THE CALCULUS
WITH ANALYTIC GEOMETRY

Fifth Edition

Louis Leithold

THE CALCULUS
WITH ANALYTIC GEOMETRY

Fifth Edition

Louis Leithold
Pepperdine University

1817

HARPER & ROW, PUBLISHERS, New York
Cambridge, Philadelphia, San Francisco,
London, Mexico City, São Paulo, Singapore, Sydney

Sponsoring Editor: Ann Trump
Project Editor: David Nickol
Text and Cover Design: Edward A. Butler
Text Art: Fineline Illustrations, Inc.
Production: Debra Forrest
Compositor: Syntax International Pte. Ltd.
Printer and Binder: Kingsport Press

Library of Congress Cataloging-in-Publication Data
Leithold, Louis.
The calculus with analytic geometry.
Includes indexes.
1. Calculus. 2. Geometry, Analytic. I. Title.
QA303.L428 1986 515'.15 85-22037
ISBN 0-06-043926-2

HARPER INTERNATIONAL EDITION
ISBN 0-06-350413-8

85 86 87 88 9 8 7 6 5 4 3 2 1

To the pursuit of excellence in teaching and to the following teachers who influenced me the most:

Florence Balensiefer; English, Lowell High School, San Francisco
Ivan Barker; Mathematics, Lowell High School, San Francisco
Alan McKeever; Journalism, Lowell High School, San Francisco
Benjamin Bernstein; Mathematics, University of California, Berkeley
Pauline Sperry; Mathematics, University of California, Berkeley
Virginia Wakerling; Mathematics, University of California, Berkeley

ACKNOWLEDGMENTS

Reviewers of the fifth edition of *The Calculus with Analytic Geometry*
Linda H. Boyd, DeKalb Community College, Clarkston
Robert A. Chaffer, Central Michigan University, Mt. Pleasant
Jacqueline M. Dewar, Loyola Marymount University, Los Angeles
Nicholas J. DeLillo, Manhattan College, New York City
Larry S. Dilley, Central Missouri State University, Warrensburg
Linda L. Exley, DeKalb Community College, Clarkston
W. W. Mitchell, Jr., Phoenix College, Phoenix
Roger B. Nelsen, Lewis and Clark College, Portland
Raymond Plankinton, Central Community College, Columbus
George W. Schultz, St. Petersburg Junior College, Clearwater
Delores A. Williams, Pepperdine University, Malibu

Assistants for Answers of Exercises
Charles Stone, DeKalb Community College, Clarkston
Carol Stanton, University of California, Riverside

Cover and Chapter Opening Artist
Joyce Treiman, Pacific Palisades
Courtesy of Tortue Gallery, Santa Monica

To these people, to the staff of Harper & Row, and to all the users of the first four editions who have suggested changes, I express my deep appreciation.

L.L.

THE COVER AND CHAPTER OPENING ARTIST

Joyce Treiman had her first solo exhibition in Chicago in 1943. Since then she has become a well-known American painter with a multitude of exhibitions, prizes, and awards.

In 1985, Joyce Treiman attended some of my calculus classes at Pepperdine University where she observed me and the students, and the interaction among us. During these classes she made sketches and took photographs, and from them she created the drawings that are reproduced on the cover and the chapter opening pages.

L.L.

PREFACE

The fifth edition of *The Calculus with Analytic Geometry,* like the other four, is designed for prospective mathematics majors as well as for students whose primary interest is in engineering, the physical and social sciences, or nontechnical fields. As with previous editions, I have endeavored to achieve a healthy balance between the presentation of elementary calculus from a rigorous approach and that from the intuitive and computational point of view. Bearing in mind that a textbook should be written for the student, I have attempted to keep the presentation geared to a beginner's experience and maturity. I desire that the reader be aware that proofs of theorems are necessary and that these proofs be well motivated and carefully explained so that they are understandable to the student who has achieved an average mastery of the preceding sections of the book. If a theorem is stated without proof, I have generally augmented the discussion by both figures and examples, and in such cases I have always stressed that what is presented is an illustration of the content of the theorem and is not a proof.

In this edition, the structure of the text has been altered. The material is now divided into four segments:

Prelude

Part 1 Functions of a Single Variable

Part 2 Infinite Series

Part 3 Vectors and Functions of More Than One Variable

There are fifteen sections appearing at the ends of some of the chapters that are designated as *supplementary.* They are self-contained and can be included or omitted without affecting the understanding of subsequent material. These supplementary sections are of three types:

1. Additional subject matter that is not necessarily part of the traditional syllabus of a calculus course: Sections 3.12, 7.5, 8.11, 8.12, 10.9, 13.9, and 18.5.
2. More applications of the calculus: Sections 3.11, 4.10, 5.8, 5.9, and 16.6.

3. Further theoretical discussions including proofs of some theorems: Sections 1.9, 1.10, and 15.8.

"Prelude" consists of the single Chapter 0, "Topics in Precalculus." It contains basic facts about the real-number system, and this treatment is less detailed than in previous editions. The introduction to analytic geometry in this chapter includes the traditional material on straight lines as well as that on the circle. The definition of a function, operations with functions, and particular kinds of functions are discussed. The presentation of the six trigonometric functions here allows their early use in examples of differentiation and integration of nonalgebraic functions

Part 1, "Functions of a Single Variable," is comprised of Chapters 1 through 10. A thorough discussion of limits and continuity is contained in Chapter 1. The definition of limit is stated in the form "if a then b" rather than its logical equivalent "b whenever a," and this form of the definition is used throughout the text. Proofs of some theorems on limits appear in Supplementary Sections 1.9 and 1.10. The treatment of continuity has been modified, and there is an added discussion of continuity of the trigonometric functions. Some additional geometrical interpretations are included.

In Chapter 2, "The Derivative and Differentiation," before giving the formal definition of a derivative I have defined the tangent line to a curve to demonstrate in advance its geometrical application. The derivatives of all six trigonometric functions are presented here, and they are then available as examples for the initial presentation of the chain rule. Section 2.7, "The Derivative of a Composite Function," has been rewritten with more examples and illustrations. The chain rule is now stated with composite function notation and the proofs for both the special case and the general case are given with this symbolism. The discussion of notations for the derivative has been rewritten with the Leibniz notation $\frac{dy}{dx}$ introduced earlier than in previous editions. The section on the differential has been moved forward to this chapter.

Chapter 3 gives the traditional applications of the derivative to problems involving maxima and minima, as well as to curve sketching. There has been a reordering of topics in this chapter. Concavity and points of inflection appear before the second-derivative test, and all the graphing techniques are presented before the second section on absolute extrema. Infinite limits are introduced here because they are useful for graphing. They also can be applied when determining extreme function values. New to this edition are the discussion of oblique asymptotes in Section 3.8 and the presentation of Newton's method in Supplementary Section 3.12.

The topics of antidifferentiation and the definite integral are combined in Chapter 4. Section 4.2, "Some Techniques of Antidifferentiation," has been rewritten with more examples and illustrations, and the antidifferentiation technique of "changing the variable" is used instead of "substitution." The chain rule for antidifferentiation is now proved with composite function notation. I use the term "antidifferentiation" instead of "indefinite integration," but the standard notation $\int f(x)\, dx$ is retained. This notation will suggest that some relation must exist between definite integrals and antiderivatives, but I see no harm in this as long as the presentation gives the theoretically proper view of the definite integral as the limit of sums. In Chapter 4, there is an introduction to differential equations, and the complete discussion of area of a plane region appears here. Most of the applications of the definite integral are now in a single chapter (Chapter 5) as was the situation in the first three editions; two of them are in supplementary sections.

The coverage of inverse functions in the first two sections of Chapter 6 has been revised. Also in Chapter 6, applications of the natural exponential function include the laws of growth and decay as well as some new material on bounded growth involving the learning curve. In Chapter 7, the domains of the inverse secant and inverse cosecant functions have been redefined so that the formulas for their derivatives do not involve absolute value. The section on inverse hyperbolic functions has been designated as supplementary.

In Chapter 8, the exercise sets have been expanded to include more applications of the various techniques of integration. Section 8.6 contains new coverage of logistic growth with applications to biology and sociology. Numerical integration has been moved to this chapter. A new section, labeled supplementary, involves the use of a table of integrals.

Chapter 9, "Indeterminate Forms, Improper Integrals, and Taylor's Formula," has been moved forward to precede the one on analytic geometry topics. A discussion of the probability density function has been added to give another application of improper integrals. Chapter 10, "Polar Coordinates and the Conic Sections," contains a major rewrite of the corresponding material that appeared in the fourth edition. The cartesian equations of the conics are obtained first. The polar equations do not appear until Section 10.8, where they occur as part of a unified treatment of conic sections. New to this edition is a discussion of when the limaçon has a dent and when it hasn't.

The presentation of infinite series appears in a separate part (Part 2) to make it more apparent that it is self-contained and can be covered anytime after the completion of Part 1. Some of the material has been rewritten with more motivation added. The treatment is now in two chapters instead of one, and some of the longer sections in the fourth edition have been split into two. New to this edition is a discussion of the root test. Also new is a summary of tests for convergence of an infinite series. The exercise sets in Chapters 11 and 12 have been expanded to include more applications.

Part 3 contains the calculus of functions of more than one variable and vector calculus with a vector approach to solid analytic geometry. In Chapter 13, "Vectors in the Plane and Parametric Equations," and Chapter 14, "Vectors in Three-Dimensional Space and Solid Analytic Geometry," there has been a rewriting and reordering of some of the material. The discussions of scalar projection, vector projection, and cross product have been expanded and the triple vector product introduced. Additional applications have been incorporated and new examples and illustrations have been added.

In Chapter 15, "Differential Calculus of Functions of More Than One Variable," the treatment of functions, limits, and continuity has been revised with new examples and illustrations. The material on differentiability and the total differential has been rewritten. The proof of the theorem that gives sufficient conditions for a function of two variables to be differentiable at a point has been deferred to Supplementary Section 15.8. New topics in Chapter 16 include exact differential equations, appearing in Section 16.5, and the method of least squares, which is the subject of Supplementary Section 16.6.

Chapter 17, "Multiple Integration," remains essentially the same as in the fourth edition, although some new exercises have been incorporated. The section on Green's Theorem that appeared in the chapter on multiple integration in the fourth edition is now part of Chapter 18, "Introduction to the Calculus of Vector Fields," which has an expanded coverage of vector calculus. The discussion of line integrals has also been moved to Chapter 18. The new topics here are surface integrals, divergence and curl in three dimensions, Gauss's Divergence Theorem, and Stokes's Theorem. The approach in Chapter 18 is intuitive and the applications are to physics and engineering.

There are now over 7000 exercises that have been revised and graded to provide a wide variety of types that range from computational to applied and theoretical problems. The answers to the odd-numbered exercises are given in the back of the book, and the answers to the even-numbered ones are available in a separate booklet. Detailed step-by-step solutions for nearly half the even-numbered exercises (those having numbers divisible by four) appear in a supplement to this text, *An Outline for the Study of Calculus*, by John H. Minnick, published by Harper & Row, in three volumes.

Louis Leithold

CONTENTS

PRELUDE

PRELUDE

You must enter into the study of calculus with a knowledge of certain mathematical concepts. In the first place, it is assumed that you have had courses in high school algebra and geometry. Secondly, there are particular topics that are of special importance. You may have studied these topics in a precalculus course or you may be exposed to them here in Chapter 0 for the first time.

You need to be familiar with facts about the *real numbers* and have facility with operations involving inequalities, and this material forms the subject matter of the first section. The next three sections contain an introduction to some of the ideas of *analytic geometry* that are necessary for the sequel.

The notion of a *function* is one of the important concepts in calculus, and it is defined here as a set of ordered pairs. This idea is used to point up the concept of a function as a correspondence between sets of real numbers. You have probably studied *trigonometric functions* in a previous course, but a review of the basic definitions is presented, and important formulas you will need for the purposes of calculus are provided. There is also an application of the tangent function to the slope of a line.

Dependent upon your preparation, Chapter 0 may be covered in detail, treated as a review, or omitted.

Z E R O

TOPICS IN PRECALCULUS