feet per second or 350 meters/sec. It is clear that all these values suffer from the lack of reference to the temperature, humidity and wind velocity conditions. It was not until 1740 that the Italian Branconi showed definitely by some experiments performed at Bologna that the velocity of sound in air increases with the temperature. Probably the first open air measurement of the velocity of sound that can be reckoned at all precise in the modern sense was carried out under the direction of the Academy of Sciences of Paris in 1738, using cannon fire. When reduced to 0°C the result was 332 meters/sec. Careful repetitions during the rest of the 18th century and the first half of the 19th century gave results differing by only a few meters per second from this value. The best modern value is 331.36 + 0.08 meters per second in still air under standard conditions of temperature and pressure (0°C and 76 cm of Hg. pressure).