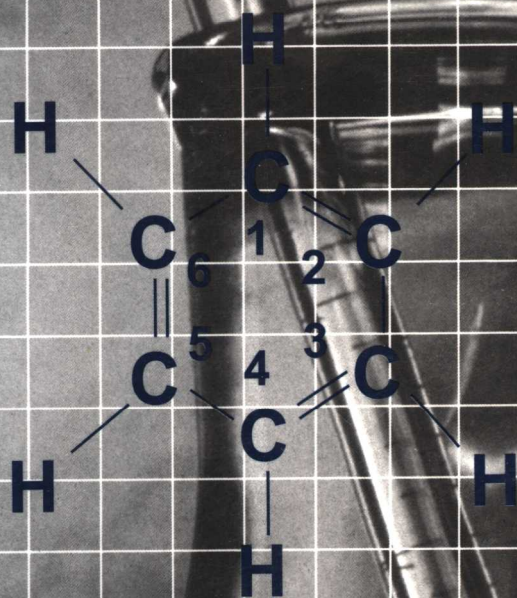


Fourth Edition

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Chemical Skills

Fourth Edition

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Chemical Skills

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PREFACE

The purpose of Chemical Skills is to be to students a "tutor" that is always available, at any time and at any place, during the early stages of learning chemistry. The course's reputation as a "difficult" course is well known. We who teach it know that the "difficulty" often lies not in chemistry, but in weak mathematical skills or not knowing how to translate a problem into a workable calculation setup. A thousand-page textbook does not have space to develop basic skills for its readers. Nor does it have room to guide students gradually through the problem-solving methods that arise in the course. This book is designed to overcome both of these obstacles to starting out successfully in chemistry.

Chemical Skills addresses first poor mathematical skills. Part I, the first five chapters, ranges from a review of basic arithmetic and algebra to graphing and logarithms. These chapters will be used selectively, depending on the level of the math background, whether or not graphing skills are needed in the laboratory, and whether or not the course reaches a need for logarithms.

Chapters 2 and 3 cover mathematics and its application to measurement. Chapter 4 introduces an approach to solving chemistry problems that is used throughout the book. It includes the usual advice found in most textbooks, but we formalize that approach by "analyzing" each problem. It also "explains" quickly and in a minimum of space how example problems are solved. More about that in a moment.

Part II, chapters 6 through 13, begins with the basics of atoms, elements, chemical formulas, equation writing, and calculations based on chemical formulas and equations. Part II then concludes with a more advanced treatment of atoms, and the fundamentals of gas laws and thermochemistry. Lack of understanding of these basic chemical skills presents a barrier to successful problem solving which threatens students' survival in the chemistry course itself. Our purpose is to remove this barrier.

Part II, the remaining seven chapters, guides the student to skill in solving the problems that commonly appear a bit later in the course. In all of the above areas, Chemical Skills offers a more gradual, learning-oriented development than is possible in a textbook. Nearly all students benefit from such an approach, and for many it means survival itself.

In Chemical Skills, principles that underlie a particular skill are presented in the usual textbook format, but example problems are programmed. Rarely, when a student wants help in learning how to "do" something—solve a problem, write an equation, or perform any other act that employs what we have called a "chemical skill"—do we simply "show" him or her how to do it. More often we ask questions that will guide the student into doing it himself or herself, trusting that the act of doing will produce learning. That is what we have done in writing the programmed examples in this book. The success of this method is attested to by the large number comments received from users of earlier editions of this book and of other books that use programmed examples.

We referred above to a "formalized" problem solving approach. Most textbooks offer brief suggestions on how to solve problems. Writing down what is given and what is wanted is usually the first step. Many books do not complete that first step in their worked out examples. (We do it constantly.) Procedures vary after the given and wanted quantities are identified. In this book we recommend next a determination of which of two problem solving strategies are required by a specific example:

1. **Dimensional analysis** is used whenever the given and wanted quantities are related by one or more proportionalities. The necessary conversion factors are identified and a unit path is plotted.
2. **Algebra** is used if the relationship is fixed by a mathematical equation. In most cases, the equation is solved for the wanted quantity first. With more complicated equations, the given quantities—including units—are substituted into the original equation and the result calculated.

We are not so naive as to believe a student will write down the given and wanted quantities, followed by an equation or a list of conversion factors and a unit path for each problem. Nor do we want them all to be written, *unless the student is unable to get started on a particular problem*. (We sometimes ask that they be written when a new topic is introduced.) But we do want the student *always to think of these steps*, not necessarily as sequential "steps," but rather as a means of analyzing the problem. To encourage this, we show the analysis of nearly every problem example. Usually, when the steps are written, the pattern for solving the problem is obvious and no further explanation is needed.

Perhaps you would like to see how this approach is developed. It first appears in Section 2.3, on page 36. Note particularly the last paragraph in the section. In Section 2.4 we introduce two other abbreviations that are used throughout the book. For problems solved by dimensional analysis, FACTOR identifies conversion factors and PATH refers to a unit path. A reminder of the dimensional analysis procedure appears in the middle of page 58, and again near the top of pages 149. The symbol EQN is first used on an equation-type problem on page 44. A reminder appears at the bottom of page 65, just before the next application for temperature conversions.

We have placed our description of the structure of Chemical Skills in a special "To the student" section entitled HOW TO LEARN CHEMISTRY FROM THIS BOOK. It follows this preface. Note particularly the instructions on how to solve programmed examples on page iv.

You might find it interesting to see the programmed format as a student sees it by a few minutes of role playing. If so, tear out one of the periodic tables to be used as a shield and reference source for atomic masses. A glance at the text just above Example 8.5 (page 152) will show you the form in which molar mass calculations have been developed. Then try the example, looking up the atomic mass on the periodic table and writing the needed information in the book. Example 8.8 (page 155) gives you a chance to apply molar mass to a mol \rightarrow g conversion just as a student will do it for the first time. Continuing through several examples, here or elsewhere in the book, shows how the student is guided into learning by doing.

Among the nicer things about writing a chemistry book are the comments and suggestions that come from instructors who review the manuscript. It is reassuring when a review praises what has been written; it is, perhaps, even more valuable when it does not. We are challenged to compare our time-honored ways with something someone else thinks is better. Sometimes we adopt the new method, and sometimes we stay with the old. Either way, we benefit from the challenge and the re-evaluation. We want to express our sincere appreciation to the following who have contributed to this edition in this way:

Ildy Boer	County College of Morris, NJ
Robert Ouellette	Ohio State University
Jean Shankweiler	El Camino College
Eric Snyder	Arcadia, California
Danny White	American River College

In addition, we greatly appreciate the assistance and encouragement of Karen Hughes, our faithful editor at McGraw-Hill.

Edward I. Peters

William T. Scroggins

To the student:

HOW TO LEARN CHEMISTRY FROM THIS BOOK

¹tutor n: . . . a private teacher . . .

²tutor vt: . . . to teach or guide usu. individually in a
special subject for a specific purpose . . .

Webster

Meet your tutor: this book that you hold in your hands. This is not as impersonal as it seems. This book expresses in print what we, the authors, have given to students like you when they have been stuck on some problem or skill as they are beginning to learn chemistry. Even though we cannot meet you personally, we hope you will let us help you as we have helped others in one-to-one tutoring sessions.

A tutor can help you to learn, but it is you who must do the learning. All the studying that you do and all the assignments you complete are wasted if you do not *learn* the material. Demand of yourself that you *learn* how to do what is required.

This book has been designed to help you learn. It does this by guiding you into putting into practice what you are learning while you are learning. If you use the book as it is intended—if you work the examples as suggested on the next page—you will learn more chemistry in less time. Your reward will be better test scores and more time for fun and games. Enjoy both; you will have earned them. Here are some of the features of this book that will help you to learn:

Prerequisites Most chapters open with a list of things you are expected to know or be able to do before you study the present chapter. These are things which we expect that you learned in an earlier chapter. Section references are given so you can check back if you need a refresher in a particular skill. And do check back when necessary. You will not learn something new if you have not already mastered what you need for the new learning.

Chemical Skills This is a list of things you will learn how to do when you study this chapter. Use this list as a preview of the chapter so you will know where to focus your attention as you reach different topics in your study. When you finish that topic, return to the particular chemical skill and ask yourself, "Can I do that?" If the answer is truly yes, go on. If not, go back to the section and study it some more. Solve more end-of-chapter problems. Do whatever else is necessary for you to be able to look again at that chemical skill and say with confidence, "I can do that." Then, and only then, have you *learned* that topic.

Text and Examples Here's where the actual learning takes place. We'll look at this item more closely in a moment.

End-of-Chapter Questions and Problems The only way you can be sure you have learned how to do something is to do it. The end-of-chapter questions and problems give you that opportunity. About two-thirds of the questions are answered in the back of the book. Answers to problems include complete calculation setups. Your instructor has answers to the remaining questions. The answered questions appear first, and they are separated from the unanswered questions by a bar:

+++++

Additional Problems You will occasionally want to challenge your chemical skills with more advanced problems. The questions in this section require you

to apply your knowledge to new situations which may not have been covered with an example in the chapter.

Chapter Test This is a list of questions such as those which might appear on a test based on the chapter. We recommend that you answer these questions under test conditions. That is, use only a pencil, paper, calculator, and periodic table. Do not look back to the text or examples for help. Answers are in the back of the book.

Now let's look more closely at the **Text and Example** section of each chapter, the place where the real learning occurs. To save you time, or to learn more in less time, we strongly urge you to use this section in the following way. To do this you will need four things: a pencil, a calculator, an opaque shield, and a periodic table for references. If you must buy a calculator, you will be interested in the discussion of calculators in Chapter 1. We have combined the shield and the periodic table as a tear-out card elsewhere in this book. On the side opposite the periodic table is a summary of the instructions you are about to read.

After being given the theory or technique behind a particular skill, you will practice that skill immediately in blank spaces in the book itself. These practice spaces are identified as **Examples**, set apart like this:

When you reach such a point, you should glance down the page until you find a pair of T-bars on each side:

This is where the opaque shield is placed—just below the T-bars so it covers the printing beneath that point. Read the question or example. Usually, it is followed by some comment or suggestion about how to proceed. Follow that suggestion, writing the answer or solving the problem in the space provided.

When this is done, move the shield down to the next set of T-bars, or the end of the example if there are no more T-bars. The first thing you expose on moving your shield down is the correct response to the question you have just answered. Compare your answer with the one in the book. If they are the same, as they will be most of the time, proceed as the book directs. If your answer and the book answer are not the same, find out why. Usually the comments that accompany the answer will be the only explanation you will need. If not, restudy the text material preceding the problem, or any other earlier material you may not have understood that is responsible for your incorrect answer. When you have corrected whatever needs to be corrected, proceed to the rest of the example. If you have reached the end of the example, it will close like this:

The importance of your active participation in solving example problems this way cannot be stated strongly enough. It is the key to *learning* chemistry from this workbook. Remember to demand that you master each step in the book. Satisfying that demand is the difference between merely *doing* your homework and *learning* chemistry—and the only thing that counts is what you learn.

Edward I. Peters

William T. Scroggins

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PART I. MATH AND PROBLEM SOLVING SKILLS

1. CALCULATORS AND MATH REVIEW

1.1 INTRODUCTION

Chemistry is a quantitative science. You, as a student, will use solutions prepared from chemicals measured in the storeroom, and you will measure out quantities of different chemicals in performing experiments. Part of studying chemistry is learning how to calculate the quantities that are needed and the quantities that will be produced. To do this, you must know how to perform certain math operations. In most cases this will mean knowing how to use your calculator.

If you are about to buy a calculator, you will find many from which to choose. A "scientific" calculator that is acceptable for lower division chemistry courses will be able to do all of the following:

MATH OPERATION	KEY SYMBOLS
(a) addition, subtraction, multiplication, and division;	$+$, $-$, \times , \div
(b) work in exponential notation;	EE or EXP
(c) do base 10 logarithms and antilogs;	$\log x$, INV $\log x$ or 10^x
(d) raise any base to any power;	y^x
(e) find reciprocals (inverses);	$1/x$
(f) find squares and square roots; and	x^2 , \sqrt{x}
(g) work with natural (base e) logarithms;	$\ln x$, INV $\ln x$ or e^x

A calculator with the above capabilities will also have trigonometric functions that are used in physics courses. One or more memory storages are also desirable.

Two types of operating systems are in common use: the Algebraic Operating System (AOS), used on most brands of calculators, and the Reverse Polish Notation (RPN), used mainly on Hewlett-Packard calculators. Note that the order of operations (addition, multiplication, etc.) is the same in both operating systems, but there is a difference in the order in which numbers and operations are keyed. Calculators also differ in the form of the display, particularly in the number of digits displayed. In this chapter, displays will show up to eight digits.

This chapter is limited to the mathematical operations and corresponding calculator techniques required to solve the problems in this book. After each math operation is presented, specific directions will be given for the calculator techniques to solve problems using that math operation. We suggest that you perform each operation as you read it. If any operation cannot be performed as described, consult the instruction manual that came with your calculator.

1.2 ENTERING A NUMBER

To introduce a number into your calculator, simply press the number keys in their proper order. This includes the decimal. If the number is negative, press the +/- key (sometimes identified by CHS or some other symbol) after the last digit. The number will appear in the display window of the calculator as it is entered.

Some calculators permit you to "fix" the number of digits that are displayed after the decimal point. Or you can allow the decimal to "float," that is, to show only the minimum digits needed to display the answer.

Most calculators can display eight or more digits. Very small numbers, with more than eight digits to the right of the decimal, such as 0.000000834, may have some of the digits lost in the display. This may cause the number to appear as 0.0000008, for example.

Numbers that are quite large or quite small should be entered in **exponential notation**, also known as **scientific notation**. This shows the number as the product of a coefficient, N , and an exponential, 10 raised to some integral (whole number) power, x , as $N \times 10^x$. In the standard form of exponential notation, N is equal to or more than 1, but less than 10. If you enter N outside of this range, the calculator adjusts the display to this form. Exponential math is discussed in detail in Section 1.4.

The procedure for entering a number in exponential notation is:

1. Type the coefficient, N . If the number is negative, type +/-.
2. Type EE (or whatever key is used for entering exponents).
3. Type the exponent. If the exponent is negative, type +/-.

Note: Most calculators will display an entered number in exponential notation only if (a) the calculator has been changed from "FIX" mode to "SCI" or "EXP" mode, or (b) if the entered number is too large or too small to fit the eight-digit display.

EXAMPLE 1.1: Enter the following numbers into your calculator:

10^4 ; 5.6709×10^{-8} ; -9.87×10^6 ; -5.43×10^{-2} .

10^4			5.6709×10^{-8}		
Enter	Press	Display	Enter	Press	Display
1	EE	1 00	5.6709	EE	5.6709 00
4		1 04	8	+/-	5.6709-08
-9.87×10^6			-5.43×10^{-2}		
Enter	Press	Display	Enter	Press	Display
9.87	+/-, EE	-9.87 00	5.43	+/-, EE	-5.43 00
6		-9.87 06	2	+/-	-5.43-02

Did you get 100,000 or 10^5 instead of 10,000 or 10^4 ? Enter exponential numbers that don't have coefficients as " 1×10^x " not as " 10×10^x ." Were your displays in decimal rather than exponential notation? Were digits missing? When some calculators run out of display space for a very small decimal number, they show only the zeros after the decimal, or perhaps the first one or two nonzero digits. The calculator carries the other digits, but hides them from view. Correctly calculated answers are displayed incorrectly. Be sure to change to exponential notation so that what you see is acceptable.

1.3 INVERSE

A common math operation is to take the **inverse** or **reciprocal** of a number. For any number x the inverse of that number is $1/x$, sometimes written x^{-1} . (In the expression x^{-1} the "-1" is the exponential of x . Exponentials are explained in the next section.)

To find an inverse on a calculator, enter the number whose inverse you desire, and push the $1/x$ key. For example, the inverse of 40 is $1/40 = 0.025$.

1.4 POWERS AND ROOTS: EXPONENTIALS

An **exponential** is a number in which a base, y , is raised to a power, x , as in y^x . Our number system is a base 10 exponential system. For example, $1000 = 10^3$ and $0.01 = 1/100 = 10^{-2}$. In the expressions 10^3 and 10^{-2} , 10 is called the **base**, and the superscripts 3 and -2 are the **exponents**, or **powers**, to which the base is to be raised. Two types of operations involve exponentials: raising a number to a power and taking the root of a number.

Raising a Number to a Power To square a number, multiply that number by itself. The square of 3 is $3 \times 3 = 3^2 = 9$. If an exponent is a **positive integer**, it represents the number of times the base is to be multiplied by itself. When a base is multiplied by itself three times, the result is the cube of the number. For example, "2 cubed" is the same as "2 to the third power" or: $2^3 = 2 \times 2 \times 2 = 8$. If an exponent is a **negative integer**, it is the inverse of the power of that integer: $5^{-2} = 1/5^2 = 1/25 = 0.04$.

Finding the Root of a Number If one mathematical operation is exactly the opposite of another, they are called **inverse operations**. The inverse of squaring a number is finding the square root of a number. The square root of X is the number that, when multiplied by itself, will give X as the product. A positive number has two square roots, one positive and one negative. The square root of 9 is +3 or -3, because each number, when multiplied by itself, gives 9 as the product: $(+3)(+3) = 9$, and $(-3)(-3) = 9$. There are no occasions in this book to use the negative root of any number. Hereafter we will disregard such roots without comment.

There are two ways to write the extraction of a root of a number. One is to use the square root symbol, $\sqrt{}$. If the cube root is desired, or any other n -th root, that number is written as a superscript in front of the root symbol: $\sqrt[n]{}$ for the third root, and $\sqrt[n]{}$ for the n -th root.

A more general way to represent the extraction of a root is to write it as an exponential. The base is the number whose root is to be found, and the exponent is the fraction $1/n$, where n is the desired root. The exponent may be written either as a typical fraction, or as a decimal fraction. Accordingly, the square root of 25 and the fifth root of 32 are

$$\sqrt{25} = 25^{1/2} = 25^{0.5} = 5$$

$$\sqrt[5]{32} = 32^{1/5} = 32^{0.2} = 2$$

If your calculator has a "y^x" key, that key can be used to find the value of any base raised to any power. The general procedure is

1. Enter the base, y, the number that is to be raised to a power.
2. Press y^x.
3. Enter the exponent, x, the power to which the base is to be raised.
4. Press =.

Note that, in the answers to the examples that follow, the display on your calculator may not be the same as that shown in the book. When working with exponentials, some calculators display all answers in exponential notation, while others use exponential notation only if the number of digits is more than the calculator can display. Also, the number of digits the calculator can show varies among different brands.

EXAMPLE 1.2: Calculate: $123^2 = \underline{\hspace{2cm}}$; $\sqrt[3]{123} = \underline{\hspace{2cm}}$;

$5.67^{0.25} = \underline{\hspace{2cm}}$; $(8.91 \times 10^{-2})^{3.4} = \underline{\hspace{2cm}}$.

123 ²			$\sqrt[3]{123} = 123^{1/3}$		
<u>Enter</u>	<u>Press</u>	<u>Display</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
123	x ²	15129	123	y ^x	123
(or y ^x , 2, =)			0.33333333	=	4.9731899
5.67 ^{0.25}			(8.91 × 10 ⁻²) ^{3.4}		
<u>Enter</u>	<u>Press</u>	<u>Display</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
5.67	y ^x	5.67	8.91	EE	8.91 00
0.25	=	1.543106	2	+/-, y ^x	8.91-02
			3.4	=	2.689-04

1.5 ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION

Addition and Subtraction When one quantity is to be added to another, they are written with a plus sign between them, as $12.84 + 6.93$. The result of an addition is the **sum**, or **total**.

When one quantity is to be subtracted from another, they are written with a minus sign between them, as in $12.84 - 6.93$. The result of a subtraction is the **difference**. In effect, subtraction is the same as the addition of a negative number: $12.84 + (-6.93) = 12.84 - 6.93$. Unless there is a reason for doing otherwise, the addition of negative numbers will always be written as a subtraction.

The subtraction of a negative number is the same as the addition of a positive number. The operation: $14.28 - (-2.71)$ is the same as the operation: $14.28 + 2.71$. Unless there is a reason for doing otherwise, the subtraction of a negative number will always be written as an addition.

Multiplication Quantities that are to be multiplied by each other are called **factors**. Multiplication may be indicated in several ways. One way is to write a multiplication sign, \times , between them, as 12.84×6.93 . Another way is to enclose the factors in parentheses, as $(12.84)(6.93)$. In algebra, where letters are used to represent numbers, factors may be written side-by-side: ab means $a \times b$ or $(a)(b)$. Sometimes a raised dot is used to indicate multiplication: $ab = a \cdot b$.

When a numerical factor is multiplied by a letter factor, the numerical factor is called the **coefficient** of the letter factor. In $12.3m$, 12.3 is the **coefficient** of m . The result of a multiplication is called the **product**. The sign, positive or negative, of a product is governed by the following rule:

A product is positive if both factors have the same sign—both positive or both negative; a product is negative if one factor is positive and the other factor is negative.

Division In almost all mathematical problems you will encounter, a division such as $12.84 \div 6.93$ will be written as a fraction such as

$\frac{12.84}{6.93}$ or $12.84/6.93$ The number above the line is the **numerator**.
The number below the line is the **denominator**.

The result of a division is a **quotient**. The sign of a quotient is set by the following rule:

A quotient is positive if the numerator and denominator have the same sign, either positive or negative; a quotient is negative if the numerator and denominator have different signs.

The general procedure on an AOS calculator for the one-step addition, subtraction, multiplication, or division of two numbers, A and B, is

1. Enter A.
2. Press required function key, $+$, $-$, \times or \div .
3. Enter B.
4. Press $=$.

EXAMPLE 1.3: Calculate: $12 + 345 = \underline{\hspace{2cm}}$; $12 - 345 = \underline{\hspace{2cm}}$;
 $12 \times 345 = \underline{\hspace{2cm}}$; $12 \div 345 = \underline{\hspace{2cm}}$.

12 + 345			12 - 345		
<u>Enter</u>	<u>Press</u>	<u>Display</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
12	+	12	12	-	12
345	=	357	345	=	-333

12 × 345			12 ÷ 345		
<u>Enter</u>	<u>Press</u>	<u>Display</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
12	×	12	12	÷	12
345	=	4140	345	=	0.0347826

1.6 LOGARITHMS AND ANTILOGARITHMS

Math operations with logarithms and antilogarithms, commonly called logs and antilogs, are sufficiently complicated to merit their own chapter, Chapter 5. Logs and antilogs are found on a calculator by entering a number and just pushing one key. The procedure for performing these operations is outlined below.

1. Enter the number x.
2. Press log x for the logarithm of x. Press 10^x (or INV log x) for the antilogarithm of x.

Natural logarithms, which use the base "e" instead of base 10, are found with the calculator keys $\ln x$ and e^x (or INV $\ln x$). See Chapter 5 for a discussion of natural logarithms.

EXAMPLE 1.4: Calculate: $\log 123 = \underline{\hspace{2cm}}$; $\text{antilog } -2.32 = \underline{\hspace{2cm}}$;

$\log 123$			$\text{antilog } -2.32$		
<u>Enter</u>	<u>Press</u>	<u>Display</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
123	log x	2.0899051	-2.32	10^x or INV log x	0.0047863

1.7 CHAIN CALCULATIONS

A chain calculation is one in which two or more operations are performed, one after the other. The order in which the operations are done may be critical. Most AOS calculators perform simple calculations in the proper order when numbers and operations are entered exactly as they appear in the calculation setup. (This is not true with RPN calculators.) The calculator automatically multiplies and/or divides before adding and/or subtracting. For example, when your calculator computes $0.5054 \times 78.92 + 0.4946 \times 80.92$, it first multiplies 0.5054 by 78.92. Then 0.4946 and 80.92 are multiplied. Finally the two products are added together.