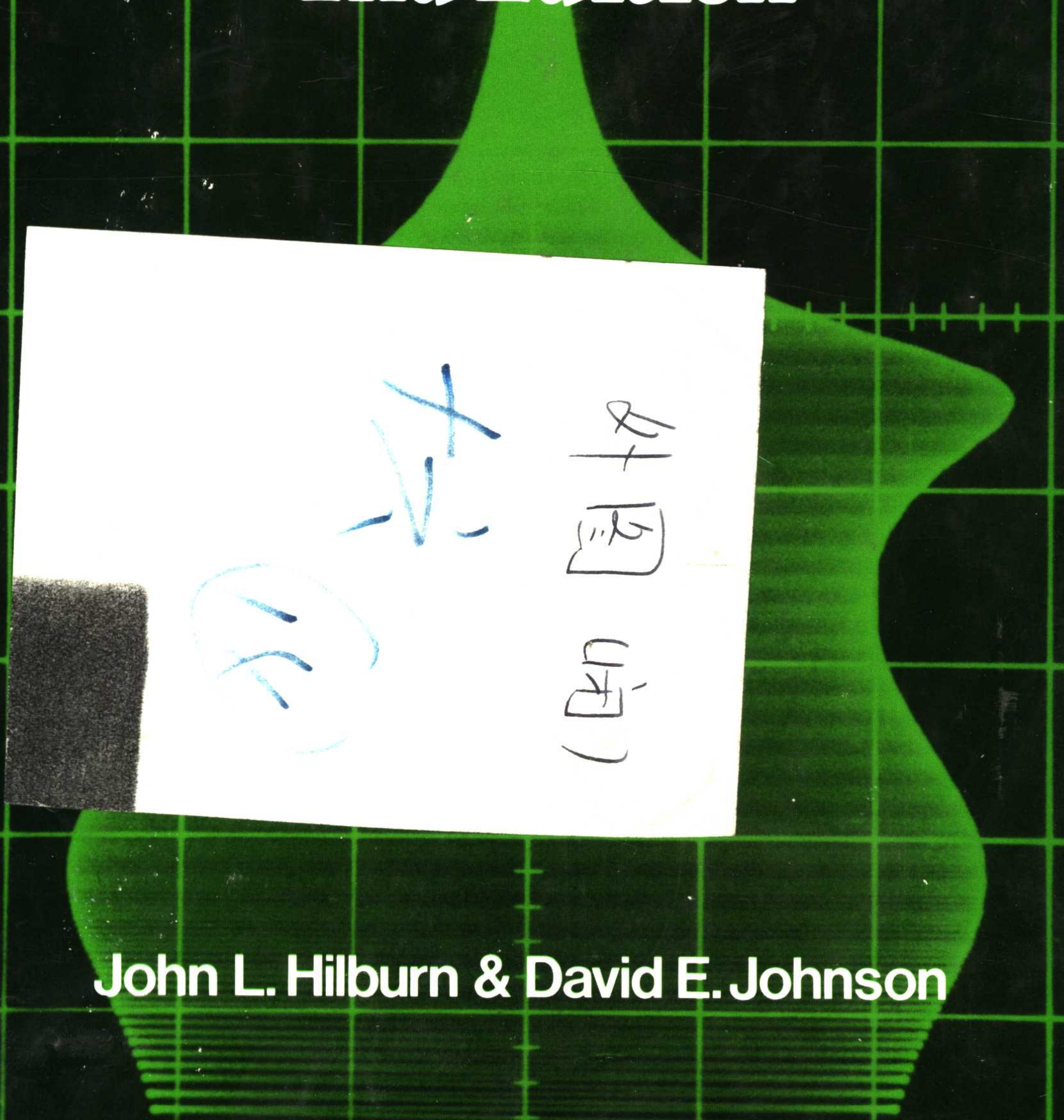
MANUAL OF ACTIVE FILTER DESIGN

2nd Edition



John L. Hilburn & David E. Johnson

MANUAL OF ACTIVE FILTER DESIGN

JOHN L. HILBURN

President, Microcomputer Systems, Inc.

DAVID E. JOHNSON

Department of Electrical Engineering,
Louisiana State University

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PREFACE

In this book we present simplified methods for obtaining a complete, practical filter design by inspection of a graph, requiring no computations whatsoever. The book therefore is useful to all filter designers, from the novice to the expert. The filter circuit elements used are operational amplifiers and standard values of resistances and capacitances.

The type of filters which one may construct using the graphs are the following:

1. Low-pass (Butterworth or Chebyshev of second or fourth orders)
2. High-pass (Butterworth or Chebyshev of second or fourth orders)
3. Band-pass (second and higher orders)
4. Band-reject (notch)
5. Phase-shift or delay (all-pass or Bessel)

Each of the filter types is discussed in a separate chapter. At the end of each chapter the design procedure is summarized and the appropriate graphs are presented. Practical design suggestions are given for each circuit considered.

Examples are given of every type of filter considered and actual photographs of the results are included. A detailed example is presented in Sec. 2.3, which may be used as a design guideline. However, it is not necessary to read the chapters in order to use the handbook, since all the necessary information is presented on the summary pages of each chapter.

In this second edition we have retained the popular filter designs that have been widely used and well accepted in the first edition. We have added, in the cases of the low-pass and high-pass filters, the very useful 0.1 dB Chebyshev designs. The other major addition is the inclusion, in the cases of low-pass and band-pass filters, of the important biquad filter designs, which have superior performance characteristics and are extremely easy to tune. It is probably no exaggeration to say that the biquad is the best of all possible filter designs available.

John L. Hilburn
David E. Johnson

About the Authors

JOHN L. HILBURN, Ph.D., P.E., is president of Microcomputer Systems, Inc., a firm specializing in the design and fabrication of microcomputer-based industrial instrumentation. A former professor of electrical engineering at Louisiana State University, Dr. Hilburn is the author of 12 books and numerous technical papers and a member of IEEE.

DAVID E. JOHNSON, Ph.D., P.E., is professor of electrical engineering at Louisiana State University, where he has been since 1962. A member of IEEE, Dr. Johnson is the author of 17 books and some 40 technical papers.

CONTENTS

1 INTRODUCTION

1.1 Active Filters

A filter is a device which passes signals of certain frequencies and rejects or attenuates those of other frequencies. Passive filters are constructed with inductors, capacitors, and resistors, but for certain frequency ranges inductors, because of their size and practical performance limitations, are undesirable. Consequently there has been, for some years, a trend toward replacing inductors by active devices which simulate the effect of inductors. This trend has accelerated with the advances in miniaturization which have made the active devices available at prices competitive with, and in many cases cheaper than, those of inductors.

In this manual we present simplified methods for constructing a variety of active filters having specified characteristics with standard element values. The active device we use is the integrated circuit (IC) operational amplifier, which is briefly described in the remainder of this chapter. Graphs are presented for each type of filter, and, depending on the specifications, the designer may simply choose the appropriate graph and read off the circuit element values. For the designer interested in the theoretical details, there is a chapter of background material with numerous references provided for each filter type. However, to use the manual, one needs only to refer to the summary sheet preceding the graphs at the end of each chapter.

1.2 The Operational Amplifier

The basic element we shall use in the construction of an active filter is the operational amplifier (op-amp), the symbol for which is shown in Fig. 1.1. Only three terminals are shown in the figure, the inverting input terminal ($-$), the noninverting input terminal ($+$), and the output terminal. However, a practical op-amp is actually a multiterminal device. The purposes of the other terminals are specified by the manufacturer and include, in general, power supply connections, frequency compensation terminals, and offset null terminals.

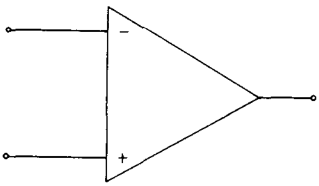

Fig. 1.1 A differential op-amp.

The equations we have derived in the following chapters are obtained assuming zero voltage between the two input terminals of the op-amp and zero current into the two input terminals. This is true of the ideal op-amp, and is closely approximated by practical op-amps, if used according to the manufacturer's specifications.

Numerous publications are available describing in detail the characteristics and uses of commercially available operational amplifiers. (See, for example [1]–[14].)* In addition, most manufacturers publish detailed catalogs containing information on their specific op-amps. An extensive list of manufacturers is given in [1]. Some well-known manufacturers include Burr-Brown Research Corp., Fairchild Semiconductor, Motorola, National Semiconductor, RCA, Signetics Corp., and Texas Instruments.

The op-amp of Fig. 1.1 is a differential-input amplifier, which is a commonly manufactured type. In general, for stable operation, IC op-amps require frequency compensation. Some, such as the 741, are internally compensated. The μA741, AD741, MC1741, LM741, CA741, etc. are all type 741 op-amps. The different representations are used to identify the manufacturer. Other types of compensated op-amps include the 536, 107, 5556, 740, and 747 (dual 741). Other op-amps require external compensation, specified by the manufacturer, but generally are useful for much higher frequencies and gains. Some examples of these are the types 709, 748, 101, and 531.

For best results in the circuit configurations given in the following chapters, the designer should use op-amps which perform adequately for the

*References thus cited are listed in the Bibliography.

gains and frequency ranges of interest. For example, the open-loop gains as specified by the manufacturer should be at least 50 times the filter gain [4]. Other suggestions will be made on the summary sheets at the end of each chapter.

1.3 Resistors and Capacitors

There are three types of resistors in common use. The carbon composition resistor is the most widely used and is acceptable in most noncritical filter applications. This is particularly true if the filter is used at room temperature. In all our examples the filters were constructed with 5% tolerance carbon composition resistors. These were used because they are the most economical and commonly available. For high-performance applications, or in instances where temperature is important, one should use either metal-film or wire-wound resistors.

In the case of capacitors, the ceramic disk capacitor is a very common and economical type. However, these should be used in the most noncritical applications. A more acceptable common type is the Mylar capacitor, which is the type we used in most of our examples. For critical applications and high performance, polystyrene and Teflon capacitors are good choices in most cases.

For a good discussion of resistors and capacitors, the reader is referred to [2], pp. 317–319.

1.4 Basic Op-Amp Circuits

The filters that are designed in the book perform best when the input signals are from low-output-impedance sources. If the signal that is to be filtered is not of this type, it may be desirable to *precondition*, or *preamplify*, the signal with an op-amp circuit as shown in Fig. 1.2. The circuit consists of an op-amp and two resistors R_a and R_b, and has the property that its output voltage V_1 is related to its input voltage V_{in} by

$$V_1 = \mu V_{in} \tag{1.1}$$

where

$$\mu = 1 + \frac{R_a}{R_b} \tag{1.2}$$

The circuit is called a *voltage-controlled voltage source* (VCVS) and μ is its *gain*.

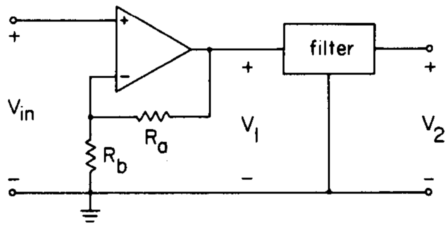

Fig. 1.2 A VCVS circuit cascaded with a filter circuit.

In the case of Fig. 1.2, the VCVS is cascaded with a filter circuit with input voltage V_1 (the output of the VCVS) and output voltage V_2. The advantages of this arrangement are that the VCVS approximates the zero-output impedance we need and, by adjusting the gain μ, we may obtain a predetermined level of the filter output voltage V_2. The filter itself is designed to provide certain predetermined levels, or *gains*, but with the added flexibility of adjusting μ we may restrict the filter designs to a relatively small number of filter gains.

As an example, if the filter of Fig. 1.2 is designed to give at a certain frequency,

$$V_2 = 2V_1$$

and we wish to have

$$V_2 = 6V_{in}$$

then we need

$$V_1 = 3V_{in}$$

Thus the VCVS must have a gain of

$$\mu = 1 + \frac{R_a}{R_b} = 3$$

or

$$R_a = 2R_b$$

If we arbitrarily select $R_b = 10\ k\Omega$, then we require $R_a = 20\ k\Omega$.

If in the VCVS of Fig. 1.2 we make $R_a = 0$ (a short circuit) and R_b infinite (an open circuit), then we have the circuit of Fig. 1.3. By Eq. (1.2) its gain is $\mu = 1$, so that

$$V_1 = V_{in} \tag{1.3}$$

Thus the output voltage V_1 is the same as the input voltage V_{in}, but there is the important difference that V_1 is the output of a zero-output-impedance device whereas V_{in} may be any voltage. In addition the device draws no current because of the presence of the op-amp, and thus it acts as a *buffer* stage between V_{in} and V_1. For this reason it is referred to as a *buffer amplifier*, or, because the output V_1 *follows* the input V_{in}, as a *voltage follower.* The buffer amplifier is useful when we need to precondition the filter input signal but all the gain we need is provided by the filter itself.

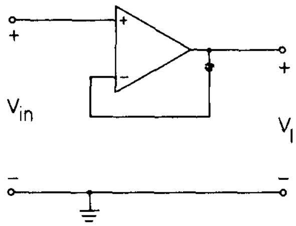

Fig. 1.3 A buffer amplifier.

Another popular op-amp circuit is that of Fig. 1.4, for which

$$V_{out} = -\frac{R_a}{R_b} V_{in} \tag{1.4}$$

This circuit thus is a nearly zero-output-impedance device that provides a gain of R_a/R_b and *inverts* (changes the sign of) the input signal. For this reason it is called an *inverting amplifier,* or simply an *inverter.*

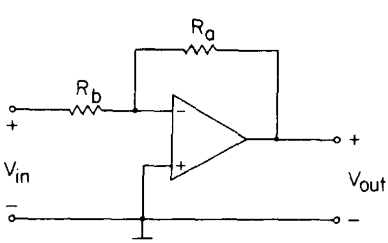

Fig. 1.4 An inverter.

The VCVS, buffer amplifier, and inverter circuits thus are useful in preconditioning the input signals when necessary. As we will see, they are also very popular components of many filter circuits.

2 LOW-PASS FILTERS

2.1 General Circuits and Equations

A low-pass filter is a device which passes signals of low frequencies and suppresses or attenuates those of high frequencies. Its performance may be illustrated by its amplitude response, which is a plot of the amplitude $|H(j\omega)|$ of its transfer function $H(s)$ versus frequency ω (radians/sec) or f (Hz), where $\omega = 2\pi f$. In all cases we shall take $H(s) = V_2(s)/V_1(s)$, where V_2 is the output voltage and V_1 is the input voltage. An example is shown in Fig. 2.1, where the response represented by the broken line is the ideal case and the response represented by the solid line is a realizable approximation of the ideal. The value ω_c (or in Hz, $f_c = \omega_c/2\pi$) is the cutoff frequency defined as the point at which $|H(j\omega)|$ is $1/\sqrt{2} = 0.707$ times its maximum value, shown here as A. The passband is the range $0 \leq \omega < \omega_c$ and the stopband is the range $\omega > \omega_c$.

Alternatively the amplitude response may be plotted as amplitude in decibels (dB), which we denote by α, versus frequency ω (or f), or versus

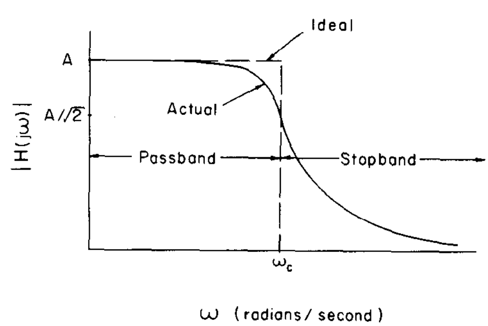

Fig. 2.1 Low-pass amplitude response.

log ω (or log f). An example is shown in Fig. 2.2, where it may be seen that cutoff corresponds to $\alpha = -3$ dB.

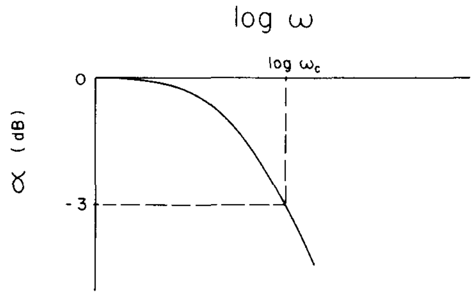

Fig. 2.2 Amplitude response in dB.

A second-order approximation to an ideal low-pass filter is achieved by the transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{K}{s^2 + as + b} \tag{2.1}$$

where a and b are properly chosen constants and K is a constant [15]. The term *second-order* refers to the degree of the denominator polynomial. Higher order transfer functions are like Eq. (2.1) except that the denominator is of higher degree. The gain of the low-pass filter is the value of its transfer function at $s = 0$, and is given in the case of Eq. (2.1) by gain $= K/b$.

There are any number of ways of obtaining low-pass filters using active devices in lieu of inductors. (See, for example [2], [6], [15], [16].) One method we use is that of Sallen and Key [17], in which the active device is an operational amplifier (op-amp), described in Chapter 1. A Sallen and Key second-order low-pass filter is shown in Fig. 2.3, where the resistors and capacitors are properly chosen to realize given values of a and b in Eq. (2.1). The op-amp, together with the resistors R_3 and R_4, constitutes a voltage-controlled voltage source (VCVS), and hence the Sallen and Key network is of the VCVS type.

Higher order filters may be obtained by cascading two or more second-order filters. For example, a fourth-order low-pass filter is obtained by cascading two networks such as that of Fig. 2.3, as we shall see in Sec. 2.6.

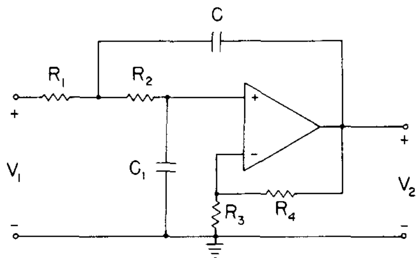

Fig. 2.3 A second-order VCVS low-pass filter.

Analysis of Fig. 2.3 shows that it achieves Eq. (2.1) with

$$K = \frac{\mu}{R_1 R_2 C C_1}$$

$$a = \frac{1}{R_2 C_1}(1 - \mu) + \frac{1}{R_1 C} + \frac{1}{R_2 C} \tag{2.2}$$

$$b = \frac{1}{R_1 R_2 C C_1}$$

where

$$\mu = 1 + \frac{R_4}{R_3}$$

The quantity μ is the gain of the VCVS, and is also the gain of the filter since $K/b = \mu$.

Another low-pass filter circuit that we consider is the so-called *biquad* circuit [18] of Fig. 2.4. It has more elements than the VCVS circuit, but it is much easier to adjust, or *tune*, and is much more stable, particularly in higher order cases. Analysis of Fig. 2.4 shows that Eq. (2.1) is achieved with

$$K = \frac{1}{R_1 R_4 C^2}$$

$$a = \frac{1}{R_2 C} \tag{2.3}$$

$$b = \frac{1}{R_3 R_4 C^2}$$

The gain K/b is therefore R_3/R_1.