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(in alphabetical order)

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SERIES FOREWORD

The fields of signal processing, optimization, and control stand as well-developed disciplines with solid theoretical and methodological foundations. While the development of each of these fields is of great importance, many future problems will require the combined efforts of researchers in all of the disciplines. Among these challenges are the analysis, design, and optimization of large and complex systems, the effective utilization of the capabilities provided by recent developments in digital technology for the design of high-performance control and signal-processing systems, and the application of systems concepts to a variety of applications, such as transportation systems, seismic signal processing, and data communication networks.

This series serves several purposes. It not only includes books at the leading edge of research in each field but also emphasizes theoretical research, analytical techniques, and applications that merit the attention of workers in all disciplines. In this way the series should help acquaint researchers in each field with other perspectives and techniques and provide cornerstones for the development of new research areas within each discipline and across the boundaries.

The analysis of adaptive systems is a particularly appropriate topic for a book in this series, given its relevance to so many problems of interest to researchers in signal processing and systems and control and given the considerable intensification of interest in and research on this subject in recent years. As the reader is probably aware, the field of adaptive systems is filled with a seemingly endless variety of methods, with assumptions and conditions whose fundamental significance may be far less apparent than the less than completely satisfying fact that they seem to make things work, and with controversy concerning the performance of these methods in practical applications. Given this background, the nonspecialist might, with some justification, view a book with eight authors as a fitting contribution to the cacophony someone once referred to as the Grand Bazaar of adaptive systems. This nonspecialist would be wrong. Brian Anderson, Robert Bitmead, C. Richard Johnson, Petar Kokotovic, Robert Kosut, Iven Mareels, Laurent Praly, and Bradley Riedle's book *Stability of Adaptive Systems: Passivity and Averaging Analysis* is a most impressive achievement and an extremely important and welcome addition to the literature on adaptive systems.

As the authors point out, this book is somewhere between a research monograph and a textbook on stability of adaptive systems. The book actually deserves both descriptors. Without doubt the results and methods presented in the book represent extremely important contributions to the theory of adaptive systems. As is often the case, the most difficult theoretical problems involving signals and systems center on issues related to making a methodology work in practice. In the case of adaptive systems this translates into the difficult problem of ensuring stable operation of an adaptive system in a real environment. The authors have taken this problem square on and have produced a most impressive theory of robust stability of adaptive systems answering many of the questions and challenges raised in recent years. This achievement certainly makes this book an important one for researchers and practitioners in adaptive systems.

What is perhaps equally impressive about this book is the cohesiveness of the treatment, a fact that is deserving of praise in a single-author textbook on a well-documented topic and a source of amazement when there are as many authors as there are words in the title and the topic is as recent in its development as this one is. This book can most definitely be used as the basis for a course on adaptive systems, and furthermore, the fundamental nature of the results presented should ensure this book a long life.

Starting from a basic total stability theorem and several of the fundamental principles underlying much of stability analysis -- passivity, small gain theorems, and averaging methods -- the authors develop a comparatively (and laudably!) small number of extremely general and powerful stability results for adaptive systems. These few results provide considerable insight into the mechanisms of stability and instability in adaptive systems and into the central role played by conditions on system positivity and richness of reference signals. These few results then provide the basis for analyzing the stability and robustness of a wide variety of adaptive systems. Through these analyses, the notes and references provided at the ends of chapters and at numerous other points in the development, and the most welcome inclusion of a design example and the remarkable commentary that accompanies and follows it, the reader develops an appreciation not only for the theory developed in this book but also for the nature of adaptive systems and the rich "folklore" and tricks of the trade for which the methods and insights developed in this book provide a theoretical interpretation and a rational basis. Without question the sophisticated development in this book presents a challenge to the reader, and mastery of the material requires a commitment of some time and energy. However, as the reader progresses -- or, should I say, bounds -- from chapter to chapter of *Stability of Adaptive Systems*, I think that he or she will find more than an ample return on the investment.

Alan S. Willsky

PREFACE

The tempting vision of engineering designs and algorithms which adapt to an ever changing environment is being pursued by many researchers trying to achieve convergence not only of adjustable parameters, but also of their diverse theories. This volume is itself a convergence result: its eight authors have converged to a set of passivity and averaging concepts organized into a pedagogical sequence of chapters, rather than a collection of papers. Each chapter is the result of a cooperative draft-review-rewrite effort, based almost exclusively on the current research of the authors. Notes and References stress this fact and do not attempt to overview all other important works.

This book offers insights into the behavior of adaptive systems from the viewpoint of stability theory, employing both input/output and Lyapunov stability concepts. While passivity theory shows input/output stability, the method of averaging significantly relaxes the restrictive passivity conditions. The stability analysis is extended to reveal the causes and mechanisms of instability and to suggest means to counteract them. The emphasis is on methodology and basic concepts, rather than on details of adaptive algorithms. Hence, the adaptive algorithms considered are of the simplest form, clearly exhibiting common properties of more elaborate schemes. Simultaneous treatment of continuous and discrete-time systems stresses the similarity of the results.

The presentation starts with the concepts of the tuned system, linearization, and total stability in Chapter 1, time-varying linear analysis in Chapter 2, linear averaging theory in Chapter 3, and then extends them to nonlinear continuous systems in Chapter 4 and nonlinear discrete systems in Chapter 5. Chapter 6 offers an extensive illustrative example with comments and a suggestion for algorithm modification based on regressor filtering. Simple examples are presented throughout to illustrate the uses and limitations of the theory.

The perceived place of this book in the adaptive control literature is somewhere between a purely research monograph and a basic pedagogical exposition. This is because we develop a *robustness theory* for adaptive systems without all the specifics of the algorithms, which are the province of the textbooks by Landau (1979), Ljung and Soderstrom (1983), and Goodwin and Sin (1984). These texts show that, under some *ideal modeling assumptions,* most of the basic algorithms possess global convergence properties. On the other hand, the more specialized monographs of Egardt (1979) and Ioannou and Kokotovic (1983) demonstrate that such global properties are not robust and can be lost due to disturbances, lack of excitation, violation of positive realness (passivity) conditions, unmodeled dynamics and similar modeling imperfections always present in actual systems. These robustness issues are also addressed in this book, but with new, sharper tools which, hopefully, have produced clearer results. With these tools it was possible first to decipher many of the cryptic notions of adaptive control -- strictly positive real conditions, persistence of excitation, etc. -- and then to replace them with more robust signal dependent positivity conditions which remain valid, at least locally, in the presence of disturbances and unmodeled dynamics. Thus, while the intent and content of our work here is more in line with the monographs of Egardt (1979) and Ioannou and Kokotovic (1983), we pace our development to suit a less specialized audience.

Perhaps the most remarkable aspect of this book is the number of its authors -- a direct result of the friendship and technical collaboration established over an extended period. While, unfortunately, some of the authors have never met face to face with all of the others, the emergence of this manuscript was assisted by travel exceeding a quarter of a million miles.

In this epoch of formation of national research centers, it might sound a discordant note that cooperative research can flourish at disperse locations. The genesis of the whole concept was in August 1984 in Bozeman, Montana, USA -- *home to none of us* -- but made so by the warm hospitality of Don Pierre of Montana State University. Subsequent venues and approximate dates, where intensive joint work on the book was part of an otherwise hectic schedule, were as follows: Urbana, Illinois (October '84 and December '85); Canberra, Australian Capital Territory (November '84, May '85 and February '86); Ithaca, New York (August-December '84, June '85); Las Vegas, Nevada (December '84); Los Angeles, California (May '85); Boston, Massachusetts (June '85); Bozeman, Montana (August '85); Lund, Sweden (August '85); Palo Alto, California (November '85); Fort Lauderdale, Florida (December '85).

In this process of wandering we inconvenienced not only our families, but also many friends and colleagues imprudent enough to share our enthusiasm for this project. This way of thanking them for support is inadequate but the volume of writing down all the gratitudes would be overwhelming. Caught in the web were our editor, Alan Willsky, and publisher, Frank Satlow, who, while continuously puzzled by the phenomenon of eight authors writing a book, managed to produce it without delay. This accomplishment was made possible by the expert typing of Patricia Krokel of Stanford University who stoically endured the frenzy of eightfold correcting and proofreading. Her patient assistance and perserverance have been irreplaceable.

And, to conclude with an even more remarkable facet of this project: at its completion all friendships and the enthusiasms for the research in adaptive systems not only survived, but have continued to grow.

"It is an error to imagine that evolution signifies a constant tendency to increased perfection. That process undoubtedly involves a constant remodeling of the organism in adaptation to new conditions; but it depends on the nature of those conditions whether the direction of the modifications effected shall be upward or downward."

-- *T.H. Huxley*

CONTENTS

Chapter 1

ROBUST STABILITY FORMULATION

1.1 INTRODUCTION

Adaptive systems are a response to engineering design problems arising from uncertainty. In the representative areas of adaptive control, communications, and signal processing, uncertainty means an imprecise knowledge of the current system. The aim of adaptation is to provide on-line modification of system behavior in response to current measured performance. Thus, the formulation of adaptive systems involves concepts of performance measures, adjustment rules, and desired objectives, together with an appreciation of the role of uncertainty in describing the system.

In characterizing the effectiveness of adaptive methods one is led immediately to questions concerning the nature of the response of the adaptive system vis-a-vis a hypothetical nonadaptive system designed according to the same objectives but with complete system knowledge, or more properly, with a much reduced level of uncertainty. The primary question raised by this characterization is whether the adaptive system achieves a performance close to that of the hypothetical "ideal." Thus, *stability* arises as an issue in adaptive systems and deals with the adaptive system per se or its deviation from the platonic ideal. It describes the signals in the system and their asymptotic properties such as boundedness or convergence. When we discuss *robust stability* in this framework, we include into our assessment the preservation of these stability notions when a type of uncertainty is allowed to be present in the system which is not explicitly accounted for in the adaptive mechanism. Specifically, we will be concerned with a system or plant which is variously characterized by *structured* parametric uncertainty and *unstructured* modeling error. The adaptive system responds directly to the structured uncertainties by the adjustment of selected system parameters. By its nature, unstructured uncertainty is not modelled in the analysis, and so must be quantified and introduced in a different fashion.

Historically, the developments in adaptive systems -- essentially, parameter adaptive systems -- have concentrated on the issues of maintaining good bounds between adaptive performance and ideal nonadaptive performance ("reference model") in the absence of unstructured uncertainty. For the more complicated areas such as adaptive control, it has not been possible to account fully for the behavior with unstructured uncertainty because of the occurrence of nonlinear dynamical equations whose solutions have been difficult to qualify. The emphasis in this area has therefore been primarily to develop convergence theories to demonstrate the ability of adaptive schemes to overcome structured parametric uncertainty. This has led to a class of important results presenting global convergence and asymptotic optimality. With the inclusion of additional unstructured effects it proves no longer possible to preserve these global results in a broad range of situations. Consequently, the development of robust stability theory for adaptive systems in its most general form must take place in a local context, although some global results are still achievable in some instances.

Our central objective in this treatise is to develop, in as cogent a manner as possible, a theory of the robust stability of adaptive systems. Our thrust will primarily be directed towards a qualitative understanding of the mechanisms of robust local stability so that questions concerning the suitability of adaptation in certain practical circumstances can be addressed. More quantitative characterizations then are derived subsequently from this basis. In an attempt to promulgate our creed most widely, we stress the pedagogical aspects of the theory at the same time as deriving new results and concentrate on developing the conceptual and intuitive fundamentals. This focus comes at the expense of an exhaustive analysis of many specific variants of adaptive algorithms and of the presentation of the most flexible but less transparent supporting results. Typically, sophisticated extensions possess an identical conceptual basis.

Summary of Chapters: In the rest of this chapter we provide a derivation of the general equations describing adaptive systems and an introduction of our main tool for the underlying theory of robust stability in the face of unstructured uncertainty -- the Total Stability Theorem. The specific approach taken is to describe the evolution of the signals in the adaptive system in terms of their deviation from nominal "tuned" values which exemplify the ideal design discussed above. Thus, we do not consider the nonlinear dynamical equations of the adaptive system directly, but rather the equations of an error system. These equations are still nonlinear but, subject to certain regularity conditions, do admit a linearization about zero error. From the Total Stability Theorem, exponential stability of the linearized system implies local exponential stability of an associated nonlinear system. This, in turn, establishes conditions for the local boundedness of the state of the complete nonlinear error system in terms of restrictions on the magnitude of initial conditions and unstructured error signals. This presentation is in general terms of Banach spaces, fixed point theorems, operator notations, etc., and supports the body of our work where we tie our analysis more closely to standard adaptive methods, at which stage we utilize state-variable descriptions more than operators. We conclude Chapter 1 with the development of the standard form of the dynamical equations governing the *adaptive error system,* including the underlying linearized form, i.e., the *linear error system*.

We next turn to the problem of deriving sufficient conditions for the exponential stability of the linear error system. This is done in Chapters 2 and 3 using the separate techniques of passivity analysis and averaging, respectively. In these analyses, the effect of the unstructured uncertainty is reflected in imprecision of the knowledge of the linear operator appearing in the linearized equations. In Chapter 2 the stability of the linear equation is analyzed using passivity and small gain methods -- these encompass the renowned strictly positive real (SPR) condition of adaptive systems, and reconstitute this positivity requirement to more general signal dependent operator forms. This *signal dependent positivity condition*, also referred to as a *dominant richness condition*, enlarges the notion of persistence of excitation of input signals to the adaptive system, and is a major theme of the book. For example, when the unstructured uncertainty reflects high frequency dynamics, the condition is composed of a low-frequency positivity requirement on the operator coupled with a nondegeneracy and dominant low-pass stipulation on the signals within the adaptive system.

In Chapter 3, the alternative techniques of averaging and explicit time-scale separation further develop specifications of sufficient conditions for exponential stability of the linear equation. This approach makes clear the role played in the theory by both the dominant richness assumption and slow adaptation rate. These techniques produce stability and instability conditions for the linear equation and provide the most transparent statement of the dominant richness condition in terms of an average positivity condition of signal products, i.e., a *signal dependent "average SPR" condition*. The theory of averaging is extended in this chapter to include signals with *sample averages*. Illustrative examples include the linearized analysis of an output error algorithm.

Chapters 4 and 5 contain derivations of the specific error equations which arise in particular, but typical, examples of adaptive identification and control: equation error, output error, and model reference adaptive control. The continuous-time case is studied in Chapter 4, while Chapter 5 contains the discrete-time case. Both chapters provide a local stability analysis of the adaptive error