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Financial Analysis with an Electronic Calculator

Third Edition

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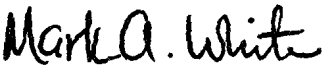
PREFACE TO THE THIRD EDITION

Rapidly changing information and communication technologies have had an enormous impact on the financial world. Today's fast-moving markets place a premium on "running the numbers" swiftly and accurately. Financial calculators are affordable and easy-to-learn tools for accomplishing both of these objectives.

The purpose of this book is to provide information and procedures enabling students to master their financial calculators while simultaneously gaining a deeper understanding of financial mathematics. It is designed to be used as a self-contained supplement in the introductory financial management course. Complete instructions are included for solving all major problem types on three common models of financial calculator – Hewlett Packard's HP-10B, Sharp Electronics' EL-733A and Texas Instruments' BA II Plus. The third edition contains 40 new problems drawn from several of the leading introductory finance texts.

This book owes a tremendous debt to the hundreds of finance students I've had the pleasure of teaching at Michigan State University and the University of Virginia over the past thirteen years. Several colleagues were also helpful in its development and revision: Janet Todd (University of Delaware), Harry White, Jr. (University of Detroit-Mercy), Rick Osborne (University of Colorado), Rich DeMong (University of Virginia), Jerry Stevens (University of Richmond) and Jennifer Frazier (James Madison University). I have thoroughly enjoyed working with Senior Sponsoring Editor Gina Huck and Editorial Assistant Paula Krauza in preparing this revised edition. This book is dedicated to my wife, Susie, and our two wonderful children, Katie and Skye.



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Chapter One

INTRODUCTION

"But the age of chivalry is gone. That of sophisters, economists and calculators has succeeded ..."

– Edmund Burke

Financial calculators are a relatively recent yet widespread addition to modern finance. By providing students with greater speed and accuracy in problem-solving, they can, if used correctly, greatly enhance the learning process. Mastering the field of finance requires practice and often repetitious computation. Financial calculators can eliminate the tedium associated with these calculations and allow you to better focus on the overall analysis. Furthermore, financial calculators allow instructors to introduce more relevant "real world" examples without fear of overwhelming students with numerical details. Indeed, the dependence of the financial industry on calculating machines of all types – financial calculators, personal computers, and mainframes – might lead you to wonder how the financial world got along without these 'electronic brains' in the first place.

Suffice it to say that finance has benefited tremendously from advances in electronic technology. The pocket calculators of today run faster, contain more memory and are hardwired to perform far more advanced calculations than Philadelphia's legendary ENIAC computer.[1] Tables of so-called interest rate 'factors,' designed to ease calculations by presenting the results of various combinations of time periods and interest rates, are well on their way to becoming a thing of the past. Financial calculators can provide more accurate solutions to a greater universe of problems than is possible by using tables alone. Some problems requiring iterative solutions are practically impossible to solve without electronic means. You can, of course, use a financial calculator to derive all the factors found in the various Present Value/Future Value tables.

For all of this, calculators pose hidden dangers for beginning students. They are not and can not be substitutes for clear-headed analysis. Far too many individuals tend to treat financial calculators as "black boxes" that provide answers to those initiates fortunate enough to press the secret key sequences. Calculators are a means, not an end. According to the author of a popular finance textbook,

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[1]The Electronic Numerical Integrator and Calculator (ENIAC) is widely regarded as the first successful high-speed electronic digital computer. Developed by John Eckert and John Mauchly at the University of Pennsylvania, it was completed in 1946. The computer used over 18,000 vacuum tubes, stood two stories high and occupied 1,500 square feet of floor space. It required so much electricity that surrounding city lights dimmed when it was running.

> "If finance was merely calculating numbers, taking a finance course and reading this text would be a waste! All you would have to do is purchase a good quality financial calculator, read its owners manual, and start making financial decisions. As you progress through this text you will see that the proper setup of the problem is the difficult part. Once this step is accomplished, the calculations are easily done ..."[2]

Learning finance requires dedication and a willingness to try to understand the *relationships* among various financial variables. Financial calculators can assist in this process but mere possession of one does not confer instant knowledge.[3] Ownership of a financial calculator is not a license to stop thinking!

Calculators vs. Computers

The use of a calculator to solve financial problems is frequently disparaged in favor of computers running analytical software. There is no doubt that personal (and larger) computers are capable of analyzing much more complex situations than you might wish to tackle on a pocket calculator. On the other hand, the vast majority of problems encountered in the classroom are generally quite simple. Financial calculators require a much smaller investment in capital and training time, as compared with personal computers. Thus, they are likely to be used in greater numbers than personal computers.

Different situations require different tools. In some cases, only paper and pencil are necessary to reach a "ballpark" figure. In others, the greater accuracy afforded by a financial calculator is appropriate. More complicated analyses may require a personal computer, and even greater levels of sophistication may call for the use of a mainframe or supercomputer. Similar circumstances are found in other fields. Consider, for instance, the different types of saws used in building a house. Financial calculators might be likened to the handsaw, a versatile tool that performs many jobs. Personal computers, with their greater bulk and greater capabilities, are akin to a Skilsaw™ or portable circular saw. Finally, mainframe computers share many features with tablesaws (permanent mounting, optimized for fast processing of large jobs, etc). The point is that different tools are used to solve different problems, and that the most powerful tool is not always the most appropriate one.

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[2]Ramesh K. S. Rao, *Fundamentals of Financial Management*. (New York: Macmillan Publishing Co., 1989), 808 pp.

[3]The author has first-hand acquaintance with a number of students who bought an electronic calculator the night before their first exam, apparently expecting the mysteries of the financial universe to be revealed at the appropriate time. The results were (predictably) dismal.

Calculator Models Covered in this Book

Three models of financial calculator are discussed in this book: Hewlett-Packard's HP-10B, Sharp Electronics' EL-733A and Texas Instruments' BA II Plus. These calculators are the most common models found in the introductory finance course. They possess roughly the same set of features and sell for approximately the same price (Table 1-1).

Table 1-1. Comparative Features of Financial Calculators

Feature	Hewlett-Packard HP-10B	Sharp Electronics EL-733A	Texas Instruments BA II Plus
Arithmetic Functions	yes	yes	yes
Time Value of Money	yes	yes	yes
Data Entry Format	algebraic	algebraic	algebraic
Memory	yes	yes	yes
Multiple compounding	yes		yes
Interest Rate Conversions	yes	yes	yes
Loan Amortization	yes	yes	yes
Cash Flow Analysis	yes	yes	yes
Bond Pricing			yes
Depreciation			yes
Break-Even Analysis			yes
Descriptive Statistics	yes	yes	yes
Weighted Averages	yes		
Regression Analysis	yes	yes	yes
Percentage Change	yes	yes	yes
Margin/Markup	yes	yes	yes
Date Functions			yes
Trigonometric Functions			yes
Suggested Retail Price	$50	$46	$45

More advanced calculators are also available and in many cases a good investment, particularly if a student intends to pursue further studies in finance. Many of the techniques and problem-solving methods discussed in this book apply equally well to other models by the same manufacturer.

Hewlett-Packard HP-10B

Hewlett-Packard's HP-10B is in some ways the successor to the firm's venerable HP-12C, which was first introduced in 1981. It is an entry-level calculator offering a surprisingly complete array of features, including the ability to analyze uneven streams of cash flows. Potential users put off by Hewlett-Packard's traditional use of "Reverse Polish Notation"[4] or RPN logic in its calculators will appreciate the switch to an algebraic mode of entry. The HP-10B is a finely made, dependable calculator that provides easy access to a wide array of financial functions.

Sharp Electronics EL-733A

In 1992, Sharp Electronics introduced the EL-733A, a slightly modified version of its popular EL-733 financial calculator. The two models are functionally equivalent, but the larger keys on the EL-733A make it much easier to use. Its features are nearly identical to the HP-10B, although it can be often be found at a slightly lower price. The EL-733A has an interesting display designed to increase its acceptance in international circles. Instead of the comma (,) normally used to designate multiples of 1,000, the EL-733A uses an apostrophe (').

Texas Instruments BA II Plus

The BA II Plus, released in 1991, is the latest incarnation of Texas Instruments' original Business Analyst calculator. It boasts more features than either of the other two models covered in this book, and uses a unique system of menus for data entry and problem solution. It is priced similarly to the HP-10B.

Notation

The following conventions are used to identify key sequences:

- *Numerical entries* are shown without modification, e.g., `23.00`
- *Primary keys* have a physical location and appear in boxes, e.g., [PMT]
- *Secondary keys* are enclosed within curly braces, e.g., {NPV}. Pressing a modifier key, e.g., [2nd], invokes the secondary function generally printed slightly above the physical key location.[5]

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[4]The term "Reverse Polish Notation" honors its developer, the Polish logician Jan Lukasiewicz (1878-1956). Hewlett-Packard's implementation differs from Lukasiewicz' original notation, which placed the operators *before* the numbers or variables. Hewlett-Packard calculators require the operators be entered *after* the variables, hence, 'reverse' polish notation.

[5]The HP-10B's modifier key is a large orange square. It appears in this book as [ ].

Incidentally, this book is not intended as a substitute for the operating manual that came with your financial calculator. Its goal is to show how a financial calculator can assist you in learning the techniques of *financial analysis*, not reiterate the specifics of each and every key on each and every calculator. Margin/markup functions are not included because they are fairly uncommon in the introductory finance course. Several useful features peculiar to the BA II Plus, e.g., depreciation, advanced bond pricing, dates and trigonometric functions, have also been left out for reasons of space and because they are only hardwired into one of the three calculators discussed in this book. What's left is a series of examples carefully chosen to reflect the types of problems you are likely to meet in an introductory finance course. Your mastery of this tool will be rewarded by a deeper understanding of finance and (possibly!) higher grades.

Housekeeping Functions

Before solving financial problems, it's best to familiarize yourself with the basic housekeeping functions, e.g., clearing memory registers, setting display formats, and determining the compounding and payment modes. These are explained in the following paragraphs.

Memory Registers

All calculators have areas of memory, called registers, where variables and intermediate results of calculations are stored. There are essentially three groups of registers: 1) Display registers, 2) Financial registers and 3) Data registers. As their name suggests, display registers are used to display data. The financial registers are used to solve problems involving the time value of money (TVM) and are sometimes known as the TVM registers. Depending on the problem type, data registers may hold intermediate results of computations, statistical data or cash flow information.

It is extremely important to CLEAR or "zero out" the requisite memory registers before beginning a new calculation.[6] Most errors, particularly in time value of money problems, are due to a failure to clear the appropriate memory registers. Registers can be cleared individually, as a group, or all at once. Table 1-2 identifies the sets of registers peculiar to different calculator models and the keystrokes necessary for clearing them.

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[6]Some memory registers can be "cleared" by storing zeros in them.

Table 1-2. Clearing Entries and Memory Registers

Function	Hewlett-Packard HP-10B	Sharp Electronics EL-733A	Texas Instruments BA II Plus
Clear last digit	[←]	[→]	[→]
Clear display	[C]	[C·CE]	[C/CE]
Clear error messages	[←] or [C]	[C·CE]	[C/CE]
Clear M(emory) registers	[ ] {CLEAR ALL}	0 [X→M]	[2nd] {MEM} [2nd] {CLR Work}
Clear Markup registers	[ ] {CLEAR ALL}	[2ndF] {CA}	[2nd] {CLR Work}
Clear TVM registers	[ ] {CLEAR ALL}	[2ndF] {CA}	[2nd] {CLR TVM}
Clear Cash Flow registers	[ ] {CLEAR ALL}	[2ndF] {CA}	[2nd] {CLR Work}
Clear Statistical registers	[ ] {CLΣ}	[2ndF] {CA}	[2nd] {CLR Work}
Delete statistical data	[ ] {Σ–}	[CD]	[2nd] {DEL}
Clear all registers	[ ] {CLEAR ALL}	[2ndF] {CA}*	[2nd] {QUIT}**
Quit worksheet mode	n/a	n/a	[2nd] {QUIT}
Reset default modes	***	n/a	[2nd] {Reset} [ENTER]

* This key sequence does NOT clear the **M**(emory) register; even turning the calculator on and off won't do that. Clear the **M** register by pressing 0 [X→M] or [RM] [+/–] [M+]

** This sequence only works in standard calculator mode. Use [2nd] {CLR Work} to clear a specific worksheet before returning to standard calculator mode.

*** To reset all calculator modes, press and hold the [C] key, then simultaneously press and hold down the [N] and [Σ+] keys at each end of the top row. Releasing all three should display 'ALL CLr' and reset the calculator's internal defaults.

Display Formats

The HP-10B and BA II Plus provide an option for displaying numbers in either US or European formats.[7] Press [ ]{.,/,} to toggle between these formats on Hewlett Packard's HP-10B. On the BA II Plus, press [2nd]{Format} and select the 'EUR' separator format by pressing [2nd]{SET} once you've scrolled to the decimal separator section of the format worksheet.

Setting the display to a fixed number of decimal places makes it easier to enter data and to read results. Two decimal places are most commonly used, although you may display other numbers of fixed digits by substituting a different number (from 0 to 9) in place of the '2' shown below. Answers will be rounded to conform to the desired number of displayed digits, although all digits are retained when performing internal calculations. Very large numbers are displayed using scientific notation. To set the display format to *two* decimal places, use the following key sequences:[8]

- HP-10B: [ ]{DISP} 2
- EL-733A: [2ndF]{TAB} 2
- BA II Plus: [2nd]{Format} 2 [ENTER]

Use the following keystrokes to reset displays to all possible decimal places (floating point format):

- HP-10B: [ ]{DISP} [.]
- EL-733A: [2ndF]{TAB} [.]
- BA II Plus: [2nd]{Format} 9 [ENTER]

Sign Changes

Financial transactions often consist of an initial cash flow followed by subsequent cash flows of opposite sign. For instance, a bank might make a loan to an individual (a *cash outflow*) expecting to receive a series of loan payments (*cash inflows*) in the future. From the borrower's perspective, he/she receives an initial sum of money (a cash inflow) by agreeing to repay the loan according to some schedule (cash outflows). Sign changes are used to identify the direction of these cash flows. Cash outflows are entered as negative numbers and cash inflows are entered as positive numbers. To enter a negative number on any of the calculators

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[7]In the United States, decimal points are indicated by periods and commas are used as digit separators. In Europe, the situation is reversed. Hence, 1,958,000.00 (US) = 1.958.000,00 (European). The BA II Plus can also display *dates* in either US or European formats, i.e., month first versus date first. These options are changed in the Format worksheets.

[8]Some instructors recommend *four* decimal places for increased accuracy. It's always a good idea to use four decimal places when working with interest rates.

discussed in this book, first press the appropriate digit keys and *then press the change sign key*, [+/-]. Do **NOT** use the minus sign key, [-], as its effects are quite unpredictable.

Annuities and Annuities Due

In finance, timing is everything. It makes a difference whether monies are paid at the beginning or end of a period. *Ordinary* annuities assume payment is made at the *end* of a period. *Annuities due* assume payment is made at the *beginning* of a period. In general, most problems (in the classroom and in real life) assume payment is made at the end of a period. During the introductory course it is probably best not to even use the annuity due feature, as you may forget to reset the calculator for subsequent problems. Moreover, all annuity due problems can be solved using techniques applicable to ordinary annuities. However, for those who must, use the following keystrokes to adjust for payments occurring at the beginning of a period:

- HP-10B: [ ] {BEG/END}
- EL-733A: [BGN]
- BA II Plus: [2nd] {BGN} [2nd] {SET} [2nd] {QUIT}

In all cases, these keystrokes act as toggles, i.e., pressing the same sequence again returns you to ordinary annuity or "END" mode.

Compounding Frequency

One last issue that must be dealt with is the compounding frequency per period. For both pedagogical and practical reasons, most problems in the introductory finance course assume compounding occurs **once** per period. Although there are a few exceptions (semiannual bonds are a notable example), it is often more confusing to switch between different compounding frequencies, i.e., daily, monthly, quarterly and semiannually than it is to simply leave your calculator set to compound interest once per period.

Sharp's EL-733A provides users no choice – all time value of money and cash flow computations assume a compounding frequency of once per period. Hewlett-Packard's HP-10B and Texas Instruments' BA II Plus allow you to set different compounding periods. Unfortunately for beginning students, *the default mode on each of these calculators is twelve times per period (monthly).* Therefore, it is very important that users of these models know how to determine and change the compounding frequency. Both models allegedly 'remember' the most recent setting, even after the power has been turned off. Experience has shown, however, that these settings can sometimes be changed bouncing around in a backpack or purse.

To check the setting for compounding frequency on the HP-10B, press the [ ] key, then press and briefly hold the [INPUT] key. This performs two valuable

functions; it clears the Time Value of Money registers *and* displays the compounding frequency. The default frequency is 12 times per period, displayed as "12 P_Yr". To change it, enter the desired compounding frequency, e.g., 1, then press [ ]{P/YR}. It's best to confirm that this has occurred before beginning any lengthy calculations.

You can specify both payment frequency *and* compounding frequency on the BA II Plus. Problems involving a discrepancy between the two are rare in the introductory finance curriculum, and payment frequency and compounding frequency should normally be set to the same number. To set both to once per period, first press [2nd]{P/Y} 1[ENTER], then press [↓] 1[ENTER] (to set the compounding frequency). Pressing [2nd]{QUIT} returns you to standard calculator mode.

One last word on compounding frequency – as noted previously, most calculator errors arise from a failure to properly clear the memory registers. On the HP-10B and BA II Plus, incorrect compounding regimes run a close second.

Troubleshooting

If despite all your best efforts you can't get the correct answer, or the dreaded "Error" message appears, quickly run through the following checklist:

1. Did you CLEAR all the registers?

2. Is the compounding frequency set to one time per year? [HP-10B and BA-II Plus only]

3. Is the calculator in END mode?

4. Did you enter negative numbers using the [+/-] key?

If the answer is "No" to any of these questions, clear and reset your calculator and try again. Other possible sources of error include neglecting to make one set of cash flows negative and one set of cash flows positive when solving for an unknown interest rate or number of periods, and on the EL-733A, forgetting in which mode (Standard, Financial or Statistics) the calculator is set.

Chapter Two

THE TIME VALUE OF MONEY

"I will gladly pay you Tuesday for a hamburger today."

– Wimpy

According to economists, most people display *positive time preference* with regard to decisions about money. Given a choice, individuals would rather receive a dollar (or pound or Deutschemark or ringgit or whatever) sooner rather than later. Thus, we say that money has "time value" – a dollar to be paid in the future is not worth as much as a dollar paid now. This is because a dollar received in the present can be invested to earn interest. Suppose, for instance, you wished to have $100 in your bank account at the end of this year. Because your bank pays interest, you need to deposit some lesser amount today. The interest you earn over the next year makes up the difference between your initial deposit (*present value*) and the final amount (*future value*). Financial mathematics deals with relationships between Present Value, Future Value, Payment, Interest Rate and Time. If you know any three of these variables, it's possible to solve for a fourth.[1] Although the basic equations governing transformations between present money and future money have been known for literally thousands of years,[2] the advent of the electronic calculator has alleviated much of the tedium and imprecision associated with former methods. This chapter provides an introduction to basic problem-solving using the time value of money (TVM) registers on a financial calculator.

How To Solve Financial Problems

Beginning finance students sometimes freeze up upon learning that they will be expected to solve so-called "story problems." High school memories of trying to determine where to bury the survivors from a train wreck involving two trains traveling at x miles per hour for y hours and meeting at point z can cause a relapse of *terror mathematicus*.[3] Fortunately, there are a few "tricks" you can employ to get a handle on most financial story problems. The first involves drawing a time line, or cash flow diagram.

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[1]Two of the four variables involved in a particular calculation must be Interest Rate and Time. Sometimes it is possible to solve for a fifth variable (if one knows the other four).

[2]Aristotle (384 - 322 BC) condemned the charging of interest as an evil practice.

[3]As Foster Leghorn might say, "It's a joke, son ... you don't bury *survivors*."