ENSEMBLE MODELING

Inference from Small-Scale Properties to Large-Scale Systems

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Preface

To set this book in perspective, and to suggest the proper audience for it, we note that a particular modeling scheme is followed in several specific areas. This puts it in general systems theory, if that term is understood as the theory of midlevel modeling. This definition is essentially that proposed in 1956 by Boulding*, who described general systems theory as the study of modeling conducted at levels of abstraction lower than that of pure mathematics, but higher than those used in specific disciplines. Several other threads of emphasis lace through the work: (1) The modeling approach taken juxtaposes stochastic and deterministic processes; (2) we explicitly introduce an observer's perspective into the modeling process by the use of populations of models (called "ensembles" in what follows); and (3) we maintain a rather sharp representative model versus theoretical model distinction, coming down on the side of theory by our insistence on simple models. Hence we address ourselves especially to

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*K. Boulding, General systems theory--The skeleton of science. *Management Science, 2,* 197-208 (1956). Also found in J. Buckley (Ed.), *Modern Systems Research for the Behavioral Scientist*. Chicago: Aldine, 1968, pp. 3-10.

general systems theorists, modeling theorists, statisticians, and probabilists.

Since we work with switching nets, and do some combinatorics (on Boolean functions), readers with backgrounds in these areas might find something of interest --switching theorists primarily because we try to extend the interpretive reach that their models have.

This work is marked by our interests in the properties of formal models. Nevertheless, we devote a fair amount of effort to interpretation. Our persuasion is that the subject-matter specialist deserves at least some hints as to how the abstract systems examined here might be linked to the real world, however wide of the mark our efforts may turn out to be. We discuss an existing biological interpretation, and go on to suggest applications of ensemble-based network modeling in organizational and marketing contexts. In addition to these motives in presenting interpretive material, we have been impressed with the clarity that the effort of interpreting simple models seems to bring to the subject matter at hand, and for that reason recommend the exercise to substantive theorists.

The scope of our discussion is limited. We do not attempt to review all uses of network models. The bulk of the literature on graph theory, for example, is not of interest here. Our emphasis is on how locally specifiable properties of models influence overall model behavior. In this modeling the behavioral characteristics of the network elements themselves, Boolean transformations, are of focal importance.

Limited as our subject is, at present pieces of it appear in widely scattered sources. In this book we wanted to bring these pieces together, add to them where we could, but especially to provide a framework in which these separate contributions could be seen as more of an intellectual unity.

We have tried to make the mathematics used accessible to the nonspecialist. We introduce the mathematical section in such a way that only relatively basic tools in probability and mathematical statistics will suffice for understanding. Readers resolutely disinterested in mathematical details are encouraged to concentrate on the interpretive material. Such readers may find Chapters 1, 3, and 6 of most interest.

To summarize content by chapters: Chapter 1 states our general goals, then gives our view of modeling, pointing out that complex things, simple models, and ensembles go usefully together. In Chapter 2 we try to characterize more formally our ensemble approach by showing how it differs from conventional data-fitting approaches. We are not estimating parameter values in the traditional sense, but selecting plausible subsets of models. Chapter 3 introduces terminology and gives a rudimentary development of the models used, together with a preview of the application areas described later. Chapter 4 gives a technical development of the mathematics now available for deducing network behavior. Chapter 5 summarizes simulation results most pertinent for our discussion. Chapter 6 displays the potentially wide scope of a net ensemble modeling approach using examples in several substantive areas.

Finally, this manuscript is a testimonial to collaborative research work. We are two scientists with very different training who individually would have been limited in vision or in technical skill, but who have jointly flourished. Each of us made contributions in accordance with our expertise and in the end we hope the reader will find that a sound, coherent, provocative tract has emerged. Needless to say, we are solely responsible for any inaccuracies or errors.

We acknowledge the Office of Naval Research through Herbert Solomon at Stanford University, the Center for Real Estate and Urban Economic Studies at the University of Connecticut, and the University of Connecticut Research Foundation for partial support of this project.

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Introduction

1.1 Main Questions

Our overall purpose is to examine several broad questions in general systems theory, and to suggest areas of application for the approach used. The main questions are: In systems where details matter, what does limited specification of those details mean for the predictability of overall system dynamics? For example, do details of system structure (in a well-defined sense) matter? Is explanation of system dynamics decisively thwarted by lack of knowledge of underlying details? Could explanation perhaps be *aided* instead? If so, in what circumstances might that be? Can certain limitations on specification be usefully interpreted in the real world? Would *control* of system behavior make practical sense in such settings?

The success of statistical mechanics in, say, the study of gases suggests an answer to at least some of these questions. Predictability without using all details is, to a degree, possible. Explanation is also possible.

What we are doing here that is different is to show how a statistical approach to these questions can still be applied to systems in which their stochastic nature is less apparent than their determinism. In this book we are concerned with "nongaseous" complex systems, systems in which transactions among their many parts follow unchanging (but complicated and otherwise largely unknown) interaction paths. In addition to discussing an established application in biology, we intend to move this statistical method--an ensemble approach--into an important realm of internally organized and selective systems: the social realm.

Among the broad findings made using this approach is that some behavioral regularities are robust with respect to changing internal organizational details. Additionally, control methods and explanatory variables can be identified in situations where internal change, or an observer's ignorance of internal detail, may appear overwhelming.

More specifically, we first examine and extend an existing ground-breaking model of the genetic control system. It is argued (see, e.g., Kauffman, 1974) that the biochemical identity of a biological cell is specified by the sequential activity of the genes within it. These activities are determined by the genes themselves through the intricate interaction pattern by which they repress or "derepress" one another's productions. Approximating this genetic apparatus with a binary switching net model, and studying the properties of the model, one can directly give reasons why it is advantageous, in the evolutionary sense, for each gene to be affected by only a very limited number of other genes, why tissue types differentiate into no more than a half-dozen or so of other tissue types, why genetic activities are largely homeostatic, and why the number of tissue types increases as a fractional power of the number of genes in an organism.

In the managerial realm we argue that in some organizations, control mechanisms can usefully be seen as switching nets. So construed, we are able to present evidence that the productive routine of such organizations can be maintained in a wide variety of structural circumstances, provided that certain loosely specified constraints on other aspects of organizational detail obtain. We will suggest parallels between characteristics of our generic models (switching nets) and existing or potential real-world management principles. Those we examine are: span of control, the exception principle, the scalar principle of management, and consensus-level management. Using our modeling approach we are able to find relationships among and move toward quantification and theoretical clarification of these management principles. We point out, for example, how the general utility of the exception principle can be explained in this context. We examine the interesting question of how organizations built only by reference to small scale detail should differ from organizations constructed on a fully planned basis. We show how psychological constructs related to organizational climate might be handled in our scheme, and, in a frankly speculative effort, suggest the possible psychological effects of the managerial control styles with which we are dealing.

Moving our model networks into a marketing and advertising context where nets are seen as groups of interacting consumers, we show how changes in net characteristics can be interpreted as types of advertising message content. Those we define are: primary persuasion, imitative persuasion, and consensus persuasion. We then examine the theoretical effects that advertising campaigns making use of these different persuasive thrusts would have on (1) buyer group disposition toward an opinion target and on (2) those buyer groups' brand loyalty, as interpreted in our modeling scheme. Our model suggests, for example, that where relatively enduring enhancement of opinion is sought, an extended advertising campaign making use of consensus or imitative persuasion is desirable. Where a quick but possibly transient opinion boost would suffice, a short campaign making use of high intensity primary persuasion is indicated.

Since the network ensemble modeling approach we use may be unfamiliar, we begin with a general discussion of modeling as a way of introducing the specifics of our approach. The reader preferring to move directly into the model development may wish to proceed to Chapter 3. The balance of Chapter 1 considers the philosophy of modeling, while Chapter 2 clarifies the inferential framework of our modeling approach. In any event, the reader should feel free to read selectively in what follows.

1.2 Models

Dictionary definitions of "model" are surprisingly varied. The term can signify a small replica of something, a standard that should or may be followed, or something which is to be copied. Common usage allows the term to refer both to a copy of something and to something that is to be copied. Our focus will be on models in the first sense: as means by which things are duplicated in different forms.

It is significant that it is difficult to specify what a model is and what it is not. Part of the difficulty lies in the fact that a model is a model of something, and for some purpose. Burks (1975) points out that a model is a triadic relation involving that which is modeled, the model itself, and the purpose of the model. All three parts are needed for a complete specification. For example, is a 150-pound sack of sand a model? Not necessarily, of course, but it could be a model of a human being in a study of automobile dynamics. This is to say that a model isolated in a sense does not exist. It is just another thing: perhaps a collection of statements or equations.

An interesting illustration of this triadic relation in common use is provided by some riddles. In a riddle, a thing or a situation is given, usually in verbal form. The game can be to provide the unstated parts of the triad so as to make the thing become a model, and hence, to make sense. What is white and black, has one horn, and gives milk? What moves on four legs in the morning, two at noon, and three in the evening?*

Modeling is a very pervasive human activity. Any time we describe something, to ourselves or to someone else, we are modeling. Description provides a way in which the thing modeled can be brought into the social order in a controlled, accessible, and transferable manner. That is, a model of something, its description-for-some-purpose, provides for selective, focused perception of the thing in forms that allow symbolic manipulation, both public and private. In short, a model allows us to discuss the thing, with others, or with ourselves. Modeled, the thing can participate in a variety of public processes. In this form, our perceptions of it can be debated and thereby sharpened or broadened. Finally, in this form the thing can be understood. That description provides such benefits is obvious. That descriptions of things are possible is one of the true

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*A milk truck and a man, respectively. The first is a children's riddle; the second is from Greek mythology. In these riddles the modeling purpose appears to be description: in the first case a simple visual description is intended, in the second, a less obvious temporal relationship. Clearly, an important part of the entertainment function of riddles lies in an apparent misspecification of the original by the stated model.

natural wonders.

Description involves abstracting, selecting features for emphasis from among the myriad of possibilities inherent in the thing itself. But abstractions, to be communicated, must be expressed in some way. Hence a model may be an artifact (a toy train, a set of equations on a chalkboard) or a natural object (a tree offered as a model of a river). In either case, whether constructed or found, the thing offered as a model serves as a model by virtue of its expression of some abstracted content. (For an enjoyable illustration of some geometrical similarities, see Stevens, 1974.) Models, then, have binary nature. They are both real things and abstractions.* Take the example of a tree offered as a model of a river. The abstractions being carried may be obscured by being left unremarked on. The model clearly has other features. Are the leaves of interest, the roughness of the bark, birds' nests? In the present example, it is surely a two-dimensional representation of the tree's branching pattern that is being pointed to. Moreover, it is almost certainly the case that what is common among the patterns of many trees is being remarked on as resembling what is common in many river systems' branching patterns. Some models' abstracted content is more implied than explicit, and

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*This can be a matter of some importance in psychopathology. "The schizoid confusion of symbols with the thing symbolized . . ." (Murphy, 1967, p. 424) may be relevant in this discussion of models, in that models, while being representative, can also be used as symbols (which ordinarily need not decisively resemble the thing symbolized). The schizoid person, owing to difficulty in social functioning, may not perceive the purposive aspect of models. That is, he or she may fail to separate model and original because of an inability or refusal to deal with the other-person aspect of models: hence a person who speaks in riddles.

further, there may also exist the implication of an abstractive process that is statistical in nature.

1.3 Models Versus Theories

Modeling or theorizing: which are we doing? Both, it turns out, but perhaps more of the second than the first. This should be explained, since the terms "model" and "theory" are often used interchangeably (Simon and Newell, 1956).

Models of real things are abstractions, simplifications of reality. So are theories, in our view. In that both are representative simplifications, models and theories are similar. It is possible that a set of abstractions might be at the same time a (representative) model and a theory, depending on the purpose for which the set is being used at the given time. From our point of view, however, they are importantly different. What distinguishes a theory from a model is the analyst's purpose. Theories are meant to be understood: to explain something. Models are meant to be apprehended: to display something. What is to be displayed may be as routine as a simple physical appearance, or as complex as a intricate behavioral sequence. The best model, best in the sense of resembling the original, is an arbitrarily close duplicate of the original. That is, the "best" model is a set of abstractions elaborated to the point where no differences can be detected between the model and the original. But in being virtually indistinguishable from the original, it has at the same time contracted virtually all the complexity of the original as well. It has become useless for transmitting an explanation of the original, and therefore, it is now as bad a theory of the original as can be devised. To the extent that a model is only required to mimic, while a theory is additionally supposed to