A. BOUWERS

ACHIEVEMENTS IN OPTICS

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by

A. BOUWERS

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AUTHOR'S PREFACE

This monograph intends to give a survey of work on optics in the Netherlands during the last war or immediately before. Part of the results mentioned have already been published, but they have not of course penetrated into the English speaking countries. However, part of the work referred to was carried out secretly and the achievements were purposely not disclosed during the German occupation. The text was prepared and ready for printing before the liberation of our low countries. Owing to some purely technical reasons however, the appearance had to be postponed till about a year afterwards. So some papers or similar subjects as covered in this volume have not been paid attention to. Especially the article by MAKSUTOV in the Journal of the optical Society of America in May 1944 should have been referred to, since this author apparently obtained — at a later date than the author of the present volume — many of the results mentioned in the first chapters.

An attempt has been made to arrange the subjects in such a way that continuation of thought is maintained wherever possible: New optical systems, New optical instruments, Geometrical optics and Physical optics.

The author wishes to express his gratitude to those workers who have contributed in any way to this volume and especially to IR F. HEKKER.

It is hoped that the reader may find real interest in some of the subjects and also that his interest may be awakened for future results from the country where the cradle stood of a VAN LEEUWENHOEK, a HUYGHENS and a SNELLIUS.

CONTENTS

I. NEW OPTICAL SYSTEMS

§ 1. Introduction: the concave spherical mirror . . . 1
§ 2. The Schmidt-camera and its aberrations . . . 8
§ 3. A new optical mirror system . . . 16
§ 4. The field of view . . . 22
§ 5. The concentric mirror system . . . 25
§ 6. The chromatic aberrations . . . 34
§ 7. The corrected concentric system . . . 40

II. NEW OPTICAL INSTRUMENTS

§ 1. A new microscope . . . 46
§ 2. New telescopes . . . 53
§ 3. New cameras . . . 60

III. GEOMETRICAL OPTICS

§ 1. Geometrical methods in optical design . . . 67
§ 2. Researches on aberrations . . . 73
§ 3. Discussion of geometrical aberrations . . . 80
§ 4. On the luminosity of optical systems . . . 88
§ 5. On the light distribution in the image point . . . 94

IV. PHYSICAL OPTICS

§ 1. The diffraction theory of aberrations . . . 104
§ 2. Phase contrast . . . 116

I. NEW OPTICAL SYSTEMS

§ I. INTRODUCTION: THE CONCAVE SPHERICAL MIRROR

a. As an optical system the spherical mirror has some real advantages over a lens, or even a combination of lenses used as such. Perhaps most important and certainly the most obvious of these is the complete absence of chromatic aberrations which with lens systems can never be completely eliminated, although they may be rendered ineffective by suitable combinations of lenses of different glass.

Another feature of the spherical mirror as compared with a single lens is the smallness of the spherical aberration, being about 8 times smaller than with a single lens of equal aperture and focal distance, even should the latter be so shaped as to show a minimum spherical aberration.

If a diaphragm is placed in the centre of curvature of a spherical mirror, no such aberrations as coma, astigmatism, or distortion can occur, for the straight line containing the centre of curvature, parallel to any given ray of light, may be regarded as an axis of the system. Any direction may therefore be considered as an axial direction. There is, of course, curvature of the image, the radius of curvature being the focal distance for objects at infinite distance and very nearly so for any object plane at a finite distance.

The curvature of the image is hardly if at all disturbing when the field of view is restricted, as, for example, in most telescopes. Moreover, if the optical system is to be used for photographic work, a curved film may be used.

If the spherical mirror is to be combined with lenses, the curvature of the image may even be an asset, for the image of the mirror is convex towards the direction from which the rays are incident (negative curvature), whereas positive lenses show a curvature in the opposite direction (positive curvature). An appropriate combination may thus be practically free of image curvature.

An inconvenience of the mirror used as an optical system is the reversal in direction of the rays, which usually causes a certain loss of light, as the object or image receiver cannot always be prevented from intercepting part of the useful beam. This "shadow effect" can be serious if the field of view is extensive and the relative aperture moderate, but it may be unimportant with moderate fields of view and a large relative aperture.

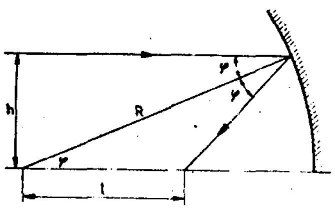

Fig. 1. Spherical mirror

In the SCHMIDT-camera, which consists essentially of a spherical mirror with a diaphragm and a non-spherical lens or correction plate at the centre of curvature, the relative aperture may be of the order of unity for a field of view of say 15°, in which case the shadow effect is negligible.

The SCHMIDT-camera will be discussed in more detail later on. Its performance will then be compared with that of the new systems which are the subject of this chapter.

b. As we have seen, the only aberration of a spherical mirror, if the diaphragm is situated at its centre of curvature, is the spherical aberration. It is easy to compute the axial aberration ΔF for an infinite object point. From Fig. 1, it follows at once that a ray, incident at a distance h from the axis, meets the axis after reflection at a distance l from the centre of curvature which is given by:

$$\frac{h}{l} = \sin 2\varphi$$

$$\varphi = \arc \sin \frac{h}{R}$$

From this we derive by series expansion:

$$\Delta F = l - F = \frac{h^2}{8F} + \frac{3}{128}\frac{h^4}{F^3} \qquad (1)$$

F is the focal length of the spherical mirror.

The first term is the well-known primary or third order longitudinal aberration. Multiplication by h/F gives the transversal spherical aberration, i.e., the radius of the aberration circle in the paraxial focal plane of the third order:

$$\varrho_h = \frac{h^3}{8\,F^2} \tag{2}$$

Owing to the smallness of the next higher order coefficient, the higher order terms may usually be neglected as long as $h < \frac{F}{2}$.

The third order approximation will at any rate be sufficient for our immediate purpose, although third order approximation in general is by no means sufficient to compute the aberrations of the optical systems yet to be considered.

In view of some subsequent considerations we shall deduce the third order spherical aberration in a different way, trying at the same time to find its value for any given finite object distance. Let us therefore consider Fig. 2. The curves indicated by P, E, and C respectively are a parabola, an ellips, and a circle, all having in common the vertex V and also in this vertex the radius of curvature R = VM. The curves may represent sections of a parabolical, an elliptical, and a spherical mirror.

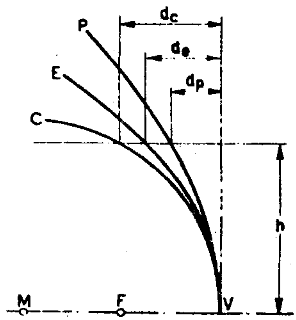

Fig. 2.
Parabola, ellips, and circle

It is well-known that, in the case of the parabolic mirror, the focus of rays incident in axial direction is the point half way between V and M. For paraxial rays the same focus appears with the circular and elliptical mirrors, but we want to find the axial meeting point of the reflected rays incident at some distance from the axis. We must therefore consider the separations in axial direction between the three curves of Fig. 2, at a given distance h from the axis. Now the distance (abscissa) d_e between the ellips with the half axes a and b at distance h (ordinate) from the a-axis and the tangent plane in V is easily found to be:

$$d_e = \frac{h^2}{2R} + \frac{h^4}{8R^3}\frac{b^2}{a^2} + \ldots\ldots \qquad (3)$$

For the parabola we thus find:

$$d_p = \frac{h^2}{2R} \qquad (3a)$$

and for the circle:

$$d_c = \frac{h^2}{2R} + \frac{h^4}{8R^3} + \ldots\ldots \qquad (3b)$$

The distance between circle and ellips is therefore

$$d_{ec} = \frac{h^4}{8R^3}\frac{c^2}{a^2} + \ldots\ldots \qquad (3c)$$

where $c^2 = a^2 - b^2$.

The behaviour of incident light waves for the three different kinds of mirrors can now be discussed:

a. The parabolic mirror.

The plane wave of incident light parallel to the axis meets the parabolic mirror at the distance h from the axis before reaching the vertex V. It has, indeed, already returned from the point of incidence at distance h from the axis and covered a distance backward equal to $\frac{h^2}{2R}$ (to the third order approximation) at the moment of meeting the mirror in the vertex V. This is just what is needed to create the spherical wave front converging at the focus F.

β. The elliptical mirror.

We observe a similar behaviour for those waves starting from the distant focal point of the ellips at a distance a + c from V. They do not meet in F, however, but in the second focal point of the ellips at a distance a — c from V.

γ. The spherical mirror.

A wave of incident light parallel to the axis meets the mirror at the axial distance h according to (3a) and (3b) earlier than it would meet the parabolic mirror; the corresponding advance of the wave front being $\frac{h^4}{4R^3}$.

By differentiation with respect to h we find the difference in direction of wavefronts or between the rays (as normals to the wave fronts) thus:

$$\alpha_h = \frac{h^3}{R^3} = \frac{h^3}{8F^3} \qquad (4)$$

This is the third order angular spherical aberration.

$$\varrho_h = F\alpha_h = \frac{h^3}{8F^2} \text{ in accordance with (2).}$$

A beam of light starting from a finite axial point F at a distance s from the mirror surface should meet the axis in a point at a certain distance s′, if the mirror were elliptical and if the semi-axes a and b were given by:

$$a + c = s$$
$$a - c = s'$$
$$c^2 = a^2 - b^2.$$

Now the mirror being spherical, the advance in path of the wave front at a distance h from the axis, according to (3c), is $\frac{h^4}{4R^3}\frac{c^2}{a^2}$, and the angular spherical aberration therefore:

$$\alpha_h = \frac{h^3}{R^3}\frac{c^2}{a^2} = \frac{h^3}{8F^3}\frac{c^2}{a^2} \qquad (5)$$

If m is the ratio between object and image dimensions, $-\beta = \frac{1}{m}$ being the lateral magnification, we have

$$\frac{a + c}{a - c} = m$$

and therefore

$$\frac{c}{a} = \frac{m - 1}{m + 1}$$

Instead of (5) we may then write

$$\alpha_h = \frac{h^3}{8\,F^3}\left(\frac{m-1}{m+1}\right)^2 \qquad (6)$$

which reduces to (4), for $m = \infty$ as it should do.

c. As the spherical aberration of a spherical mirror is relatively small, and coma, astigmatism, and distortion can be avoided by means of a diaphragm at the centre of curvature, it is of interest to investigate if a spherical mirror can be used as an optical system of reasonable aperture, or more precisely what will be the admissible aperture of the spherical mirror of a given radius.

To solve this problem, we may, for example, start from RAYLEIGH's condition that in the focal point the deviation from the wave fronts of different zones do not exceed a quarter of the wave length of the light used. From Fig. 2, p. 3, and formula (3), p. 4, it follows that the advance in wave front from the spherical mirror compared with the parabolic one is $\frac{h^4}{4\,R^3}$. Consequently:

$$\frac{h^4}{4\,R^3} = \frac{\lambda}{4} \text{ or } \frac{h^4}{F^3} = 8\,\lambda \qquad (7)$$

appears to be the condition from which the maximum aperture of a mirror of given radius may be deduced.

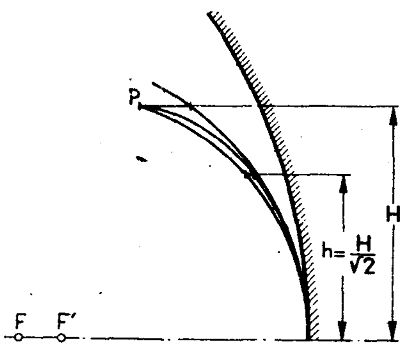

Fig. 3.
Wave front reflected from spherical mirror

There is, however, a more favourable point, a little nearer the mirror surface, for which the difference in path is only a quarter of the value found for the paraxial focal point F. The proof of this statement may be deduced from Fig. 3. The separation between the circle through the vertex V of the mirror, with its centre in F, and the wave front emerging from the spherical mirror is $\frac{h^4}{4\,R^3}$. Let us consider now the circle through a point P of the wave front at the axial distance H and through the vertex V, having its centre F′ on the axis. The separation between the two circles with centre F and F′ respectively is $\frac{h^2}{H^2}\frac{H^4}{4R^3}$. Thus, the separation between the circle with centre F′ and the wave front is $\frac{h^2}{4R^3}(h^2 - H^2)$, which has a maximum value for $h = \frac{H}{\sqrt{2}}$, amounting to $\frac{H^4}{16R^3}$, or a quarter of the path difference in the focal point F. So, instead of (7), we write

$$\frac{h^4}{F^3} = 32\,\lambda \tag{8}$$

More generally, for an object at a finite distance, we find from (3c), p. 4:

$$\frac{h^4}{F^3} = 32\left(\frac{m+1}{m-1}\right)^2 \lambda \tag{8a}$$

Another approach to the solution of the same problem may be obtained in the following way: We know that the diameter d_m of the aberration circle, sometimes called the least circle of aberration, is only a quarter of the diameter at the focal point:

$$d_m = \frac{h^3}{16F^2} \tag{8b}$$

It seems a reasonable assumption that this diameter should not be larger than the diameter of the unavoidable AIRY-diffraction disc, thus, provided $m \gg 1$:

$$\frac{h^3}{16F^2} = \frac{F}{h}\,1{,}22\,\lambda \quad \text{or approximately} \quad \frac{h^4}{F^3} = 20\,\lambda \tag{9}$$

It is interesting to note, that both methods give approximately equal results, which is shown still more clearly if we take the focal distance F as a given quantity and write the relative aperture $\alpha = \frac{2h}{F}$ as a function of F, thus, putting $\lambda = 5{,}5 \times 10^{-5}$ cm, instead of (8) and (9) respectively:

$$\alpha = \frac{0{,}4}{\sqrt[4]{F}} \qquad \text{and} \qquad \alpha = \frac{0{,}36}{\sqrt[4]{F}}$$

where F is measured in cm.

To be on the safe side, we take the latter and smaller of the two values and obtain finally for the admissible relative aperture:

$$\alpha = \frac{2h}{F} = \frac{0{,}36}{\sqrt[4]{F}} \tag{10}$$

Putting F equal to 1 cm, it follows from (10) that $\frac{F}{h} = 5.5$, which is very nearly equivalent to a numerical aperture of 0.18.

We thus arrive at the somewhat surprising result that a simple spherical mirror with 2 cm radius may be used as a microscope-objective, if the numerical aperture is limited to about 0.2.

A microscope with such a simple objective will be described later on.

§ 2. THE SCHMIDT-CAMERA AND ITS ABERRATIONS[1]

a. The SCHMIDT-camera was suggested and produced by B. SCHMIDT as long ago as 1930. Its theory and very fine performance have already been described in several papers, so we shall restrict ourselves here to a brief description only. This is partly by way of further introduction and partly because we want to compare its performance with one or two of the new optical systems which are the main subject of this chapter.

The SCHMIDT-camera is essentially a spherical mirror with a non-spherical correcting element, located at the centre of curvature of the mirror. The correcting element is a glass or other transparent disc, the thickness of which increases with the axial distance h. It is easy to compute to first approximation to what degree the decrease in thickness is required if the light rays parallel to the axis after reflection from the mirror are to be directed to the focal point F. For we have already

[1] B. SCHMIDT, *Mitt. Hamb. Sternwarte, Bergedorf*, 7 (1932) 36.

found that a plane wave of light, after reflection from a spherical mirror, shows an advance in path of $\frac{h^4}{4R^3}$ as compared with the spherical wave front after reflection from a parabolic mirror.

A spherical wave front and thus perfect focussing will be obtained after reflection from the spherical mirror, if a retardation of the light wave, increasing with axial distance h, can be caused to such a degree that this advance in path $\frac{h^4}{4R^3}$ is compensated. This amount of retardation will be brought about by a transparent disc in front of the mirror of a thickness

$$D = \frac{h^4}{4(n-1).R^3} \tag{11}$$

in which h is again the distance from the axis and n is the refractive index of the disc material.

Indeed, the deviation ε caused by a disc of the shape represented by (11) is, to the same approximation:

$$(n-1)\frac{dD}{dh} = \frac{h^3}{R^3} + \ldots = \frac{h^3}{8F^3} + \ldots. \tag{12}$$

This is exactly the angular spherical aberration α_h according to (4), p. 5. It is not very difficult to compute D to a further approximation, but this is not necessary for our immediate purpose[1].

Schmidt has improved upon this corrector of continually increasing thickness, which may be called the "first Schmidt-system".

The corrector of the "second Schmidt-system" has a shape as indicated in Fig. 4, in which the variations in thickness are considerably exaggerated for the sake of clarity. This improved corrector directs the parallel rays not towards the focal point, but to the point of the axis where the aberration circle has its minimum diameter, which is known to be only a

[1] See e.g.: Carathéodory, Elementare Theorie des Spiegelteleskops von B. Schmidt, *Hamb. Math. Einzelschriften*, Nr 28 (1940).

quarter of the diameter of the aberration circle at the focal point. The maximum deviation of the marginal ray caused by the corrector of the second SCHMIDT-system is therefore only a quarter of the deviation necessary for marginal rays with the first SCHMIDT-system, and its maximum inclination is therefore also smaller to the same proportion.

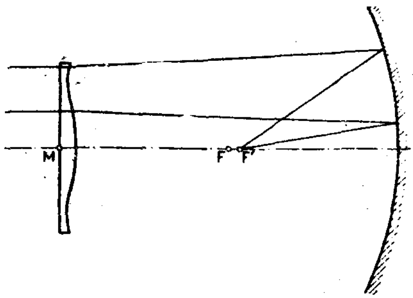

Fig. 4. Second SCHMIDT-system

The improved corrector may be represented to first approximation by adding to (11) a term with h^2, viz.:

$$(n - 1)\, D_1 = -\frac{3}{8}\,\frac{H^2}{R^3}\,h^2 + \frac{1}{4R^3}\,h^4 + \ldots\ldots \qquad (13)$$

where H is the semi-diameter of the corrector.

The first term expresses that the displacement of the paraxial focus, caused by the corrector is $\frac{3H^2}{16R}$, the second term represents the third order aberration.

We verify also by differentiating (11) and (13):

$$\frac{dD_1}{dh} = \frac{1}{4}\,\frac{dD}{dh} \qquad (14)$$

for $h = H$ and

$$\frac{dD_1}{dh} = 0, \quad \text{for} \quad h = \frac{1}{2} H \sqrt{3} \qquad (15)$$

(14) confirms the four times smaller marginal inclination of the second system; (15) shows that the corrector has parallel surfaces at about 0.86 of its diameter.

Obvious advantages of the second over the first system are:

1° The chromatic aberrations, already small in the first system, are reduced to about a quarter and become quite negligible.

2° The aberrations caused by oblique rays will also prove to be reduced.

It is of importance to estimate the amount of aberration of oblique rays, both meridian and sagittal, as it is mainly by these aberrations that the excellent performance of the SCHMIDT-camera is limited. Their computation will be the subject of the following section.

b. To compute the meridian aberration of oblique rays, we have first to find the deviation by the corrector of oblique rays making a certain angle with the axis but lying in a plane containing the axis. Moreover, it must be remembered that such an oblique ray does not require the full amount of correction because of its smaller distance from the centre of curvature of the mirror. This second point will be dealt with separately.

To find the deviation of oblique rays, we must first consider the deviation caused by a prism of small refracting angle A for rays of light lying in a principal section. Let the rays be incident at the angle φ and $A \ll \varphi$. We then obtain (to third approximation):

$$\varepsilon_\varphi = (n - 1)\, A \left(1 + \frac{n + 1}{2n}\, \varphi^2\right) \tag{16}$$

For incident rays parallel to the normal on the prism surface ($\varphi = 0$), the deviation is of course $\varepsilon_0 = (n - 1)\, A$, and the additional deviation, which is of special interest for our purpose, is:

$$\varepsilon_a = \frac{n^2 - 1}{2n}\, A \varphi^2 \tag{17}$$

It is indeed small, compared with ε_0, as long as $\varphi^2 \ll 1$, but acquires an appreciable value comparable to ε_0, as soon as $\varphi = \frac{1}{4}$ or thereabouts, say $\varphi = 15°$.

We have yet to consider, however, that the distance to the centre of curvature of the mirror of such a marginal ray of light, meeting the corrector at an angle φ, is only $h \cos \varphi$ or approximately $h_\varphi = h\left(1 - \frac{\varphi^2}{2}\right)$. It does not require the full amount of correcting $(n - 1)A$, therefore, but only the deviation:

$$\varepsilon'\varphi = (n - 1)\, A \left(\frac{h\varphi}{h}\right)^3$$

or

$$\varepsilon'\varphi = (n - 1)\, A \left(1 - \frac{3}{2}\varphi^2\right) \qquad (18)$$

From (16) and (18) it follows that the total excess in deviation, which is the meridian angular aberration:

$$\alpha\varphi = \varepsilon\varphi - \varepsilon'\varphi = \left(\frac{n+1}{2n} + \frac{3}{2}\right)\varphi^2 (n - 1)\, A \qquad (19)$$

To find the length of the resulting aberration figure l_m in meridian direction, we have to multiply this with $2F = R$, and substitute in the case of the first SCHMIDT-system for A the value $\left(\frac{dD}{dh}\right)_{h = H} = \frac{H^3}{(n - 1)\, R^3}$, so finally:

$$l_m = \frac{4n + 1}{2n}\, \varphi^2 \frac{H^3}{R^2} \qquad (20)$$

where H is the radius of the corrector [1].

To find the aberrations of oblique rays for the second SCHMIDT-system, it is convenient to first consider the more general case of a corrector, the thickness of which is an arbitrary function $D = f(h)$. A meridian ray, incident at an angle φ with the axis and with the height of incidence h, will obtain the deviation

$$\delta = (n - 1)\, f'(h) \left(1 + \frac{n+1}{2n}\varphi^2\right) \qquad (21)$$

where $f'(h) = \frac{dD}{dh}$; but the distance from this incident ray to the centre of curvature is only $h \cos \varphi$, and therefore the correction required is

[1] CARATHÉODORY, in the book already referred to, arrives at twice this value, presumably owing to a numerical error, made in an earlier part of his computations.

$$\delta_1 = (n - 1)\, f'\left(h - \frac{h\varphi^2}{2}\right) \tag{22}$$

We may expand (22) in a TAYLOR power series as $\varphi \ll 1$, hence

$$\delta_1 = (n - 1)\left[f'(h) - \frac{h\varphi^2}{2} f''(h) + \ldots\ldots\right] \tag{23}$$

The excess in deviation ε is therefore:

$$\varepsilon = \delta - \delta_1 = (n - 1)\,\frac{\varphi^2}{2}\left[\frac{n + 1}{n} f'(h) + h\, f''(h)\right] \tag{24}$$

Applying (24) to the SCHMIDT-corrector of the second kind (13), p. 10, we obtain:

$$\varepsilon = \left(\frac{n + 1}{n} + 9\right)\frac{\varphi^2}{8}\,\frac{H^3}{R^3} \tag{25}$$

The length of the corresponding aberration figure is

$$l'_m = \frac{10n + 1}{8n}\,\varphi^2\,\frac{H^3}{R^2} \tag{26}$$

which is about $\frac{4}{7}$ of the value found for the SCHMIDT-corrector of the first kind.

c. To estimate the sagittal aberration, we have to study the deviation of the ray, incident at the border of the corrector, making an angle φ with the plane containing the axis and the normal of the point of incidence. It is thus necessary to find the deviation caused by a prism of a ray incident at an angle φ with its principal section. We have to apply, therefore, the formula:

$$\varepsilon_p = \left(n\,\frac{\cos\varphi'}{\cos\varphi} - 1\right) A^{1} \tag{27}$$

where φ and φ' are the angles between the incident ray and the principal plane in the air and inside the glass respectively, A is again the refracting angle of the prism, and ε_p is the deviation of the projection of the ray on the principal plane of the prism. The actual deviation $\varepsilon = \varepsilon_p \cos\varphi$, is thus:

[1] See, for example: CZAPSKI EPPENSTEIN, *Grundzüge der Theorie der Opt. Instr.*, J. A. Barth, 1922, p. 331.