Quantum Mechanics

(Non-relativistic Theory)

Course of Theoretical Physics
Volume 3 Third Edition

QUANTUM MECHANICS

NON-RELATIVISTIC THEORY

by

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Translated from the Russian by

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FROM THE PREFACE TO THE FIRST ENGLISH EDITION

THE present book is one of the series on *Theoretical Physics*, in which we endeavour to give an up-to-date account of various departments of that science. The complete series will contain the following nine volumes:

1. *Mechanics.* 2. *The classical theory of fields.* 3. *Quantum mechanics (non-relativistic theory).* 4. *Relativistic quantum theory.* 5. *Statistical physics.* 6. *Fluid mechanics.* 7. *Theory of elasticity.* 8. *Electrodynamics of continuous media.* 9. *Physical kinetics.*

Of these, volumes 4 and 9 remain to be written.

The scope of modern theoretical physics is very wide, and we have, of course, made no attempt to discuss in these books all that is now included in the subject. One of the principles which guided our choice of material was not to deal with those topics which could not properly be expounded without at the same time giving a detailed account of the existing experimental results. For this reason the greater part of nuclear physics, for example, lies outside the scope of these books. Another principle of selection was not to discuss very complicated applications of the theory. Both these criteria are, of course, to some extent subjective.

We have tried to deal as fully as possible with those topics that are included. For this reason we do not, as a rule, give references to the original papers, but simply name their authors. We give bibliographical references only to work which contains matters not fully expounded by us, which by their complexity lie "on the borderline" as regards selection or rejection. We have tried also to indicate sources of material which might be of use for reference. Even with these limitations, however, the bibliography given makes no pretence of being exhaustive.

We attempt to discuss general topics in such a way that the physical significance of the theory is exhibited as clearly as possible, and then to build up the mathematical formalism. In doing so, we do not aim at "mathematical rigour" of exposition, which in theoretical physics often amounts to self-deception.

The present volume is devoted to non-relativistic quantum mechanics. By "relativistic theory" we here mean, in the widest sense, the theory of all quantum phenomena which significantly depend on the velocity of light. The volume on this subject (volume 4) will therefore contain not only Dirac's relativistic theory and what is now known as quantum electrodynamics, but also the whole of the quantum theory of radiation.

Institute of Physical Problems L. D. LANDAU
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August 1956

PREFACE TO THE SECOND ENGLISH EDITION

FOR this second edition the book has been considerably revised and enlarged, but the general plan and style remain as before. Every chapter has been revised. In particular, extensive changes have been made in the sections dealing with the theory of the addition of angular momenta and with collision theory. A new chapter on nuclear structure has been added; in accordance with the general plan of the course, the subjects in question are discussed only to the extent that is proper without an accompanying detailed analysis of the experimental results.

We should like to express our thanks to all our many colleagues whose comments have been utilized in the revision of the book. Numerous comments were received from V. L. Ginzburg and Ya. A. Smorodinskiĭ. We are especially grateful to L. P. Pitaevskiĭ for the great help which he has given in checking the formulae and the problems.

Our sincere thanks are due to Dr. Sykes and Dr. Bell, who not only translated excellently both the first and the second edition of the book, but also made a number of useful comments and assisted in the detection of various misprints in the first edition.

Finally, we are grateful to the Pergamon Press, which always acceded to our requests during the production of the book.

L. D. LANDAU
E. M. LIFSHITZ

October 1964

PREFACE TO THE THIRD RUSSIAN EDITION

THE previous edition of this volume was the last book on which I worked together with my teacher L. D. Landau. The revision and expansion that we then carried out was very considerable, and affected every chapter.

For the third edition, naturally, much less revision was needed. Nevertheless, a fair amount of new material has been added, including some more problems, and relating both to recent research and to earlier results that have now become of greater significance.

Landau's astonishing grasp of theoretical physics often enabled him to dispense with any consultation of original papers: he was able to derive results by methods of his own choice. This may have been the reason why our book did not contain certain necessary references to other authors. In the present edition, I have tried to supply them as far as possible. I have also added references to the work of Landau himself where we describe results or methods that are due to him personally and have not been published elsewhere.

As when dealing with the revision of other volumes in the *Course of Theoretical Physics*, I have had the assistance of numerous colleagues who informed me either of deficiencies in the treatment given previously, or of new material that should be added. Many useful suggestions incorporated in this book have come from A. M. Brodskii, G. F. Drukarev, I. G. Kaplan, V. P. Krainov, I. B. Levinson, P. E. Nemirovskii, V. L. Pokrovskii, I. I. Sobel'man, and I. S. Shapiro. My sincere thanks are due to all of these.

The whole of the work on revising this volume has been done in close collaboration with L. P. Pitaevskii. In him I have had the good fortune to find a fellow-worker who has passed likewise through the school of Landau and is inspired by the same ideals in the service of science.

Moscow E. M. LIFSHITZ
November 1973

NOTATION

Operators are denoted by a circumflex: $\hat{f}$

dV volume element in coordinate space

dq element in configuration space

d^3p element in momentum space

$f_{nm} = f_m^n = \langle n|f|m\rangle$ matrix elements of the quantity f (see definition in §11)

$\omega_{nm} = (E_n - E_m)/\hbar$ transition frequency

$\{\hat{f}, \hat{g}\} = \hat{f}\hat{g} - \hat{g}\hat{f}$ commutator of two operators

$\hat{H}$ Hamiltonian

δ_l phase shifts of wave functions

Atomic and Coulomb units (see beginning of §36)

Vector and tensor indices are denoted by Latin letters i, k, l

e_{ikl} antisymmetric unit tensor (see §26)

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Mechanics = Vol. 1 (*Mechanics*, third English edition, 1976).
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RQT or *Relativistic Quantum Theory* = Vol. 4 (*Relativistic Quantum Theory*, first English edition, Part 1, 1971; Part 2, 1974); the second English edition appeared in one volume as *Quantum Electrodynamics*, 1982.
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CHAPTER I

THE BASIC CONCEPTS OF QUANTUM MECHANICS

§1. The uncertainty principle

WHEN we attempt to apply classical mechanics and electrodynamics to explain atomic phenomena, they lead to results which are in obvious conflict with experiment. This is very clearly seen from the contradiction obtained on applying ordinary electrodynamics to a model of an atom in which the electrons move round the nucleus in classical orbits. During such motion, as in any accelerated motion of charges, the electrons would have to emit electromagnetic waves continually. By this emission, the electrons would lose their energy, and this would eventually cause them to fall into the nucleus. Thus, according to classical electrodynamics, the atom would be unstable, which does not at all agree with reality.

This marked contradiction between theory and experiment indicates that the construction of a theory applicable to atomic phenomena—that is, phenomena occurring in particles of very small mass at very small distances—demands a fundamental modification of the basic physical concepts and laws.

As a starting-point for an investigation of these modifications, it is convenient to take the experimentally observed phenomenon known as *electron diffraction*.† It is found that, when a homogeneous beam of electrons passes through a crystal, the emergent beam exhibits a pattern of alternate maxima and minima of intensity, wholly similar to the diffraction pattern observed in the diffraction of electromagnetic waves. Thus, under certain conditions, the behaviour of material particles—in this case, the electrons—displays features belonging to wave processes.

How markedly this phenomenon contradicts the usual ideas of motion is best seen from the following imaginary experiment, an idealization of the experiment of electron diffraction by a crystal. Let us imagine a screen impermeable to electrons, in which two slits are cut. On observing the passage of a beam of electrons‡ through one of the slits, the other being covered, we obtain, on a continuous screen placed behind the slit, some pattern of intensity distribution; in the same way, by uncovering the second

† The phenomenon of electron diffraction was in fact discovered after quantum mechanics was invented. In our discussion, however, we shall not adhere to the historical sequence of development of the theory, but shall endeavour to construct it in such a way that the connection between the basic principles of quantum mechanics and the experimentally observed phenomena is most clearly shown.

‡ The beam is supposed so rarefied that the interaction of the particles in it plays no part.

slit and covering the first, we obtain another pattern. On observing the passage of the beam through both slits, we should expect, on the basis of ordinary classical ideas, a pattern which is a simple superposition of the other two: each electron, moving in its path, passes through one of the slits and has no effect on the electrons passing through the other slit. The phenomenon of electron diffraction shows, however, that in reality we obtain a diffraction pattern which, owing to interference, does not at all correspond to the sum of the patterns given by each slit separately. It is clear that this result can in no way be reconciled with the idea that electrons move in paths.

Thus the mechanics which governs atomic phenomena—*quantum mechanics* or *wave mechanics*—must be based on ideas of motion which are fundamentally different from those of classical mechanics. In quantum mechanics there is no such concept as the path of a particle. This forms the content of what is called the *uncertainty principle*, one of the fundamental principles of quantum mechanics, discovered by W. Heisenberg in 1927.†

In that it rejects the ordinary ideas of classical mechanics, the uncertainty principle might be said to be negative in content. Of course, this principle in itself does not suffice as a basis on which to construct a new mechanics of particles. Such a theory must naturally be founded on some positive assertions, which we shall discuss below (§2). However, in order to formulate these assertions, we must first ascertain the statement of the problems which confront quantum mechanics. To do so, we first examine the special nature of the interrelation between quantum mechanics and classical mechanics. A more general theory can usually be formulated in a logically complete manner, independently of a less general theory which forms a limiting case of it. Thus, relativistic mechanics can be constructed on the basis of its own fundamental principles, without any reference to Newtonian mechanics. It is in principle impossible, however, to formulate the basic concepts of quantum mechanics without using classical mechanics. The fact that an electron‡ has no definite path means that it has also, in itself, no other dynamical characteristics.‖ Hence it is clear that, for a system composed only of quantum objects, it would be entirely impossible to construct any logically independent mechanics. The possibility of a quantitative description of the motion of an electron requires the presence also of physical objects which obey classical mechanics to a sufficient degree of accuracy. If an electron interacts with such a "classical object", the state of the latter is, generally speaking, altered. The nature and magnitude of this change depend on the state of the electron, and therefore may serve to characterize it quantitatively.

In this connection the "classical object" is usually called *apparatus*, and

† It is of interest to note that the complete mathematical formalism of quantum mechanics was constructed by W. Heisenberg and E. Schrödinger in 1925–6, before the discovery of the uncertainty principle, which revealed the physical content of this formalism.

‡ In this and the following sections we shall, for brevity, speak of "an electron", meaning in general any object of a quantum nature, i.e. a particle or system of particles obeying quantum mechanics and not classical mechanics.

‖ We refer to quantities which characterize the motion of the electron, and not to those, such as the charge and the mass, which relate to it as a particle; these are parameters.

its interaction with the electron is spoken of as *measurement*. However, it must be emphasized that we are here not discussing a process of measurement in which the physicist-observer takes part. By *measurement*, in quantum mechanics, we understand any process of interaction between classical and quantum objects, occurring apart from and independently of any observer. The importance of the concept of measurement in quantum mechanics was elucidated by N. Bohr.

We have defined "apparatus" as a physical object which is governed, with sufficient accuracy, by classical mechanics. Such, for instance, is a body of large enough mass. However, it must not be supposed that apparatus is necessarily macroscopic. Under certain conditions, the part of apparatus may also be taken by an object which is microscopic, since the idea of "with sufficient accuracy" depends on the actual problem proposed. Thus, the motion of an electron in a Wilson chamber is observed by means of the cloudy track which it leaves, and the thickness of this is large compared with atomic dimensions; when the path is determined with such low accuracy, the electron is an entirely classical object.

Thus quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation.

We may now formulate the problem of quantum mechanics. A typical problem consists in predicting the result of a subsequent measurement from the known results of previous measurements. Moreover, we shall see later that, in comparison with classical mechanics, quantum mechanics, generally speaking, restricts the range of values which can be taken by various physical quantities (for example, energy): that is, the values which can be obtained as a result of measuring the quantity concerned. The methods of quantum mechanics must enable us to determine these admissible values.

The measuring process has in quantum mechanics a very important property: it always affects the electron subjected to it, and it is in principle impossible to make its effect arbitrarily small, for a given accuracy of measurement. The more exact the measurement, the stronger the effect exerted by it, and only in measurements of very low accuracy can the effect on the measured object be small. This property of measurements is logically related to the fact that the dynamical characteristics of the electron appear only as a result of the measurement itself. It is clear that, if the effect of the measuring process on the object of it could be made arbitrarily small, this would mean that the measured quantity has in itself a definite value independent of the measurement.

Among the various kinds of measurement, the measurement of the coordinates of the electron plays a fundamental part. Within the limits of applicability of quantum mechanics, a measurement of the coordinates of an electron can always be performed† with any desired accuracy.

† Once again we emphasize that, in speaking of "performing a measurement", we refer to the interaction of an electron with a classical "apparatus", which in no way presupposes the presence of an external observer.

Let us suppose that, at definite time intervals Δt, successive measurements of the coordinates of an electron are made. The results will not in general lie on a smooth curve. On the contrary, the more accurately the measurements are made, the more discontinuous and disorderly will be the variation of their results, in accordance with the non-existence of a path of the electron. A fairly smooth path is obtained only if the coordinates of the electron are measured with a low degree of accuracy, as for instance from the condensation of vapour droplets in a Wilson chamber.

If now, leaving the accuracy of the measurements unchanged, we diminish the intervals Δt between measurements, then adjacent measurements, of course, give neighbouring values of the coordinates. However, the results of a series of successive measurements, though they lie in a small region of space, will be distributed in this region in a wholly irregular manner, lying on no smooth curve. In particular, as Δt tends to zero, the results of adjacent measurements by no means tend to lie on one straight line.

This circumstance shows that, in quantum mechanics, there is no such concept as the velocity of a particle in the classical sense of the word, i.e. the limit to which the difference of the coordinates at two instants, divided by the interval Δt between these instants, tends as Δt tends to zero. However, we shall see later that in quantum mechanics, nevertheless, a reasonable definition of the velocity of a particle at a given instant can be constructed, and this velocity passes into the classical velocity as we pass to classical mechanics. But whereas in classical mechanics a particle has definite coordinates and velocity at any given instant, in quantum mechanics the situation is entirely different. If, as a result of measurement, the electron is found to have definite coordinates, then it has no definite velocity whatever. Conversely, if the electron has a definite velocity, it cannot have a definite position in space. For the simultaneous existence of the coordinates and velocity would mean the existence of a definite path, which the electron has not. Thus, in quantum mechanics, the coordinates and velocity of an electron are quantities which cannot be simultaneously measured exactly, i.e. they cannot simultaneously have definite values. We may say that the coordinates and velocity of the electron are quantities which do not exist simultaneously. In what follows we shall derive the quantitative relation which determines the possibility of an inexact measurement of the coordinates and velocity at the same instant.

A complete description of the state of a physical system in classical mechanics is effected by stating all its coordinates and velocities at a given instant; with these initial data, the equations of motion completely determine the behaviour of the system at all subsequent instants. In quantum mechanics such a description is in principle impossible, since the coordinates and the corresponding velocities cannot exist simultaneously. Thus a description of the state of a quantum system is effected by means of a smaller number of quantities than in classical mechanics, i.e. it is less detailed than a classical description.

A very important consequence follows from this regarding the nature of the

predictions made in quantum mechanics. Whereas a classical description suffices to predict the future motion of a mechanical system with complete accuracy, the less detailed description given in quantum mechanics evidently cannot be enough to do this. This means that, even if an electron is in a state described in the most complete manner possible in quantum mechanics, its behaviour at subsequent instants is still in principle uncertain. Hence quantum mechanics cannot make completely definite predictions concerning the future behaviour of the electron. For a given initial state of the electron, a subsequent measurement can give various results. The problem in quantum mechanics consists in determining the probability of obtaining various results on performing this measurement. It is understood, of course, that in some cases the probability of a given result of measurement may be equal to unity, i.e. certainty, so that the result of that measurement is unique.

All measuring processes in quantum mechanics may be divided into two classes. In one, which contains the majority of measurements, we find those which do not, in any state of the system, lead with certainty to a unique result. The other class contains measurements such that for every possible result of measurement there is a state in which the measurement leads with certainty to that result. These latter measurements, which may be called *predictable*, play an important part in quantum mechanics. The quantitative characteristics of a state which are determined by such measurements are what are called *physical quantities* in quantum mechanics. If in some state a measurement gives with certainty a unique result, we shall say that in this state the corresponding physical quantity has a definite value. In future we shall always understand the expression "physical quantity" in the sense given here.

We shall often find in what follows that by no means every set of physical quantities in quantum mechanics can be measured simultaneously, i.e. can all have definite values at the same time. We have already mentioned one example, namely the velocity and coordinates of an electron. An important part is played in quantum mechanics by sets of physical quantities having the following property: these quantities can be measured simultaneously, but if they simultaneously have definite values, no other physical quantity (not being a function of these) can have a definite value in that state. We shall speak of such sets of physical quantities as *complete sets*.

Any description of the state of an electron arises as a result of some measurement. We shall now formulate the meaning of a *complete description* of a state in quantum mechanics. Completely described states occur as a result of the simultaneous measurement of a complete set of physical quantities. From the results of such a measurement we can, in particular, determine the probability of various results of any subsequent measurement, regardless of the history of the electron prior to the first measurement.

From now on (except in §14) we shall understand by the states of a quantum system just these completely described states.