

**MATHEMATICAL HANDBOOK**

**HIGHER  
MATHEMATICS**

**M. Vygodsky**



**М. Я. ВЫГОДСКИЙ**

**СПРАВОЧНИК  
ПО ВЫСШЕЙ МАТЕМАТИКЕ**


**ИЗДАТЕЛЬСТВО «НАУКА»  
МОСКВА**

# MATHEMATICAL HANDBOOK

## HIGHER MATHEMATICS

**M. Vygodsky**

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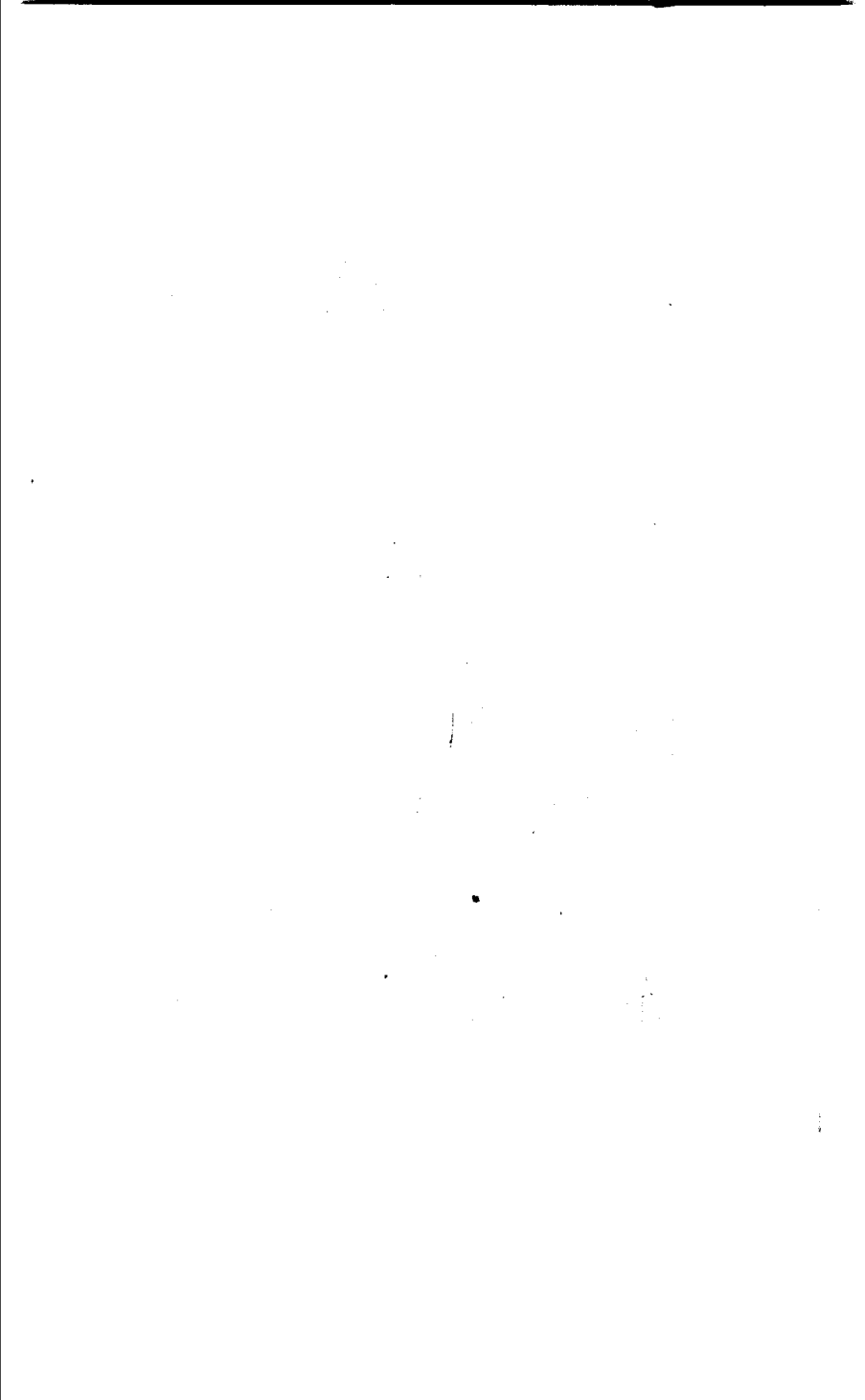
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# CONTENTS

## PLANE ANALYTIC GEOMETRY

1. The Subject of Analytic Geometry	19
2. Coordinates	20
3. Rectangular Coordinate System	20
4. Rectangular Coordinates	21
5. Quadrants	21
6. Oblique Coordinate System	22
7. The Equation of a Line	23
8. The Mutual Positions of a Line and a Point	24
9. The Mutual Positions of Two Lines	25
10. The Distance Between Two Points	25
11. Dividing a Line-Segment in a Given Ratio	26
11a. Midpoint of a Line-Segment	27
12. Second-Order Determinant	27
13. The Area of a Triangle	27
14. The Straight Line. An Equation Solved for the Ordinate (Slope-Intercept Form)	28
15. A Straight Line Parallel to an Axis	30
16. The General Equation of the Straight Line	31
17. Constructing a Straight Line on the Basis of Its Equation	32
18. The Parallelism Condition of Straight Lines	32
19. The Intersection of Straight Lines	34
20. The Perpendicularity Condition of Two Straight Lines	35
21. The Angle Between Two Straight Lines	36
22. The Condition for Three Points Lying on One Straight Line	38
23. The Equation of a Straight Line Through Two Points (Two-Point Form)	39
24. A Pencil of Straight Lines	40
25. The Equation of a Straight Line Through a Given Point and Parallel to a Given Straight Line (Point-Slope Form)	42
26. The Equation of a Straight Line Through a Given Point and Perpendicular to a Given Straight Line	43
27. The Mutual Positions of a Straight Line and a Pair of Points	44
28. The Distance from a Point to a Straight Line	44
29. The Polar Parameters (Coordinates) of a Straight Line	45
30. The Normal Equation of a Straight Line	47
31. Reducing the Equation of a Straight Line to the Normal Form	48
32. Intercepts	49
33. Intercept Form of the Equation of a Straight Line	50
34. Transformation of Coordinates (Statement of the Problem)	51
35. Translation of the Origin	52
36. Rotation of the Axes	53
37. Algebraic Curves and Their Order	54
38. The Circle	56
39. Finding the Centre and Radius of a Circle	57
40. The Ellipse as a Compressed Circle	58
41. An Alternative Definition of the Ellipse	60
42. Construction of an Ellipse from the Axes	62
43. The Hyperbola	63
44. The Shape of the Hyperbola. Its Vertices and Axes	65
45. Construction of a Hyperbola from Its Axes	67
46. The Asymptotes of a Hyperbola	67



## 8 CONTENTS

47. Conjugate Hyperbolas	68
48. The Parabola	69
49. Construction of a Parabola from a Given Parameter $p$	70
50. The Parabola as the Graph of the Equation $y = ax^2 + bx + c$	70
51. The Directrices of the Ellipse and of the Hyperbola	73
52. A General Definition of the Ellipse, Hyperbola and Parabola	75
53. Conic Sections	77
54. The Diameters of a Conic Section	78
55. The Diameters of an Ellipse	79
56. The Diameters of a Hyperbola	80
57. The Diameters of a Parabola	82
58. Second-Order Curves (Quadric Curves)	83
59. General Second-Degree Equation	85
60. Simplifying a Second-Degree Equation. General Remarks	86
61. Preliminary Transformation of a Second-Degree Equation	86
62. Final Transformation of a Second-Degree Equation	88
63. Techniques to Facilitate Simplification of a Second-Degree Equation	95
64. Test for Decomposition of Second-Order Curves	95
65. Finding Straight Lines that Constitute a Decomposable Second-Order Curve	97
66. Invariants of a Second-Degree Equation	99
67. Three Types of Second-Order Curves	102
68. Central and Noncentral Second-Order Curves (Conics)	104
69. Finding the Centre of a Central Conic	105
70. Simplifying the Equation of a Central Conic	107
71. The Equilateral Hyperbola as the Graph of the Equation $y = \frac{k}{x}$	109
72. The Equilateral Hyperbola as the Graph of the Equation $y = \frac{mx+n}{px+q}$	110
73. Polar Coordinates	112
74. Relationship Between Polar and Rectangular Coordinates	114
75. The Spiral of Archimedes	116
76. The Polar Equation of a Straight Line	118
77. The Polar Equation of a Conic Section	119

## SOLID ANALYTIC GEOMETRY

78. Vectors and Scalars. Fundamentals	120
79. The Vector in Geometry	120
80. Vector Algebra	121
81. Collinear Vectors	121
82. The Null Vector	122
83. Equality of Vectors	122
84. Reduction of Vectors to a Common Origin	123
85. Opposite Vectors	123
86. Addition of Vectors	123
87. The Sum of Several Vectors	125
88. Subtraction of Vectors	126
89. Multiplication and Division of a Vector by a Number	127
90. Mutual Relationship of Collinear Vectors (Division of a Vector by a Vector)	128
91. The Projection of a Point on an Axis	129
92. The Projection of a Vector on an Axis	130
93. Principal Theorems on Projections of Vectors	132
94. The Rectangular Coordinate System in Space	133

95	The Coordinates of a Point	134
96	The Coordinates of a Vector	135
97.	Expressing a Vector in Terms of Components and in Terms of Coordinates	137
98.	Operations Involving Vectors Specified by Their Coordinates	137
99.	Expressing a Vector in Terms of the Radius Vectors of Its Origin and Terminus	137
100.	The Length of a Vector. The Distance Between Two Points	138
101	The Angle Between a Coordinate Axis and a Vector	139
102.	Criterion of Collinearity (Parallelism) of Vectors	139
103.	Division of a Segment in a Given Ratio	140
104.	Scalar Product of Two Vectors	141
104a.	The Physical Meaning of a Scalar Product	142
105.	Properties of a Scalar Product	142
106.	The Scalar Products of Base Vectors	144
107.	Expressing a Scalar Product in Terms of the Coordinates of the Factors	145
108	The Perpendicularity Condition of Vectors	146
109.	The Angle Between Vectors	146
110.	Right-Handed and Left-Handed Systems of Three Vectors	147
111.	The Vector Product of Two Vectors	148
112.	The Properties of a Vector Product	150
113.	The Vector Products of the Base Vectors	152
114.	Expressing a Vector Product in Terms of the Coordinates of the Factors	152
115.	Coplanar Vectors	154
116.	Scalar Triple Product	154
117	Properties of a Scalar Triple Product	155
118.	Third-Order Determinant	156
119.	Expressing a Triple Product in Terms of the Coordinates of the Factors	159
120.	Coplanarity Criterion in Coordinate Form	159
121.	Volume of a Parallelepiped	160
122.	Vector Triple Product	161
123.	The Equation of a Plane	161
124.	Special Cases of the Position of a Plane Relative to a Coordinate System	162
125.	Condition of Parallelism of Planes	163
126.	Condition of Perpendicularity of Planes	164
127.	Angle Between Two Planes	164
128.	A Plane Passing Through a Given Point Parallel to a Given Plane	165
129.	A Plane Passing Through Three Points	166
130.	Intercepts on the Axes	166
131.	Intercept Form of the Equation of a Plane	166
132.	A Plane Passing Through Two Points Perpendicular to a Given Plane	167
133.	A Plane Passing Through a Given Point Perpendicular to Two Planes	167
134.	The Point of Intersection of Three Planes	168
135.	The Mutual Positions of a Plane and a Pair of Points	169
136.	The Distance from a Point to a Plane	170
137.	The Polar Parameters (Coordinates) of a Plane	170
138.	The Normal Equation of a Plane	172
139.	Reducing the Equation of a Plane to the Normal Form	173
140.	Equations of a Straight Line in Space	174
141.	Condition Under Which Two First-Degree Equations Represent a Straight Line	176
142.	The Intersection of a Straight Line and a Plane	177
143.	The Direction Vector	179

# 10 CONTENTS

144. Angles Between a Straight Line and the Coordinate Axes	179
145. Angle Between Two Straight Lines	180
146. Angle Between a Straight Line and a Plane	181
147. Conditions of Parallelism and Perpendicularity of a Straight Line and a Plane	181
148. A Pencil of Planes	182
149. Projections of a Straight Line on the Coordinate Planes	184
150. Symmetric Form of the Equation of a Straight Line	185
151. Reducing the Equations of a Straight Line to Symmetric Form	187
152. Parametric Equations of a Straight Line	188
153. The Intersection of a Plane with a Straight Line Represented Parametrically	189
154. The Two-Point Form of the Equations of a Straight Line	190
155. The Equation of a Plane Passing Through a Given Point Perpendicular to a Given Straight Line	190
156. The Equations of a Straight Line Passing Through a Given Point Perpendicular to a Given Plane	190
157. The Equation of a Plane Passing Through a Given Point and a Given Straight Line	191
158. The Equation of a Plane Passing Through a Given Point Parallel to Two Given Straight Lines	192
159. The Equation of a Plane Passing Through a Given Straight Line and Parallel to Another Given Straight Line	192
160. The Equation of a Plane Passing Through a Given Straight Line and Perpendicular to a Given Plane	193
161. The Equations of a Perpendicular Dropped from a Given Point onto a Given Straight Line	193
162. The Length of a Perpendicular Dropped from a Given Point onto a Given Straight Line	196
163. The Condition for Two Straight Lines Intersecting or Lying in a Single Plane	196
164. The Equations of a Line Perpendicular to Two Given Straight Lines	197
165. The Shortest Distance Between Two Straight Lines	199
165a. Right-Handed and Left-Handed Pairs of Straight Lines	201
166. Transformation of Coordinates	202
167. The Equation of a Surface	203
168. Cylindrical Surfaces Whose Generatrices Are Parallel to One of the Coordinate Axes	204
169. The Equations of a Line	206
170. The Projection of a Line on a Coordinate Plane	206
171. Algebraic Surfaces and Their Order	209
172. The Sphere	209
173. The Ellipsoid	210
174. Hyperboloid of One Sheet	213
175. Hyperboloid of Two Sheets	215
176. Quadric Conical Surface	217
177. Elliptic Paraboloid	218
178. Hyperbolic Paraboloid	220
179. Quadric Surfaces Classified	221
180. Straight-Line Generatrices of Quadric Surfaces	224
181. Surfaces of Revolution	225
182. Determinants of Second and Third Order	226
183. Determinants of Higher Order	229
184. Properties of Determinants	231
185. A Practical Technique for Computing Determinants	233
186. Using Determinants to Investigate and Solve Systems of Equations	236
187. Two Equations in Two Unknowns	236

188.	Two Equations in Three Unknowns	238
189.	A Homogeneous System of Two Equations in Three Unknowns	240
190.	Three Equations in Three Unknowns	241
190a.	A System of $n$ Equations in $n$ Unknowns	245

## FUNDAMENTALS OF MATHEMATICAL ANALYSIS

191.	Introductory Remarks	247
192.	Rational Numbers	248
193.	Real Numbers	248
194.	The Number Line	249
195.	Variable and Constant Quantities	250
196.	Function	250
197.	Ways of Representing Functions	252
198.	The Domain of Definition of a Function	254
199.	Intervals	257
200.	Classification of Functions	258
201.	Basic Elementary Functions	259
202.	Functional Notation	259
203.	The Limit of a Sequence	261
204.	The Limit of a Function	262
205.	The Limit of a Function Defined	264
206.	The Limit of a Constant	265
207.	Infinitesimals	265
208.	Infinites	265
209.	The Relationship Between Infinites and Infinitesimals	267
210.	Bounded Quantities	267
211.	An Extension of the Limit Concept	267
212.	Basic Properties of Infinitesimals	269
213.	Basic Limit Theorems	270
214.	The Number $e$	271
215.	The Limit of $\frac{\sin x}{x}$ as $x \rightarrow 0$	273
216.	Equivalent Infinitesimals	273
217.	Comparison of Infinitesimals	274
217a.	The Increment of a Variable Quantity	276
218.	The Continuity of a Function at a Point	277
219.	The Properties of Functions Continuous at a Point	278
219a.	One-Sided (Unilateral) Limits The Jump of a Function	278
220.	The Continuity of a Function on a Closed Interval	279
221.	The Properties of Functions Continuous on a Closed Interval	280

## DIFFERENTIAL CALCULUS

222.	Introductory Remarks	282
223.	Velocity	282
224.	The Derivative Defined	284
225.	Tangent Line	285
226.	The Derivatives of Some Elementary Functions	287
227.	Properties of a Derivative	288
228.	The Differential	289
229.	The Mechanical Interpretation of a Differential	290
230.	The Geometrical Interpretation of a Differential	291
231.	Differentiable Functions	291

232. The Differentials of Some Elementary Functions	294
233. Properties of a Differential	294
234. The Invariance of the Expression $f'(x) dx$	294
235. Expressing a Derivative in Terms of Differentials	295
236. The Function of a Function (Composite Function)	296
237. The Differential of a Composite Function	296
238. The Derivative of a Composite Function	297
239. Differentiation of a Product	298
240. Differentiation of a Quotient (Fraction)	299
241. Inverse Function	300
242. Natural Logarithms	302
243. Differentiation of a Logarithmic Function	303
244. Logarithmic Differentiation	304
245. Differentiating an Exponential Function	306
246. Differentiating Trigonometric Functions	307
247. Differentiating Inverse Trigonometric Functions	308
247a. Some Instructive Examples	309
248. The Differential in Approximate Calculations	311
249. Using the Differential to Estimate Errors in Formulas	313
250. Differentiation of Implicit Functions	315
251. Parametric Representation of a Curve	316
252. Parametric Representation of a Function	318
253. The Cycloid	320
254. The Equation of a Tangent Line to a Plane Curve	321
254a. Tangent Lines to Quadric Curves	323
255. The Equation of a Normal	323
256. Higher-Order Derivatives	324
257. Mechanical Meaning of the Second Derivative	325
258. Higher-Order Differentials	326
259. Expressing Higher Derivatives in Terms of Differentials	329
260. Higher Derivatives of Functions Represented Parametrically	330
261. Higher Derivatives of Implicit Functions	331
262. Leibnitz Rule	332
263. Rolle's Theorem	334
264. Lagrange's Mean-Value Theorem	335
265. Formula of Finite Increments	337
266. Generalized Mean-Value Theorem (Cauchy)	339
267. Evaluating the Indeterminate Form $\frac{0}{0}$	341
268. Evaluating the Indeterminate Form $\frac{\infty}{\infty}$	344
269. Other Indeterminate Expressions	345
270. Taylor's Formula (Historical Background)	347
271. Taylor's Formula	351
272. Taylor's Formula for Computing the Values of a Function	353
273. Increase and Decrease of a Function	360
274. Tests for the Increase and Decrease of a Function at a Point	362
274a. Tests for the Increase and Decrease of a Function in an Interval	363
275. Maxima and Minima	364
276. Necessary Condition for a Maximum and a Minimum	365
277. The First Sufficient Condition for a Maximum and a Minimum	366
278. Rule for Finding Maxima and Minima	366
279. The Second Sufficient Condition for a Maximum and a Minimum	372
280. Finding Greatest and Least Values of a Function	372
281. The Convexity of Plane Curves. Point of Inflection	379
282. Direction of Concavity	380
283. Rule for Finding Points of Inflection	381
284. Asymptotes	383

285.	Finding Asymptotes Parallel to the Coordinate Axes	383
286.	Finding Asymptotes Not Parallel to the Axis of Ordinates	386
287.	Construction of Graphs (Examples)	388
288.	Solution of Equations. General Remarks	392
289.	Solution of Equations. Method of Chords	394
290.	Solution of Equations. Method of Tangents	396
291.	Combined Chord and Tangent Method	398

## INTEGRAL CALCULUS

292.	Introductory Remarks	401
293.	Antiderivative	403
294.	Indefinite Integral	404
295.	Geometrical Interpretation of Integration	406
296.	Computing the Integration Constant from Initial Data	409
297.	Properties of the Indefinite Integral	410
298.	Table of Integrals	411
299.	Direct Integration	413
300.	Integration by Substitution (Change of Variable)	414
301.	Integration by Parts	418
302.	Integration of Some Trigonometric Expressions	421
303.	Trigonometric Substitutions	425
304.	Rational Functions	426
304a.	Taking out the Integral Part	426
305.	Techniques for Integrating Rational Fractions	427
306.	Integration of Partial Rational Fractions	428
307.	Integration of Rational Functions (General Method)	431
308.	Factoring a Polynomial	438
309.	On the Integrability of Elementary Functions	439
310.	Some Integrals Dependent on Radicals	439
311.	The Integral of a Binomial Differential	441
312.	Integrals of the Form $\int R(x, \sqrt{ax^2+bx+c}) dx$	443
313.	Integrals of the Form $\int R(\sin x, \cos x) dx$	445
314.	The Definite Integral	446
315.	Properties of the Definite Integral	450
316.	Geometrical Interpretation of the Definite Integral	452
317.	Mechanical Interpretation of the Definite Integral	453
318.	Evaluating a Definite Integral	455
318a.	The Bunyakovsky Inequality	456
319.	The Mean-Value Theorem of Integral Calculus	456
320.	The Definite Integral as a Function of the Upper Limit	458
321.	The Differential of an Integral	460
322.	The Integral of a Differential. The Newton-Leibniz Formula	462
323.	Computing a Definite Integral by Means of the Indefinite Integral	464
324.	Definite Integration by Parts	466
325.	The Method of Substitution in a Definite Integral	466
326.	On Improper Integrals	471
327.	Integrals with Infinite Limits	472
328.	The Integral of a Function with a Discontinuity	475
329.	Approximate Integration	480
330.	Rectangle Formulas	483
331.	Trapezoid Rule	485
332.	Simpson's Rule (for Parabolic Trapezoids)	486
333.	Areas of Figures Referred to Rectangular Coordinates	488

334.	Scheme for Employing the Definite Integral	490
335.	Areas of Figures Referred to Polar Coordinates	492
336.	The Volume of a Solid Computed by the Shell Method	494
337.	The Volume of a Solid of Revolution	496
338.	The Arc Length of a Plane Curve	497
339.	Differential of Arc Length	499
340.	The Arc Length and Its Differential in Polar Coordinates	499
341.	The Area of a Surface of Revolution	501

### PLANE AND SPACE CURVES (FUNDAMENTALS)

342.	Curvature	503
343.	The Centre, Radius and Circle of Curvature of a Plane Curve	504
344.	Formulas for the Curvature, Radius and Centre of Curvature of a Plane Curve	506
345.	The Evolute of a Plane Curve	508
346.	The Properties of the Evolute of a Plane Curve	510
347.	Involute of a Plane Curve	511
348.	Parametric Representation of a Space Curve	512
349.	Helix	514
350.	The Arc Length of a Space Curve	515
351.	A Tangent to a Space Curve	516
352.	Normal Planes	518
353.	The Vector Function of a Scalar Argument	519
354.	The Limit of a Vector Function	520
355.	The Derivative Vector Function	521
356.	The Differential of a Vector Function	523
357.	The Properties of the Derivative and Differential of a Vector Function	524
358.	Osculating Plane	526
359.	Principal Normal. The Moving Trihedron	527
360.	Mutual Positions of a Curve and a Plane	529
361.	The Base Vectors of the Moving Trihedron	529
362.	The Centre, Axis and Radius of Curvature of a Space Curve	530
363.	Formulas for the Curvature, and the Radius and Centre of Curvature of a Space Curve	531
364.	On the Sign of the Curvature	534
365.	Torsion	535

### SERIES

366.	Introductory Remarks	537
367.	The Definition of a Series	537
368.	Convergent and Divergent Series	538
369.	A Necessary Condition for Convergence of a Series	540
370.	The Remainder of a Series	542
371.	Elementary Operations on Series	543
372.	Positive Series	545
373.	Comparing Positive Series	546
374.	D'Alembert's Test for a Positive Series	548
375.	The Integral Test for Convergence	549
376.	Alternating Series. Leibniz' Test	552
377.	Absolute and Conditional Convergence	553
378.	D'Alembert's Test for an Arbitrary Series	555
379.	Rearranging the Terms of a Series	556
380.	Grouping the Terms of a Series	556

381.	Multiplication of Series	558
382.	Division of Series	561
383.	Functional Series	562
384.	The Domain of Convergence of a Functional Series	563
385.	On Uniform and Nonuniform Convergence	565
386.	Uniform and Nonuniform Convergence Defined	568
387.	A Geometrical Interpretation of Uniform and Nonuniform Convergence	568
388.	A Test for Uniform Convergence. Regular Series	569
389.	Continuity of the Sum of a Series	570
390.	Integration of Series	571
391.	Differentiation of Series	575
392.	Power Series	576
393.	The Interval and Radius of Convergence of a Power Series	577
394.	Finding the Radius of Convergence	578
395.	The Domain of Convergence of a Series Arranged in Powers of $x - x_0$	580
396.	Abel's Theorem	581
397.	Operations on Power Series	582
398.	Differentiation and Integration of a Power Series	584
399.	Taylor's Series	586
400.	Expansion of a Function in a Power Series	587
401.	Power-Series Expansions of Elementary Functions	589
402.	The Use of Series in Computing Integrals	594
403.	Hyperbolic Functions	595
404.	Inverse Hyperbolic Functions	598
405.	On the Origin of the Names of the Hyperbolic Functions	600
406.	Complex Numbers	601
407.	A Complex Function of a Real Argument	602
408.	The Derivative of a Complex Function	604
409.	Raising a Positive Number to a Complex Power	605
410.	Euler's Formula	607
411.	Trigonometric Series	608
412.	Trigonometric Series (Historical Background)	608
413.	The Orthogonality of the System of Functions $\cos nx$ , $\sin nx$	609
414.	Euler-Fourier Formulas	611
415.	Fourier Series	613
416.	The Fourier Series of a Continuous Function	615
417.	The Fourier Series of Even and Odd Functions	618
418.	The Fourier Series of a Discontinuous Function	622

## DIFFERENTIATION AND INTEGRATION OF FUNCTIONS OF SEVERAL VARIABLES

419.	A Function of Two Arguments	626
420.	A Function of Three and More Arguments	627
421.	Modes of Representing Functions of Several Arguments	629
422.	The Limit of a Function of Several Arguments	630
423.	On the Order of Smallness of a Function of Several Arguments	632
424.	Continuity of a Function of Several Arguments	633
425.	Partial Derivatives	634
426.	A Geometrical Interpretation of Partial Derivatives for the Case of Two Arguments	635
427.	Total and Partial Increments	636
428.	Partial Differential	636
429.	Expressing a Partial Derivative in Terms of a Differential	637
430.	Total Differential	638



## 16 CONTENTS

431. Geometrical Interpretation of the Total Differential (for the Case of Two Arguments)	640
432. Invariance of the Expression $\int'_x dx + \int'_y dy + \int'_z dz$ of the Total Differential	640
433. The Technique of Differentiation	641
434. Differentiable Functions	642
435. The Tangent Plane and the Normal to a Surface	643
436. The Equation of the Tangent Plane	644
437. The Equation of the Normal	646
438. Differentiation of a Composite Function	646
439. Changing from Rectangular to Polar Coordinates	647
440. Formulas for Derivatives of a Composite Function	648
441. Total Derivative	649
442. Differentiation of an Implicit Function of Several Variables	650
443. Higher-Order Partial Derivatives	653
444. Total Differentials of Higher Orders	654
445. The Technique of Repeated Differentiation	656
446. Symbolism of Differentials	657
447. Taylor's Formula for a Function of Several Arguments	658
448. The Extremum (Maximum or Minimum) of a Function of Several Arguments	660
449. Rule for Finding an Extremum	660
450. Sufficient Conditions for an Extremum (for the Case of Two Arguments)	662
451. Double Integral	663
452. Geometrical Interpretation of a Double Integral	665
453. Properties of a Double Integral	666
454. Estimating a Double Integral	666
455. Computing a Double Integral (Simplest Case)	667
456. Computing a Double Integral (General Case)	670
457. Point Function	674
458. Expressing a Double Integral in Polar Coordinates	675
459. The Area of a Piece of Surface	677
460. Triple Integral	681
461. Computing a Triple Integral (Simplest Case)	681
462. Computing a Triple Integral (General Case)	682
463. Cylindrical Coordinates	685
464. Expressing a Triple Integral in Cylindrical Coordinates	685
465. Spherical Coordinates	686
466. Expressing a Triple Integral in Spherical Coordinates	687
467. Scheme for Applying Double and Triple Integrals	688
468. Moment of Inertia	689
469. Expressing Certain Physical and Geometrical Quantities in Terms of Double Integrals	691
470. Expressing Certain Physical and Geometrical Quantities in Terms of Triple Integrals	691
471. Line Integrals	696
472. Mechanical Meaning of a Line Integral	697
473. Computing a Line Integral	698
474. Green's Formula	700
475. Condition Under Which Line Integral is Independent of Path	701
476. An Alternative Form of the Condition Given in Sec. 475	703

## DIFFERENTIAL EQUATIONS

477. Fundamentals	706
478. First-Order Equation	708
479. Geometrical Interpretation of a First-Order Equation	708
480. Isoclines	711