Francis Weston Sears

Mechanics, Wave Motion and Heat

ADDISON-WESLEY · WORLD STUDENT SERIES ·

ADDISON-WESLEY PUBLISHING COMPANY, INC.

MECHANICS, WAVE MOTION, AND HEAT

by

FRANCIS WESTON SEARS
Department of Physics
Dartmouth College

ADDISON-WESLEY PUBLISHING COMPANY, INC.
READING, MASSACHUSETTS · PALO ALTO · LONDON

THIS BOOK IS AN ADDISON-WESLEY
WORLD STUDENT SERIES EDITION

First Printing 1965

Library of Congress Catalog Card No. 58-5058

PREFACE

This book is designed as a text for the first year of a two-year course in General Physics, or for an intermediate course in Mechanics and Heat following a briefer introduction to these subjects. In order that the book can be used by a student who is concurrently taking his first course in Calculus, the use of that subject is postponed until Chapter 4.

The book follows the same general plan as the author's earlier text, *Mechanics, Heat, and Sound.* More extended treatment has been given to a number of topics, including angular momentum, elasticity, damped and forced harmonic motion, and wave motion. Vector quantities are consistently represented by bold-face type. The vector product is introduced in Chapter 3 in connection with the moment of a force, and the scalar product in Chapter 7 in connection with work.

Most of the problems are new. Answers are given immediately following odd-numbered problems.

The author wishes to express his gratitude to all those students and teachers whose criticisms and suggestions have helped to guide him in writing this book. He will appreciate having any errors called to his attention.

FRANCIS W. SEARS

April, 1958
Hanover, N. H.

FOREWORD TO THE THIRD PRINTING

It is hoped that as a result of the invitation extended in the last sentence of the Preface, most of the author's mistakes have been found and corrected. Thanks are due in particular to E. H. Clark, L. M. Clendenning, Ronald Flegel, John L. Gergen, David R. Goosman, Franklin A. Ruehl, Jr., and Edwin F. Taylor. And the invitation still holds.

F. W. S.

CONTENTS

CHAPTER 1

COMPOSITION AND RESOLUTION OF VECTORS

1–1 Units and standards. Physics has been called the science of measurement. To quote from Lord Kelvin (1824–1907), "I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of *Science*, whatever the matter may be."

The process of measuring a physical quantity consists of finding the ratio of its magnitude to that of some unit of the quantity. Thus when we say that the length of a rod is 10 centimeters, we mean that its length is 10 times as great as the unit of length called the centimeter. As a result of international collaboration over a long period, practically all of the units used in physics are now the same throughout the world.

It would be possible to define an arbitrary unit of any quantity of interest in physics. As we shall see later, however, units of three properly chosen quantities are sufficient, at least in the field of mechanics. The unit of any other quantity can then be expressed as some combination of these three (in some cases, only two are required). The choice of the three quantities is to some extent arbitrary, and in this book we shall consider them to be *length*, *mass*, and *time*.

A material object which embodies a unit is called a *standard*. The international standard of length is a bar of platinum-iridium alloy of X-shaped cross section (see Fig. 1–1) called the *standard meter*, which is kept at the International Bureau of Weights and Measures at Sevres, near Paris. The distance between two lines engraved on gold plugs near the ends of the bar, when the bar is at the temperature of melting ice, is called *one meter*, and one one-hundredth of this distance is *one centimeter*. Copies of the Paris meter have been distributed to the standardizing agencies of various nations, such as the National Bureau of Standards at Washington, D. C. The meter was originally intended to represent one ten-millionth of the earth's quadrant through Paris, but later, more accurate measurements have shown that it differs from its intended value by a small amount. The standard meter has been carefully compared with the wavelength of one particular color of light emitted by cadmium vapor in an electrical discharge, and if it were ever destroyed it could be replaced with an accuracy better than one part in a million.

The *yard*, originally embodied by a physical standard like the meter, is now defined as

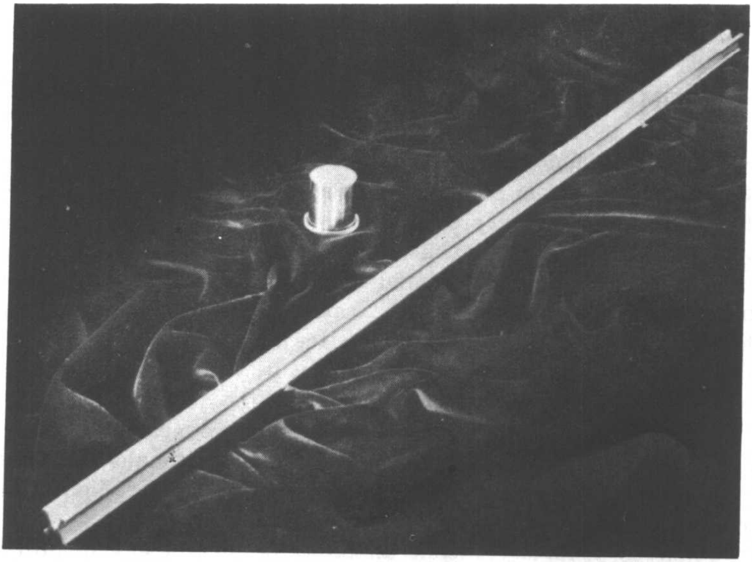

FIG. 1–1. The standard meter and the standard kilogram.

$$1 \text{ yard} = \frac{3600}{3937} \text{ meter (exactly)},$$

so that the same physical standard serves for both the yard and the meter. The *foot* is defined as one-third of a yard.

The international standard of mass* is a cylinder of platinum-iridium (see Fig. 1–1) called the *standard kilogram*. It was originally intended to have a mass equal to that of 1000 cm^3 of pure water at 4°C but, as with the standard meter, more precise measurements have shown that this is not exactly true.

The pound mass, like the yard, was formerly embodied by a physical standard of its own, but the *standard avoirdupois pound* is now defined in the U. S. as a body of mass 0.4535924277 kgm.

The standard of time is our rotating earth, and the fundamental unit of time is the *mean solar day*, the average time for the earth to make one revolution on its axis with respect to the sun. The length of a solar day increases and decreases gradually in the course of a year because of the orbital motion of the earth. The length of a solar day averaged over a year is the mean solar day.

---

* The concept of mass is discussed more fully in Chapter 5.

The unit of time used in scientific work is the *second,* defined as 1/86,400 of a mean solar day.

We shall adopt the convention that an algebraic symbol representing a physical quantity stands for both a *numerical value* and a *unit.* Thus the symbol l might represent a length of 10 ft, the symbol m a mass of 15 kgm, and the symbol t a time of 20 sec. The numerical value and the unit together constitute the *magnitude* of the quantity. The numerical value is the ratio of the magnitude of the quantity to that of the unit.

An algebraic symbol occurring in a physical equation can be replaced by the magnitude it represents but not by the numerical value alone. For example, the density ρ of a homogeneous body is defined as the quotient of its mass m and its volume V:

$$\rho = m/V.$$

Then if $m = 150$ gm and $V = 20.0\ \text{cm}^3$,

$$\rho = \frac{m}{V} = \frac{150\ \text{gm}}{20.0\ \text{cm}^3} = 7.50\,\frac{\text{gm}}{\text{cm}^3}.$$

The numerical value of the density is 7.50 and the unit is 1 gm/cm^3.

1–2 Force. Mechanics is the branch of physics which deals with the motion of material bodies and with the forces that bring about the motion. Since motion is best described by the methods of calculus and many readers of this book are just beginning their study of this subject, we shall postpone a discussion of motion until Chapter 4, and start with a study of forces.

When we push or pull on a body, we are said to exert a *force* on it. Forces can also be exerted by inanimate objects; a stretched spring exerts forces on the bodies to which its ends are attached, compressed air exerts a force on the walls of its container, a locomotive exerts a force on the train it is drawing. The force of which we are most aware in our daily lives is the force of gravitational attraction exerted on every body by the earth, and is called the *weight* of the body. Gravitational forces (and electrical and magnetic forces also) can act through empty space without contact. In this respect they differ from the forces mentioned above, where the body doing the pushing or pulling must make contact with the body being pushed or pulled.

The process of measuring a physical quantity consists of finding the ratio of its magnitude to the magnitude of some unit of the quantity. Thus to measure a force we must find the ratio of its magnitude to that of a unit of force. Two steps are therefore involved: (1) a definition of the ratio of the magnitudes of two forces, and (2) the selection of a unit force.

One effect of a force is to alter the dimensions or shape of a body on which the force acts; another is to alter the state of motion of the body. Either of these effects can be used to define the ratio of the magnitudes of two forces. In this Chapter we shall consider only the former; the latter will be discussed in Chapter 5.

The instrument most commonly used to measure forces is the *spring balance*, which consists of a coil spring mounted in a case for protection and carrying at one end a pointer that moves over a scale. The position of the pointer indicates the extension of the spring above its no-load length. When one end of the spring is kept fixed and a stretching force is exerted on the other end, the length of the spring increases and the pointer moves along the scale. *The ratio of the magnitudes of two forces F_1 and F_2 is defined as equal to the ratio of the extensions x_1 and x_2 which they produce in the same spring:*

$$\frac{F_1}{F_2} = \frac{x_1}{x_2}.$$

If the definition is to be meaningful, the same ratio must be obtained for two given forces whatever spring is used. This is found to be the case provided the spring is not stretched beyond its proportional limit of elasticity.

A unit of force *might* now be defined as the force necessary to stretch a standard spring by some specified amount. Such a definition is open to the objection that the elastic properties of the standard spring might change over a long period of time and the standard would not be permanent. To avoid this difficulty, we define a unit force as the force of the earth's gravitational attraction for some standard body at a specified point on the earth's surface. That is, the unit force is defined as the *weight* of the standard body. Three such units of force are in common use:

One kilogram-force (1 kgf) *is defined as the weight of a standard kilogram body.*

One gram-force (1 gf) *is defined as the weight of a standard gram body.*

One pound-force (1 lbf) *is defined as the weight of a standard pound body.**

That is, each of these units is defined as *the force of the earth's gravitational attraction on a specified body.* Since the earth's attraction for a given body varies slightly from point to point on the earth's surface, some particular

* We use the abbreviations kgf, gf, and lbf for these units of force, rather than kg, g, and lb, because the names kilogram, gram, and pound are also given to units of *mass*. The latter units will be abbreviated kgm, gm, and lbm.

point must be specified. The effect of the earth's rotation must also be taken into account. A more complete discussion will be found in Chapter 13.

Now suppose we wish to calibrate a spring balance in, say, pounds-force. A standard pound body is first suspended from the balance. The force stretching the spring is then equal to the weight of the standard pound body, or is 1 lbf. The position of the pointer is noted and is marked 1 lbf. The position of the pointer at twice this extension can now be marked 2 lbf, and so on.

1–3 Graphical representation of forces. Vectors. Suppose we are to slide a box along the floor by pulling it with a string or pushing it with a stick, as in Fig. 1–2. That is, we are to slide it by exerting a force on it.

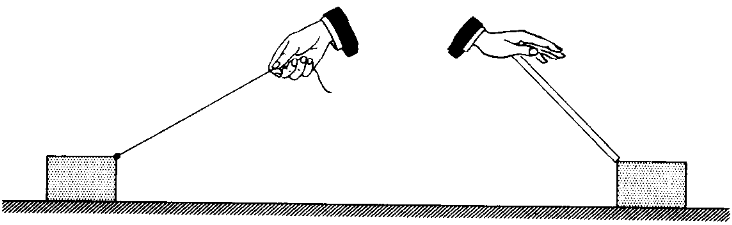

FIGURE 1–2

The point of view which we now adopt is that the motion of the box is caused not by the *objects* which push or pull on it, but by the *forces* which these exert. For concreteness, assume the magnitude of the push or pull to be 10 lbf. It is clear that simply to write "10 lbf" on the diagram would not completely describe the force, since it would not indicate the direction in which the force was acting. One might write "10 lbf, 30° above horizontal to the right," or "10 lbf, 45° below horizontal to the right," but all the above information may be conveyed more briefly if we adopt the convention of representing a force by an arrow. The length of the arrow, to some chosen scale, indicates the size or magnitude of the force, and the direction in which the arrow points indicates the direction of the force. Thus Fig. 1–3 is the force diagram corresponding to Fig. 1–2. (There are other forces acting on the box, but these are not shown in the figure.)

Force is not the only physical quantity which requires the specification of a direction in space as well as a magnitude. For example, the velocity of an aircraft is not completely specified by stating that it is 300 miles per hour; we need to know the direction also. The concept of volume, on the other hand, has no direction associated with it.

Quantities like volume, which involve a magnitude only, are called