AUTOMATIC CONTROL

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An Introduction

by

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PREFACE

This text has been prepared primarily for engineering undergraduates and for students following courses leading to the Diploma in Technology; it also covers the theoretical needs of students reading Control for Higher National Certificates and Diplomas in Mechanical and Production Engineering and of those taking certain professional examinations. It is believed that many practising engineers and applied physicists, who have not had an opportunity of following a prescribed course, will gain a clear understanding of control principles by reading the text before following up those particular references which may have a direct bearing on their duties.

The approach to the subject is via an analytical study of common transfer functions leading to the use of the root locus. Transient and harmonic analyses of closed-loop systems are shown to derive naturally from the root locus pattern. The treatment is concluded with a study of Bode and Nichols diagrams and of various techniques used for stabilization.

The book concentrates on linear single-loop systems, care being taken where necessary to point out the assumptions required for linearity. Reference is made to the Laplace transformation method for those readers who wish to make use of it, but no part of the text requires a knowledge of operational methods for its full understanding.

Whilst frequent reference is made to applications, the book does not attempt to describe manufactured components used in control systems. In the university, laboratory work serves to introduce the student to real components and systems; for this subject in particular, the author regards laboratory work as an essential adjunct to study and a list of suitable experiments is suggested in the Appendix. The problems at the end of the chapters also indicate applications of the text.

Many existing texts concentrate on one specialized field, requiring the student to select chapters from a number of books in order to acquire the essential wide background to enable him to apply the best of what is known in all fields to particular problems. The present treatment is intended to give the reader a wide and general appreciation of Control in all its fields, and the problems included at the end of each chapter cover servo-mechanisms, process control, speed governors, etc. It is hoped that the problems will encourage the student to inquire into the practical aspects of control engineering.

The author wishes to acknowledge the permission of the Senate of the University of London and of the Council of the Institution of Mechanical Engineers to publish questions from past examination papers; the University and the Institution are in no way committed to approval of any answers quoted. Certain questions have been rearranged to ensure uniformity and conformity with British Standard unit symbols. Figs. 4.2, 4.3, 4.5, 4.10, 4.15, 4.16, 5.1 and 5.2 are based upon diagrams in AUTOMATIC CONTROL by Ahrendt and Taplin, McGraw-Hill (1951), and the author wishes to acknowledge the publisher's permission to adapt these diagrams.

Finally, the author wishes to acknowledge the excellent suggestions of Professor G. D. S. MacLellan, Rankine Professor of Mechanical Engineering in the University of Glasgow and his indebtedness to his colleague, D. R. J. Mudge, Esq., for his painstaking reading of the manuscript and for suggestions which have greatly increased the clarity of the mathematical presentation.

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LIST OF SYMBOLS

A	a point
A	coefficient, gain parameter, phasor to origin, area
B	a point
B	coefficient, phasor to $(-1,0)$ point
C	a point
C	capacitance
D	differential operator d/dt
$\mathrm{D_p}$	partial differential operator $\partial/\partial t$
$D(s)$	denominator polynomial
$E(\mathrm{D})$, E	load transfer operator
E_0	$E(s)$ for $s \to 0$
$E(s)$	load transfer function
$\mathrm{F}(s)$	transform of $\mathrm{f}(t)$
$F(s)$	s-dependent part of $G(s)$
$G(\mathrm{D})$, G	forward path transfer operator
G_0	$G(s)$ for $s \to 0$
$G(s)$	forward path transfer function
$G'(s)$	equivalent $G(s)$
$G_1(s)$	controller transfer function
$G_2(s)$	plant or process transfer function
H	liquid head, hysteresis zone
I	current
J	inertia
K	scalar part of $G(s)$
$K_1, K_2, \ldots$	partial fraction coefficients
K_v	velocity constant
L	inductance, load change, an integer
M	closed-loop magnification
M_{max}	maximum value of M
$N(s)$	numerator polynomial
P	a point
P	coefficient for particular integral
P_p, P_d, P_i	controller phasor components
Q	a point
Q	flow rate, coefficient for particular integral
R	resistance, coefficient for complementary function
R_c	critical resistance

R_1	residue of zero s_1
R_a	residue of pole s_a
T_1	torque input
$T(D)$	transfer operator
$T(s)$, $T'(s)$	transfer function
U	transform of u
Y	describing function
Z	dead zone or threshold
a	(natural) damping factor
$a_0, a_1, \ldots$	polynomial coefficients
a_m	numerator coefficients
b	natural pulsatance (damped)
b_n	natural undamped pulsatance
b_n	denominator coefficients
c	viscous damping coefficient, specific heat capacitance, coefficient
c_c	critical viscous damping coefficient
d	symbol in differential operator d/dt
e	base of natural logarithms
$f(t)$	function of time
$g(t)$	unit step response
$g'(t)$	unit impulse response
h	change of liquid head
j	imaginary operator
k	proportional control factor, spring stiffness, coefficient of thermal conductivity, constant
k_1	coefficient (time constant) applied to subsidiary feedback path
l	thickness
m	order of numerator, mass
n	order of denominator, a number
p, q	Fourier coefficients
r	root in auxiliary equation, order of Fourier coefficient, radius
$r_1, r_2, \ldots$	modulus of numerator factor
$r_a, r_b, \ldots$	modulus of denominator factor
s	complex pulsatance, Laplace variable
$s_1, s_2, \ldots$	zeros, numerator roots
$s_a, s_b, \ldots$	poles, denominator roots
s_r	arbitrary point on s-plane

s', s''	closed-loop poles
t, t_1	time
t_i	instant of time
u	load change parameter
$u(t)$	unit step function
$u'(t)$	unit impulse function
x, y	co-ordinates
α	damping factor, in-phase part of s
β	damping angle
γ	phase angle
$\delta_1, \delta_2, \ldots$	phase of numerator factor
$\delta_a, \delta_b, \ldots$	phase of denominator factor
ε	pulsatance ratio in octave
ζ	damping ratio
η	thermal diffusivity
θ	error or deviation
θ_1	input or stimulus
θ_{1S}	input incurring saturation
$\bar{\theta}_1$	amplitude of input
θ_2	output or response
θ_{2m}	on–off output magnitude
θ_i	desired value
θ_o	controlled value
λ	pulsatance ratio or dimensionless pulsatance
μ	dimensionless gain, forward path gain
ν	modulus in dB
ξ	angle
ρ	instant of time
σ	time variable
τ, τ'	time constant
τ_v	finite delay time
τ_d	derivative action time (constant)
τ_i	integral action time (constant)
ϕ	closed-loop phase lag
χ	function of distance
ψ	phase angle
ω	pulsatance, quadrature part of s
ω_r	arbitrary pulsatance
ω', ω''	closed-loop pulsatance
ω_1	pulsatance for M_{max}

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Background

Automatic control, although its widespread application began in the 1930's, was probably first used several centuries ago. Examples include the use of a fantail for automatically facing the mainsail of a windmill into the wind, simple pressure cookers and the speed-governing of steam engines. The reader is referred to Macmillan[32] for an informed account of the history. The first *servo-mechanism* appears to have been the application of power steering to ships. The year 1930 saw the beginnings of the widespread application of control in the chemical industry and in oil refineries, a field which tended to develop separately and to become known as *process control*. Military requirements accelerated the development of fire-control systems during the second world war and this was accompanied by a scientific evaluation of the possibilities and limitations of automatic control techniques.

Relevant early works in English include those of Maxwell,[1] Routh,[2] Trinks,[3] Minorsky,[4] Nyquist,[5] Hazen,[6] Ivanoff,[7,8] Callender *et al.*,[9] Bode,[10] Ziegler and Nichols,[13] Hall[14] and Smith.[15] Later papers and texts are too numerous to mention in detail but a short list may be found in the bibliography; many cover more advanced work than the present volume and are suitable for further and specialized study.

1.2 The control concept

All readers will have met many applications of automatic control in their everyday experience. In the home, temperature control is available on refrigerators, cookers, domestic boilers, electric irons, washing machines, water heaters, space heaters, etc.; in road vehicles there is control of the battery charging-rate and of the engine coolant temperature; in business premises and factories, heating and ventilating are often controlled automatically.

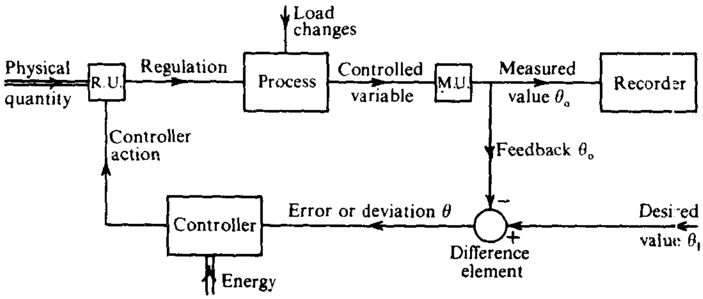
Unfortunately, the term *automatic control* is often misused in connection with devices which regulate machines; the control is said to be *automatic* whenever it sets in motion a sequence of pre-arranged operations. *Control* is used in a restricted sense in the present text, viz. a mechanism or *controller* is used which endeavours to maintain a physical variable in a plant or process at some *desired value* or to make it follow some desired law, by making continuous use of measurements of the variable under control, continuously computing the *error* or *deviation* compared with the desired value, and making continuous adjustments to the energy input to the process in a manner which results in the *deviation* remaining minimal. The essential feature of a control system is that the controller makes adjustments *based on continuous measurements of the variable it is controlling*. In other words, information is fed back to the controller to enable it to assess how well it is performing its task and to decide what action is best appropriate at every instant; for this reason, the term *feedback control* is often used.

Automatic control has been developed along similar but separate lines in various fields and one of the difficulties of pursuing a general study is the use of different terminology between one field and another. B.S. Spec. 1523/1960 on Automatic Controlling and Regulating Systems is published in several sections covering different branches and is not helpful in bringing about a composite approach. Examples from each main branch are described below to emphasize the main differences, none of which demands a separate analytical treatment. In later chapters, no distinction is made except where particular terminology and techniques find favour in special cases for convenience or by usage.

1.2.1 *Process control systems*

In this branch, interest centres on the control of such quantities as pressure, flow, level, temperature and pH value in relation to chemical processes. A diagrammatic representation of such a control system is shown in fig. 1.1, where individual elements of the system are denoted by boxes, and lines are drawn from one box to another to imply that the *output* from the one box is identical with the *input* to the other. The outputs and inputs are quantities of all kinds having widely differing dimensions and are conveniently described by the term *signal*. Such diagrams are called *block diagrams*, and are widely used to represent systems or portions of systems to avoid the need to represent each element by a detailed drawing showing its physical nature.

The box marked M.U. is the *measuring unit* which is an element which senses the magnitude of the variable under control and provides an output signal, in any convenient physical form, referred to as the *measured* or *controlled value* and denoted by θ_o. Normally, θ_o is fed to a recording instrument to provide a continuous visual record of the manner in which the control system is performing its task. More important, however, is the feedback of θ_o to the *difference element* where it is compared with the *desired value* θ_i of the



1.1 Block diagram of process control system

variable under control which is fed in manually and in the same physical form as θ_o. The task of the difference element is to perform the subtraction sum $(\theta_i - \theta_o)$ to give an output referred to as the *error* or *deviation* θ. Normally, θ_i and θ_o are measured from some convenient datum; the latter is often the steady-state value of θ_i or θ_o when θ is zero.

The deviation signal is fed to the controller, the task of which is to provide an output which is capable of regulating the process; for this purpose and to enable it to make a computation, it is supplied with energy, often in the form of compressed air or electricity. The mechanism of the controller is designed to take note of the magnitude and sense of the deviation and to regulate its output to some suitable function of the deviation; often it is a simple amplifier but where necessary (according to the complication of the process), the output signal also includes signals proportional to the rate of change of error and to the time integral of error as well as the signal proportional to the error itself. The controller output or *action* is fed to the *regulating unit* (marked R.U. in the figure) which often takes the form of a pneumatically or electrically operated valve. The physical quantity feeding the process, which may be the flow of a chemical substance or the supply of a fuel, is thus regulated so as to maintain the deviation within desired limits continuously.

In the case of process control, there is usually only one optimum value of θ_i, the desired value of temperature, pressure, etc., determined in accordance with the nature of the process, so that the control system is not normally called upon to deal with changes of θ_i. The *external load* on the process, e.g. the demand for steam from a boiler, is, however, likely to vary considerably and this is the main form of *disturbance* with which the control system is designed to deal.

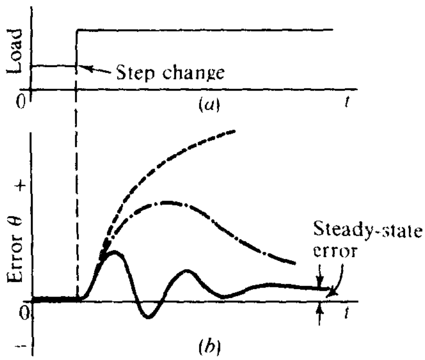
The reader should note that signals pass from one element to another in a closed loop and for this reason the term *closed-loop control* is often used. In the absence of automatic control, an operator would make manual adjustments to the regulating valve based on his observation of the measured value θ_o; the operator thus takes the place of the difference element and the controller and in effect is the link closing the loop.

It is now clear that an error must develop before the controller can take any corrective action; for this reason yet another term, *error-actuated control* is sometimes used. If the controller is sufficiently sensitive, however, a very small error will enable the controller to change the regulation of the process. For example, following a sudden and sustained load change as indicated in fig. 1.2(*a*) an error in the controlled variable will begin to develop as shown in fig. 1.2(*b*). In the absence of control, i.e. if the regulating valve was unaltered, the error would probably follow the plain-dotted curve. However, the controller senses the error and takes corrective action by changing the regulation so as to reduce the error. The error is seen to reach a maximum and then to decrease.

In the case of less sensitive control, the error might follow the chain-dotted curve; normally, however, the controller over-corrects, i.e. the controlled variable *overshoots* the desired value, resulting in an error developing in the reverse direction. Similar corrective action takes place, often resulting in a further overshoot; the *error response* curve, i.e. the error–time curve following the disturbance, often takes the form of a rapidly-damped sinusoid as indicated by the full line.

It may appear, from fig. 1.2, that the object of automatic control, i.e. of preventing excessive error arising, has not been met. However, the error scale has been exaggerated for clarity and the maximum error during the transient need only be quite small in a well-designed system.

The full curve of fig. 1.2 is shown as levelling out at a value of error other than zero; it is said that a *steady-state error* or *offset* remains as a result of the sustained disturbance. This normally occurs when the controller generates only a signal proportional to error; controllers which generate integral control in addition are able to regulate without leaving a steady-state error.



1.2 Response to load disturbance

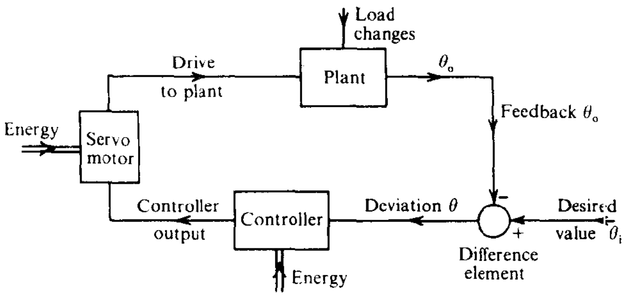
In most process control applications, the periodic time of the response curve is large; this is due to the large time lags associated with process plant. Controllers, which may therefore be designed to operate at comparatively low frequency, are usually supplied as separate self-contained instruments, often incorporating the difference element, and may be used satisfactorily to control one of a variety of different kinds of variable. In contrast, in the following branches of control, a special controller is often required for each application and this may be integrated with the process.

1.2.2 *Servo-mechanisms*

The term *servo-mechanism* is applied to systems mainly concerned with the control of the motion of a body, such as a gun, an aircraft or a machine tool, etc. The block diagram of fig. 1.3 serves to illustrate the general arrangement.

The main object of the control is to displace the plant to some specified co-ordinates in such a manner that it follows the continually changing desired value signal θ_i. Load changes sometimes occur but the main disturbances affecting the system are desired value changes; again, the controller is required to keep the deviation θ to a minimum.

Figure 1.3 again suggests a closed-loop system; the unit measuring the displacement or position of the plant is omitted for simplicity. Desired and actual values of displacement are fed to the difference element which computes the deviation signal fed to the controller. Again the controller may generate the sum of several signals, based on the information it receives, for the purpose of



1.3 Block diagram of servo-mechanism

actuating the *servo-motor.* The latter, which together with the controller may function electrically or hydraulically, is used to drive the plant to the desired position and replaces the regulating unit in the process control system.

Error responses to desired value changes are similar to that shown in fig. 1.2, but are normally of much higher frequency compared with process control applications; it follows that all elements in the loop must be designed to function properly over an entirely different frequency band.

1.2.3 *Regulators*

The term *regulator* is used for mechanical and electrical systems which are concerned with the maintenance of constant speed, constant voltage, etc., irrespective of the external load on the plant. They are like process control systems in that the main kind of disturbance is in external load and similar to servo-mechanisms in that their frequency of operation is high.

A common regulator is a system designed to control the speed of a prime mover, i.e. a *governor*. The reader should note that the mechanism which is often referred to as a governor in texts on applied mechanics is what is understood here as a measuring element, i.e. it provides a signal in the form of a movement of a link which is a function of the speed measured. A governor, in control terminology, refers to the entire speed-control system.

1.3 Alternative forms of control system

There are two classes of control system which differ from those described above in that neither are continuously variable closed-loop systems. They are sufficiently common, however, to merit a brief description.

1.3.1 *On–off control*

In many simple applications of closed-loop control, it suffices to restrict the value of the controlled variable θ_o within a specified band rather than to aim at equality with θ_i at every instant. In such cases, a simple and inexpensive controller can be employed, the output of which can regulate the physical quantity to the process at only two specified values, normally maximum and zero.

A well-known example is an electric water-heater in which an on–off thermostat acts as a switch in the circuit of the heater. The thermostat switches on the current when the water temperature falls below a given value and opens the circuit when the temperature rises above another, but higher, given value. Such a system is clearly much cheaper to manufacture than one which continuously varies the heater current; moreover, it is quite adequate for such purposes and is likely to require little or no maintenance.

On–off controls are very widely used and, for most applications, no theoretical analysis is called for. An approximate method of analysing on–off systems is included in chapter 9.

1.3.2 *Control without feedback*

Some classes of system operate without making use of measurements of the variable under control, i.e. there is no feedback of θ_o to the controller. In some circumstances such systems function quite satisfactorily. Some examples are described below to illustrate the principles involved.

Certain earlier types of gas water-heater contain a mechanism in which the gas flow to the heating element is adjusted automatically by a venturi device according to the flow rate of cold water supplied