New Trends in Magnetism

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Recife, Brazil 26-28 July 1989

Editors

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PREFACE

The meeting **New Trends in Magnetism** took place at the Departamento de Física, Universidade Federal de Pernambuco (UFPE) in Recife on July 26-28, 1989. It was attended by about a 100 participants from Argentina, Brazil, Canada, France, Germany, Israel, Italy, Japan, Norway, South Africa, Sweden, Switzerland, United Kingdom, USA and USSR.

One motivation for the meeting was the 17th International Conference on Thermodynamics and Statistical Mechanics - STATPHYS'17 - held in Rio de Janeiro in the following week. The other motivation was the fact that Magnetism and Statistical Mechanics have been major topics of research at UFPE for more than fifteen years.

Our first research group in Recife was created in 1971, and magnetism was the main subject of interest, both theoretically and experimentally. Along the years we have seen an impressive development in this field, which led Magnetism to occupy a very special place in Condensed Matter Physics. Among typical problems which have attracted the attention of a great number of investigators, one could mention: Kondo Effect, Critical Phenomena, Spin Glasses and Applications, Low Dimensional and Random Field Effects, Heavy Fermions and, very recently, the interplay between Magnetism and Superconductivity in the High T_c Copper Oxide Systems. These beautiful examples have provided not only great challenges to the imagination of Condensed Matter Physicists, but also unique systems where many ideas and concepts of Statistical Mechanics and Field Theory, such us symmetry breaking, scaling and universaliyty, have been successfully tested. In Recife we have done our best to participate in these exciting developments.

The meeting had only plenary lectures organized in the following sections: Random Systems - Spin Glass and Random Field Systems; Neutral Networks; Low Dimensional and Correlated Electron Systems; Spin Dynamics; Cellular Automata; Critical Phenomena. In addition to those appearing in this book, there were also lectures by A. Bishop, A. Caldeira, C. di Castro, J. Kosterlitz, N. Sourlas P. Wiegmann, M.E. Fisher, K. Furuya and K. Lun Yao.

We are very happy that the meeting provided a very good opportunity to enhance the interaction of Brazilian Physicists with their colleagues from all over the world.

NEW TRENDS IN MAGNETISM

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CONTENTS

STABILITY STUDY OF ISING SPIN GLASSES WITH FINITE CONNECTIVITY

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ABSTRACT

In this work we show that a class of Ising spin glass models with finite connectivity exhibits instability when replica symmetry is not broken in the condensed phase at low temperatures. These models are relevant for complex optimization problems and may also give important insights about real spin-glasses.

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1. INTRODUCTION

The study of dilute spin-glass systems with finite connectivity within the framework of mean-field theory has been undertaken recently by various workers [1-5] both for some new physics is expected to emerge from the "short-range character" of the model and its connection to the graph bipartioning problem. At finite temperatures the system was first studied by Viana and Bray [1] and close to the critical temperature T_c the system is amenable to the same perturbation expansion used in the Sherrington-Kirkpatrick model where a single family of order-parameters $q_{\alpha_1\alpha_2}$ $\alpha_i=1,\dots,n$ is sufficient to describe the transition. At low temperatures T the problem becomes highly non-standard for a whole new family of order parameters $q_{\alpha_1\alpha_2\dots\alpha_r}$, $r=1,2\dots$ has to be taken in account and one is led to introduce a global order parameter $G(\{\sigma_\alpha\})$ [4]. The properties of the model can be written in terms of the averaged local field distribution P(h) and finding the solution for the P(h) equation of motion at T=0 seems to be the most convenient approach and is directly related to solving for the global order parameter function $G(\{\sigma_\alpha\})$ [4]. In this work we study the replica symmetric (RS) solution for $G(\{\sigma_\alpha\})$ and the stability of this solution is carefully analysed. In our previous work [6] we extended the solution found by Morita [7] and Katsura [8] for the spin glass on a Bethe lattice, who used a P(h) with both a continuous part and delta-function peaks at integral multiples of J, to our system and longitudinal stability was verified. Here we show that the complete (RS) stability analysis requires breaking of replica symmetry. The work is organized as follows: in section 2 the model is introduced and the relevant equations derived;in section 3 the stability analysis in terms of $G(\{\sigma_\alpha\})$ is carried out; section 4 discusses the results.

2. THE MODEL

We consider a system described by the Ising Hamiltonian

$$H = - \sum_{(ij)} J_{ij}\, \sigma_i\, \sigma_j \qquad (1)$$

where $\sigma_i = \pm 1$ and the bonds J_{ij}'s are infinite ranged random interactions described by the probability distribution

$$P(J_{ij}) = (1 - \frac{\alpha}{N})\delta(J_{ij}) + \frac{\alpha}{N} f(J_{ij}) \tag{2}$$

where α is the average connectivity of each site, N the number of sites and $f(J_{ij})$ is normalised to unity, for example,

$$f(J_{ij}) = a\ \delta(J_{ij}-J) + (1-a)\ \delta(J_{ij}+J) \tag{3}$$

a being the fraction of ferromagnetic bonds. Following standard procedure the averaged free energy is given by

$$-\beta F = \frac{\partial}{\partial n}(\overline{Z}^n)\Big|_{n=0} \tag{4}$$

where the replicated partition function averaged over (2) is given by

$$\overline{Z^n} = \mathrm{Tr}_{\sigma_\alpha} \exp\{\sum_{(ij)} \ln\left[1 + \frac{\alpha}{N}(< \exp(\beta J_{ij} \sum_\alpha \sigma_i^\alpha\, \sigma_j^\alpha)>_f -1)\right]\} \tag{5}$$

where <...>f means average over $f(J_{ij})$ equation (3). In (5) one writes

$$<\exp(\beta J_{ij}\sum_\alpha \sigma_i^\alpha\, \sigma_j^\alpha)>_f = \sum_{r=0} \sum_{(\alpha_1\alpha_2\ldots\alpha_r)} b_r(\sigma_i^{\alpha_1}\ldots\sigma_i^{\alpha_r})(\sigma_j^{\alpha_1}\ldots\sigma_j^{\alpha_r}) \tag{6}$$

with

$$b_r = <\cosh^n(\beta J_{ij})\mathrm{tgh}^r(\beta J_{ij})>_f \tag{7}$$

Substituting (6) in (5), and neglecting terms which vanish as $N\to\infty$,

$$\overline{Z^n} = e^{-\frac{\alpha}{2}(N-1)} \mathrm{Tr}_{\sigma_\alpha} \exp\{\frac{\alpha}{2N} \sum_{r=0}^{n} \sum_{(\alpha_1 \ldots \alpha_r)} b_r (\sum_i \sigma_i^{\alpha_1} \ldots \sigma_i^{\alpha_r})^2\} \tag{8}$$

Using a Hubbard-Stratonovich transformation (8) is rewritten as

$$\overline{Z^n} = e^{-\frac{\alpha(N-1)}{2}} \int \prod_r \prod_{(\alpha_1 \alpha_2 \ldots \alpha_r)} \frac{dq_{\alpha_1 \alpha_2 \ldots \alpha_r}}{\sqrt{2\pi}} \exp\{-N\Big[\frac{\alpha}{2} \sum_r \sum_{(\alpha_1 \ldots \alpha_r)} b_r q^2_{\alpha_1 \ldots \alpha_r}$$

$$- \ln \mathrm{Tr}_{\sigma_\alpha} \exp(\alpha \sum_r \sum_{(\alpha_1 \ldots \alpha_r)} b_r\, q_{\alpha_1 \ldots \alpha_r}\, \sigma^{\alpha_1} \sigma^{\alpha_2} \ldots \sigma^{\alpha_r})\Big]\} \tag{9}$$

Finally, the integrals in (9) can be done by steepest descent method yielding for the free energy per spin

$$n\beta f = \max\{\frac{\alpha}{2} (\sum_r \sum_{(\alpha_1 \ldots \alpha_r)} b_r\, q^2_{\alpha_1 \ldots \alpha_r}\, \sigma^{\alpha_1} \ldots \sigma^{\alpha_r})$$

$$- \ln \mathrm{Tr}_{\sigma_\alpha} \exp(\alpha \sum_\alpha \sum_{(\alpha_1 \ldots \alpha_r)} br\, q_{\alpha_1 \ldots \alpha_r}\, \sigma^{\alpha_1} \ldots \sigma^{\alpha_r})\} \tag{10}$$

The parameters $q_{\alpha_1 \alpha_2 \ldots \alpha_r}$ are determined from $\partial f / \partial q_{\alpha_1 \ldots \alpha_r} = 0$, i.e.,

$$q_{\alpha_1 \ldots \alpha_r} = \langle \sigma^{\alpha_1} \ldots \sigma^{\alpha_r} \rangle = \mathrm{Tr}_{\sigma_\alpha} \{\sigma^{\alpha_1} \ldots \sigma^{\alpha_r} e^{\beta H_{eff}}\} / Z_G \tag{11}$$

where βH_{eff} is given by the exponent in (10) and

$$Z_G = \mathrm{Tr}_{\sigma_\alpha} \exp(\beta H_{eff}) \tag{12}$$

For a replica symmetric ansatz the solution (11) has the form

$$q_{\alpha_1 \alpha_2 \ldots \alpha_r} = Q_r \tag{13}$$

and for this solution to be stable the matrix

$$\partial^2 f/\partial q_{\alpha_1\ldots\alpha_r}\partial q_{\alpha_1\ldots\alpha_e} \tag{14}$$

must have non-negative eigenvalues. However, (14) is not the most convenient way of analysing the stability of the solution (13).

3. GLOBAL ORDER PARAMETER AND STABILITY EQUATIONS

We are mainly interested in the limit T=0 where all $q_{\alpha_1\ldots\alpha_r}$ are important.The following global order parameter $G_n(\{\sigma_\alpha\})$ [4] encompassing all q's is introduced

$$G_n(\{\sigma_\alpha\}) = \alpha \sum_{r=0}^{n} \sum_{(\alpha_1\ldots\alpha_r)} br\, q'_{\alpha_1\ldots\alpha_r} \sigma^{\alpha_1}\ldots\sigma^{\alpha_r} \quad . \tag{15}$$

This global order parameter generalizes to spin systems the generating function introduced by Mézard and Parisi [9] and by Orland [10] for optimization problems. The equations derived in section 2 can all be expressed in terms of $G_n(\{\sigma^\alpha\})$.The variational free energy is given by

$$\begin{aligned} - n\beta f = \frac{\alpha b_o}{2} - \frac{\alpha}{2Z_G} \mathrm{Tr}_{\sigma_\alpha} \{G_n(\{\sigma^\alpha\}) e^{G_n(\{\sigma^\alpha\})}\} \\ + \ln \mathrm{Tr}_{\sigma_\alpha} e^{G_n(\{\sigma^\alpha\})} \end{aligned} \tag{16}$$

and the equation of motion for $G_n(\{\sigma^\alpha\})$ is

$$G_n(\{\sigma^\alpha\}) = \frac{\alpha}{Z_G} \mathrm{Tr}_{\tau_\alpha} \{e^{G_n\{\tau_\alpha\}} \ln g(\sum_\alpha \tau_\alpha \sigma_\alpha)\} - \alpha b_o 2^n \tag{17}$$

where

$$g(x) = \int dJ\, f(J) \exp(\beta J x) \tag{18}$$

and Z_G is given by (12). From (15) one may notice that for replica symmetric solution the global order parameter is a function only of $\hat{\sigma} = \sum_\alpha \sigma_\alpha$:

$$G_n(\{\sigma^\alpha\}) = G_n(\hat{\sigma} = \sum_\alpha \sigma^\alpha) \tag{19}$$

The stability equation is now [4]

$$\mathrm{Tr}_{\tau_\alpha}\{ \frac{\delta^2(\beta f(G_n))}{\delta G_n\{\sigma_\alpha\}\ \delta G_n\{\tau_\alpha\}} \varphi\{\tau_\alpha\}\} = \lambda\ \varphi\{\sigma^\alpha\} \tag{20}$$

and from (16) and (17) it becomes

$$\alpha^{-1}\ \varphi\{\sigma_\alpha\} + (\alpha Z_G)^{-1}\ G_n\{\sigma_\alpha\}\ \mathrm{Tr}_{\tau_\alpha}\{\varphi\{\tau_\alpha\}\ e^{G_n\{\tau_\alpha\}}\} - \tag{21}$$

$$- \mathrm{Tr}_{\tau_\alpha}\{(\lambda + Z_G^{-1}\ e^{G_n\{\tau_\alpha\}})\ g(\beta J\sum_\alpha \tau_\alpha\sigma_\alpha)\varphi\{\tau_\alpha\}\} = 0$$

In the replica symmetric case the elements of the eigenvectors [4] have the form

$$\varphi\{\sigma_\alpha\} = \varphi_{\{\mu_\alpha\}}\ (\hat{\sigma};\ q_\sigma) \tag{22}$$

with

$$\hat{\sigma} = \sum_\alpha \sigma_\alpha \tag{23}$$

$$q_\sigma = \sum_\alpha \sigma_\alpha\ \mu_\alpha \tag{24}$$

and $\{\mu_\alpha\}$ is a spin configuration labeling the eigenvector. Thus in the replica symmetrie ansatz the stability equation becomes

$$\varphi_{\mu}(\hat{\nu},q_{\nu}) = -\frac{G(\hat{\nu})}{Z_G}\, Tr_{\tau_\alpha}\{\varphi(\hat{\tau};q_{\tau})e^{G(\hat{\tau})}\} + \tag{25}$$
$$+ \alpha\, Tr_{\tau_\alpha}\{\varphi(\hat{\tau},q_{\tau})(\lambda_{\mu} + \frac{e^{G(\hat{\tau})}}{Z_G}) \int dJ f(J) e^{\beta J \sum_{\alpha} \tau_\alpha \nu_\alpha}\}$$

Our task now is to find the λ's from (25) using the known replica symmetric solution for $G(\hat{\sigma})$ obtained from equation (17) and which has been obtained in our previous work [6]. In the limit $n \to 0$, equation (17) becomes [4,6]

$$G(x) = \alpha \int_{-\infty}^{\infty} \frac{dy}{2\pi} K_a(x,y) e^{G(y)} - \alpha \tag{26}$$

$$K_a(x,y) = \int_{-\infty}^{\infty} dJ f(J) \int_{-\infty}^{\infty} du \exp\{ -iyu + ixtgh^{-1}\left[tghu\ tgh\beta J\right] \tag{27}$$

Demanding orthogonality of $\varphi_{\mu}(\hat{\nu}, q_{\nu})$ to the constant and longitudinal eigenvectors found previously [6], we get

$$Tr_{\tau_\alpha}\{\varphi_{\mu}\{\tau_{\alpha}\}\} = 0 \tag{28}$$

$$Tr_{\tau_\alpha}\{\varphi_{\mu}\{\tau_{\alpha}\}\ \varphi_{L}\{\tau_{\alpha}\}\} = 0 \tag{29}$$

and keeping the lowest order terms in $\exp[G(y)]$, equation (25) close to the percolation threshold ($\alpha_c = 1$) with $\alpha = 1 + \varepsilon$, $a=1/2$ yields

$$-(\lambda_{\mu}+\varepsilon)\varphi_{\mu}(\xi;\sigma) = \iint_{-\infty}^{\infty} K_m(\xi,\sigma;\eta,\tau) G(\eta+\tau)\varphi_{\mu}(\eta;\tau)\, \frac{d\eta\, d\tau}{(2\pi)^2} \tag{30}$$

$$K_m(\xi,\sigma;\eta,\tau) = K_m(\xi,\eta)\ K_{-m}(\sigma;\tau) \tag{31}$$

where $\mu\beta J = 2mi$ and as $\beta J \to \infty$, we get

$$K_m(\xi,\eta) = \frac{2\sin(\eta-\xi)}{\eta-\xi} + e^{-i(\xi-\eta)}\left[\pi\ \delta(\eta+m)+i\ P(\frac{1}{\eta+m})\right]$$
$$+ e^{i(\xi-\eta)}\left[\pi\delta(\eta-m)-i\ P(\frac{1}{\eta-m})\right] \tag{32}$$

From [6] we already know the solution in lowest order for $G(\eta+\tau)$ near α_c

$$G(\eta+\tau) = a+b\cos(\eta+\tau) + c\ \ j_o(\eta+\tau) + \cdots \tag{33}$$

with $a = -16/9$, $b = c = 8/9$. From equations (30-33) we can infer a solution for $\varphi_\mu(\eta,\tau)$ in (30) and hence obtain the eigenvalues λ_μ. The eigenvectors must have the following form

$$\varphi_\mu(\eta,\tau) = A + B_+^+ e^{i\eta} + B_-^+ e^{-i\eta} + C_o^+ j_o(\eta)$$
$$+ B_+^- e^{i\tau} + B_-^- e^{-i\tau} + C_o^- j_o(\eta)$$
$$+ E_{++} e^{i(\eta+\tau)} + E_{+-} e^{i(\eta-\tau)} + E_{-+} e^{-i(\eta-\tau)} + E_{--} e^{-i(\eta+\tau)}$$
$$+ F_+^+ e^{i\eta} j_o(\tau) + F_-^+ e^{-i\eta} j_o(\tau) + F_+^- j_o(\eta) e^{i\tau} + F_-^- j_o(\eta) e^{-i\tau}$$
$$+ H\ j_o(\eta)\ j_o(\tau) \tag{34}$$

Substituting (34) in (30), together with the constraint equations (28) and (29) we obtain a set of homogeneous equations for the coefficients appearing in (34) and for λ_μ. Solving this system of equations yield the eigenvalues and eigenvectors. We defer to another publication [11] a more detailed derivation of the results published here. Now we just point out that one of the eigenvectors is

$$\varphi_o(\eta,\tau) = \sin(\eta+\tau) \tag{35}$$

with eigenvalue $\lambda=-\varepsilon/9$, as can be seen by substituting equation (35) in (30). This is the only negative eigenvalue we have found. It proves that the replica symmetric solution is unstable in the condensed phase. In our earlier work [6] we analysed the stability problem only in the longitudinal sector, i.e,one restricts the variations to the same space as that of the replica symmetric $G_n(\hat{\sigma})$ defined in equation (19) and so in (22) $\varphi_L = \varphi(\hat{\sigma})$. In the longitudinal sector all eigenvalues are positive [6], while in the transversal sector one eigenvalue is negative and so breaking replica symmetry is necessary.

4. DISCUSSION

It has been known for some time [12] that the long-range Sherrington-Kirkpatrick (SK) model [13] below T_c presents a broken replica symmetry (BRS) phase. This SK model, however, has all spins connected among themselves and may be argued not to be relevant to real spin-glass systems. Both phenomenological theories and exactly solvable models are important tools in understanding a given physical problem, but in the spin-glass case there is a deep difference between the two approaches [14,15]. Short-ranged Ising spin-glass on the Bethe lattice has also been shown to exhibit a BRS phase [16,17]. In this work we have shown that a class of diluted Ising spin-glass models with finite averaged connectivity [1] does exhibit a BRS at T=0. These systems are relevant to applications in complex optimization problems [2, 3,18].Our results show that previous replica symmetric ansätze [2,3,4, 19] even in their most general form [6] need to be modified by breaking replica symmetry. A BRS solution undertaken recently, limited to its first step of Parisi like breaking of replica symmetry [20], gives results in much close agreement with numerical simulation than the RS solution.

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