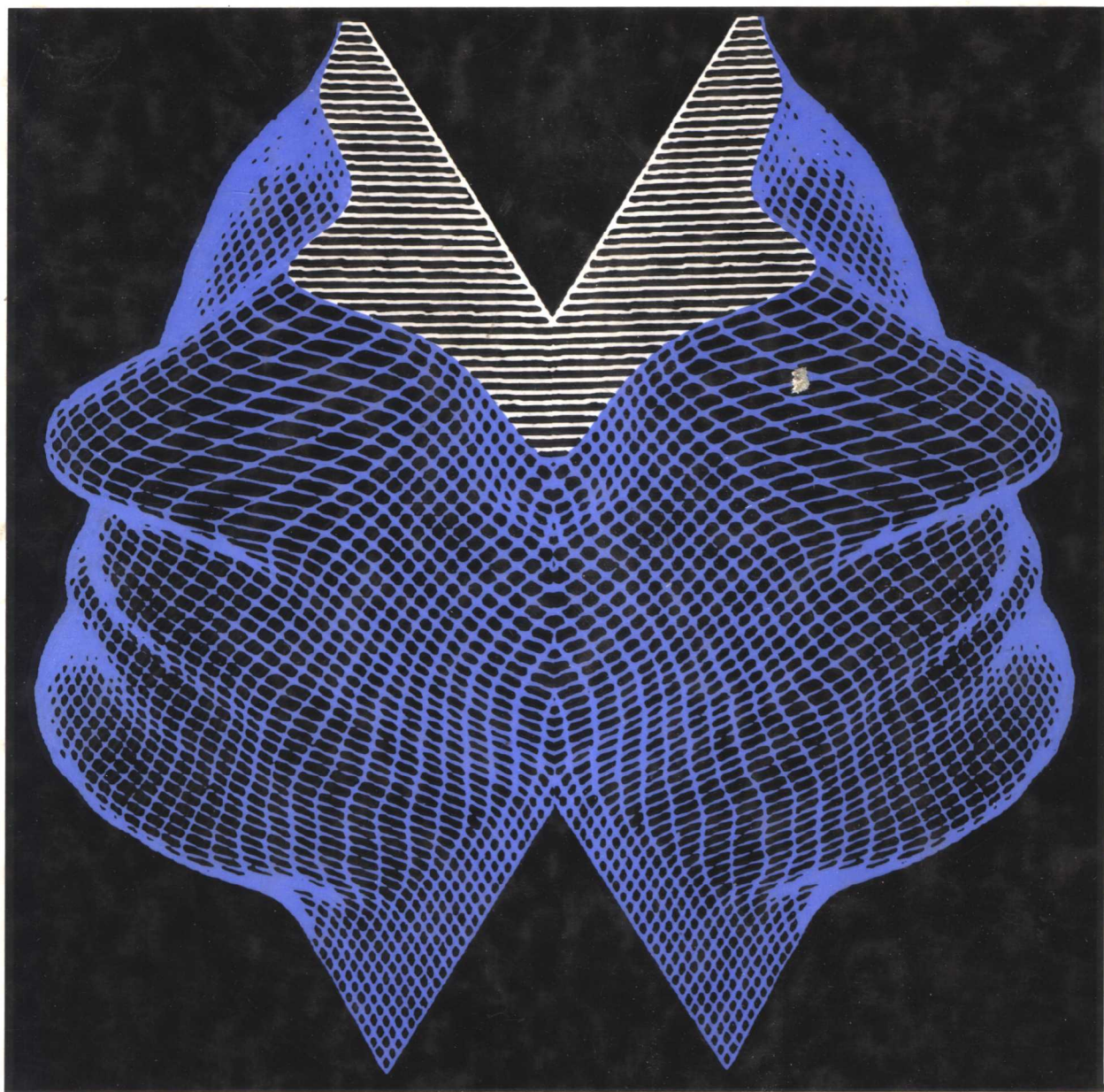

ROBERT M. KAPLAN

BASIC STATISTICS FOR THE BEHAVIORAL SCIENCES



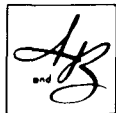
Basic Statistics for the Behavioral Sciences

ROBERT M. KAPLAN

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Preface

This book was written to provide a general introduction to statistics for the behavioral and health sciences. Beyond providing instruction in techniques, I introduce the concepts of statistical thinking. Because very few things in our lives are known for certain, statistical methods help students to think about the world in a probabilistic way and to understand consistencies and inconsistencies. Through statistical analysis they can discover how to attach a degree of certainty to statements about events and observations.

Another major aim of the book is to present the fundamentals of statistics to students who have virtually no mathematics background. The objective is to develop students' confidence and understanding in applying basic statistical procedures to concrete research situations in the behavioral and health sciences. For each key procedure, this text walks students through a step-by-step presentation of basic procedures, worked out in extremely specific detail. (These procedures are further reinforced for students through the Study Guide and its accompanying software programs.) At the same time the overall framework of research-design priorities is always kept in view to help students organize the underlying reasoning needed for statistical testing of hypotheses or exploration of data.

Although the book was written primarily for students in the behavioral sciences, most of the examples come from fields in which I have found students to be interested. For example, data from professional sports, criminal behavior, family interactions, and entertainment are used in many of the examples. In my teaching of introductory statistics, I have found that college men and women generally appreciate real numbers generated from public spectacles or everyday events.

To assist students in gaining confidence in statistical reasoning and basic procedures, a distinctive Study Guide is offered. It is accompanied by a soft-

ware disk of programs that students can use for ease of computation on problems in the text and Study Guide. One of the programs also helps evaluate the research-design conditions that are appropriate for the major statistical procedures.

Like most authors I expected the completion of the manuscript to be a brief project. However, things became more complicated. Work on the project was woven between 11 research grants and a maze of other administrative commitments. What began with a burst of enthusiasm during one summer vacation emerged as this book three years and two children later.

Many friends, colleagues, and students contributed to this effort. I am particularly indebted to the staff at the Center for Behavioral Medicine at San Diego State University. Connie Toevs made major contributions during the early phases of the project. She typed portions of the manuscript, wrote the computer programs for the workbook, and provided many other invaluable forms of assistance. Therese Cauchon made a variety of other contributions including the assemblage of permissions. Char Hook also did her share of the typing. In the later phases, Sandra Silva and Joyce Garman did a superb job of keeping many aspects of the project on track. Several San Diego State students, including Mary Bulcao, David Dickason, and Tom Meyer, read the manuscript and provided feedback.

Authors often complain about their publishers. Yet I can say with some sincerity that the relationship with Allyn and Bacon has been both amicable and enjoyable. Bill Barke was entertaining, supportive, and encouraging throughout the project. Allen Workman was consistently thorough and conscientious in helping to shape the manuscript. I am very thankful to professors Charles Hinderliter of the University of Pittsburgh at Johnson, Bill McDaniel of Georgia College, Robert Levy of Indiana State University, and especially Mike Brown of Pacific Lutheran University for their many helpful suggestions in the development of the project.

Geri Davis and Martha Morong of Quadrata, Inc., provided the editing and production services for the manuscript. They were a delight to work with and all aesthetic accomplishments of the textbook should be attributed uniquely to them. Finally, for the last several years the many versions of this manuscript shared a small house with my wife, Dr. Catherine Atkins, and two sons, Cameron and Seth Kaplan. They join me in the celebration of its completion.

R.M.K.

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Basic Statistics for the Behavioral Sciences



Introduction

Using Numbers

Mathematics is a perfectly unnatural field of study. Our species created numbers, and we may be the only living beings who consciously use and manipulate them. Unlike food and water (which are used by other animal species) we have no physiological need for numbers. Yet we have become increasingly dependent on numbers, arithmetic, and mathematics to understand the world we live in. Since numbers were created by humans, they obey well-defined sets of rules.

Quantification is the description of some quality or characteristic in terms of quantities or numbers. Quantitative studies pertain to measurement or use of measurement.

Today we could hardly survive without numbers. We count, use money, measure things, and so on. You might stop and think about how many times you use numbers each day. Numbers and quantification provide us with a very special language that allows us to express ourselves precisely. Scientific investigation requires precision, and this is gained through use of quantitative methods. These methods are human-made and precise, and follow formal logic. The precision of all other sciences is linked to mathematics and the ability to describe precise logical relationships.

We constantly process information about the world. In fact, there is so much information available that we cannot organize and interpret all of it. To make more sense of the world we must measure phenomena and be able to summarize what we have measured. Methods that are used to organize, summarize, and describe observations are called **descriptive statistics**. These might include the batting averages for Boston Red Sox players, the average

heights and weights for second-graders in the Chicago school system, and the association between watching television violence and behaving aggressively. In addition to descriptive statistics we also use **inferential statistics** for drawing general conclusions about probabilities on the basis of a sample. There are many uses for inferential statistics. In some cases we might use statistical inference to make statements about the eating habits of American citizens on the basis of the study of a small fraction of the American population. In other instances we might use inferential statistics to decide whether to attribute differences between two groups to chance or nonchance factors. For instance, suppose one group is given a treatment and another group is not treated. If small differences between the groups are observed, we need statistical methods to help us decide whether to attribute the differences to chance or to the treatment. In this book we will provide the basis for both descriptive and inferential statistical methods.

Measurement and Methods for Describing the World

Measurement requires the application of rules for assigning numbers to objects. The rules are the specific procedures for transforming qualities of attributes into numbers (Nunnally, 1978). For example, to rate the quality of wines, the wine taster must be given a specific set of rules. The wine might be rated on a 20-point rating scale in which 1 means extremely bad and 20 means extremely good. The basic feature of the system is the scale of measurement. For example, to measure the height of your classmates you might use a scale of inches; to measure their weight, you might use a scale of pounds.

In the behavioral and health sciences we do not have widely accepted scales of measurement. Nevertheless we do have numerous systems by which we assign numbers. Properties and characteristics of measurement are usually studied in courses on psychometrics and testing (see Kaplan and Saccuzzo, 1982, for an overview).

Games, Gods, and Gambling— The History of Statistics and Probability Theory

Historians have been unable to determine precisely when humankind began using number systems. There is speculation, however, that the earliest number systems were associated with the number of fingers and thumbs on the two

hands. Base-ten number systems (representing our ten fingers) are universal systems of counting for all civilized people and nearly all primitive tribes. The ancient Persians and Greeks had words for five that meant "hand." Some speculate that the Roman numeral V is for the V between the thumb and fingers of one hand. Early counting systems may have used the several joints of the fingers for notational systems. Placing the hand at certain positions on the human body expressed multiples of five and ten. By the eighth century an English historian and teacher called the "venerable Bede" was able to use the system to count to one million (David, 1962). By the ninth or tenth century people in India were writing down numbers as we do today. Yet progress in the development of arithmetic was slow for cultures such as the Greeks and Romans because their notation was very cumbersome. Arabic notation made progress in arithmetic and mathematics easier.

As civilizations progressed they became more fascinated with games and gambling. There is evidence from the Babylonians, Egyptians, Greeks, and Romans of the pre-Christian era that games of chance were prevalent. Greek vase paintings have shown boys playing a marblelike game of chance. There is also evidence from the first dynasty in Egypt (3500 B.C.) from tomb paintings depicting board games. In sum, recorded history reveals a long trail of interest in games of chance and gambling. To become better gamblers, however, they had to learn more about the rules of probability.

Rolling dice is one form of gambling that has been around for many centuries, perhaps as early as 3000 B.C. The earliest dice date to the beginning of the third millennium. There is evidence that dice games were popular among many cultures of the Mediterranean region. Some have conjectured that the dots on die representing the numbers one through six were used instead of other number systems because they were easier to carve into pottery. Over the centuries, many distinguished philosophers and mathematicians have studied outcomes of dice games. The probabilities of events in dice games were considered by Sir Isaac Newton, James Bernoulli, Galileo, Leonardo da Vinci and the French philosopher-mathematician Blaise Pascal. In fact, in the middle of the seventeenth century Pascal worked out rules of binomial power, which are essential to probability theory, in a series of letters to Frenchman Pierre de Fermat. Pascal's method is known as the Pascal triangle.

The development of games and gambling forced scholars and gamblers alike to understand the probability that events would occur by chance. A sophisticated gambler by the eighteenth century might understand the probability that his risk would pay off. In this book, you will learn about statistical methods for estimating the likelihood that observed events are more common than chance. A flip of a coin will come up heads half of the time. Similarly, a student given a true-false test will get half of the problems correct by chance. Yet would we call it chance if a student got 65% of the items correct? Statistical methods help us decide whether performance is significantly better than chance.