Handbook of linear integrated electronics for research

T. D. S. Hamilton

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Preface

'Experience is the name everyone gives to their mistakes.'
Oscar Wilde

This book is intended for people in a wide range of fields who use electronics in their experimental work. It is not particularly intended as a textbook or for professional engineers, though it could be of substantial use in both cases. Electronics is a vast subject and one cannot hope either to cover all of it in one volume, or even to know much about many aspects of it, so a severe restriction on topics must be accepted. I have chosen to omit both communication and digital applications. The former is not commonly of interest to experimenters, and the latter requires several volumes to even begin to do it justice.

The development of low cost integrated circuits constitutes, in my view, a revolution in electronics, possibly of greater magnitude than the advent of the transistor itself. It is now possible to think largely in terms of building blocks which can be assembled to form the required system, without necessarily knowing too much about the more detailed techniques of circuit design. Some knowledge of the mode of operation, however, can lead to more effective use of any device. The ready availability of many circuit functions, therefore, has been taken as the unifying feature of much of the present book.

The choice of topic and level of treatment has been strongly influenced by personal experience and by discussions with colleagues and students. As a physicist rather than an electronic engineer, one possibly sees things somewhat differently and this influences the way one understands and describes them. I have tried to concentrate on an understanding of how things work rather than on specific applications—the latter are covered by a fairly extensive collection of references to the literature and to the many excellent application notes produced by manufacturers. Though their efforts in this direction may not always be entirely altruistic, they and their authors deserve our commendation for this very substantial contribution of really useful information and guidance.

A rather larger number of references than is usual have been included and, though some of them may effectively duplicate one another, this makes it more likely that you will have access to a suitable one. More general references to a wide range of applications are given in chapter 11, followed by a bibliography of useful books. The references have been prepared directly from a computer print-out which allows them to be more up to date than would otherwise be possible. It should be noted that by no means all the references are referred to in the text, so it is worth scanning through the appropriate list to see if any others may be of use. Full titles are given in every case, so it is easier to decide whether they are worth looking up. Nowadays data sheets often contain extensive application information, so reference to these is not only an indication of the origin of the various devices.

The level of treatment poses a delicate problem—at what level does one start and how far should the development go? There is a tendency to eschew all but the most elementary mathematical ideas, but it seems to me too great a loss not to make use of some of the powerful and instructive tools available. In particular, the Laplace transform is so useful in circuit theory, and provides such a convenient description of many important situations, that it has been used wherever necessary. A short description is given of its use and application, at least to the level where a table of transform pairs can be used as readily as a set of log tables. Time spent in studying this section (1.11) will, I feel, be amply rewarded in understanding circuits, and in time saved in debugging poorly designed systems.

Chapter 1 is a resumé of useful basic circuit theory—the sort of mental furniture it is useful to have readily available. If the material is not familiar it may well repay the effort of reading this before consulting other sections. Chapter 2 begins the consideration of integrated circuits, with coverage of the forerunner of most linear devices (and still the cornerstone of systems)—the operational amplifier. This device has gone through several generations of development (now said by the admen to be in the third or is it the fourth?) and has now reached a very advanced level, with a multitude of models to suit practically every need. The introduction of negative feedback, upon which operational amplifiers depend, must rank as one of the great landmarks in the history of electronics. However effective, use of feedback requires a good understanding if disaster is to be avoided. Chapter 3 covers this topic in some detail in a form suitable for practical application.

The most common system element is probably the amplifier, operational or otherwise. Chapter 4 covers a range of the more commonly encountered types, from low-level to power and d.c. to wideband, with some discussion of servo theory *en passant*. Chapter 5 deals with oscillators, both sinusoidal and function generators. Also included are other positive feedback applications, such as the various forms of multivibrator, and the commonly used negative resistance unijunction transistor. A number of linear circuit functions are used frequently enough to warrant development of a range of IC realizations. Chapter 6 covers a number of the commonly used functions—comparators, multipliers, modulators, phase-locked loops, and sample-holds.

In every electronic circuit there is always the often somewhat neglected power supply. The whole of chapter 7 is devoted to this and related topics, covering rectification, regulation including regulator diodes, and the increasingly popular switching regulator. In recent years there has also been considerable progress in IC regulators, so that for most purposes, the provision of regulated supplies is now quite simple and in many situations more effective than before. The treatment of regulators as power operational amplifiers enables the theory of the latter, developed in chapters 2 and 3, to be used directly here.

Chapter 8 treats a number of useful discrete devices: field-effect transistors and their derivatives voltage-controlled resistors and switches, and the range of thyristor devices. Though a wide range of optical detectors has long been available, the field of optoelectronics has been somewhat limited because of the lack of convenient and fast light sources. The discovery of the light-emitting diode, in particular, has led to considerable development in the field of optoelectronics, and chapter 9 treats the more commonly used devices.

It is in the nature of research that one is frequently looking for signals hidden by noise. For this reason, particular attention has been paid to the characteristics of noise (Sec. 1.8), the performance of low-noise amplifiers (Sec. 4.1) and optical detectors (Sec. 9.3, 9.4) and, in chapter 10, the extraction of signals from noise. Though this represents by no means a comprehensive coverage, it is hoped that it will provide at least the realization of the importance of noise and of designing to minimize it. It is often no more difficult or expensive to produce a low noise rather than a noisy system.

I should like to thank my colleagues and all the correspondents who kindly answered my queries, and the semiconductor and other manufacturers who willingly supplied information, literature, and permission to copy diagrams. Also I must thank the members of the computer group of Daresbury laboratory for their assistance with my problems in setting up the reference data, Clara Nicholls for her most careful typing, and finally my family for their tolerance and understanding over the several years this book has been in the making.

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21 September 1976

Notation

a	prefix atto, 10^{-18}
A	ampere, unit of current
B	susceptance
$\mathscr{B}$	bandwidth
$\mathscr{B}_N$	noise bandwidth
C	capacitance
C	coulomb, unit of charge
C_J	junction capacity
CMRR	common-mode rejection ratio
dB	decibel
D	FET drain
E_C	capacitor voltage
E_P	peak voltage
E_T	transformer voltage
e_{os}	offset voltage
F	farad, unit of capacitance
F	noise figure
f	frequency
f	prefix femto, 10^{-15}
f_T	transition frequency
g_{fs}	transconductance
g_m	transconductance
g_{os}	output conductance
G	prefix giga, 10^{12}
G	conductance
G	FET gate
G	gain
G_{bp}	band-pass gain
G_{bs}	band-stop gain
G_d	differential gain
G_{hp}	high-pass gain
G_{lp}	low-pass gain
G_n	noise gain
G_P	power gain
$G(s)$	gain at frequency s
$G(0)$	gain at zero frequency
G_0	gain at zero frequency

$G(\infty)$	gain at infinite frequency
G_{∞}	gain at infinite frequency
H	henry, unit of inductance
Hz	hertz, unit of frequency
h_{FE}	transistor d.c. current gain
I_B	base current
I_C	collector current
I_D	FET drain current
I_{DSS}	FET drain current at zero bias
I_{DZ}	FET drain current for zero temperature coefficient
I_E	emitter current
I_G	gate current
I_H	holding current in thyristor
I_S	*P–N* junction reverse saturation current
i_b	bias current
i_{NC}	collector noise current
i_{ND}	drain noise current
i_{NE}	emitter noise current
i_{NG}	gate noise current
i_{NS}	shot-noise current
J	joule, unit of energy
K	kelvin, unit of absolute temperature
k	Boltzmann's constant, $1{\cdot}38 \times 10^{-23}$ J/K
k	prefix kilo, 10^3
L	inductance
L	load
M	prefix mega, 10^6
m	prefix milli, 10^{-3}
m	metre, unit of length
N	turns
n	prefix nano, 10^{-9}
P	power
p	prefix pico, 10^{-12}
Q	quality factor
Q	charge
Q	transistor
q	electronic charge, $1{\cdot}602 \times 10^{-19}$ C
R_{cm}	common-mode resistance
R_d	differential input resistance
R_f	feedback resistor
R_i	input resistor
R_{id}	input differential resistance
R_L	load resistance
R_{NI}	equivalent noise current resistance
R_{NV}	equivalent noise voltage resistance
R_{SV}	voltage source resistance
$R_{S(opt)}$	optimum source resistance
R_{SI}	current source resistance

R_S	source resistance
r_b	transistor base resistance
$r_{bb'}$	base spreading resistance
r_{be}	base–emitter resistance
r_e	transitor emitter resistance
r_e	diode incremental resistance
S	source
S	FET source
S	signal
s	second, unit of time
s	complex frequency
T	time constant
T	time interval or delay
T	prefix tera, 10^9
t_p	pulse width
t_r	risetime
t_{rr}	reverse recovery time
V	volt, unit of potential
V_{BE}	base–emitter voltage
V_{CC}	supply voltage
V_{CE}	collector–emitter voltage
V_{DS}	drain–source voltage
V_{GS}	gate–source voltage
V_J	*P–N* junction voltage
V_{oc}	open-circuit voltage
V_P	pinch-off voltage
V_{SS}	FET supply voltage
V_{th}	threshold voltage
V_+	positive supply voltage
V_-	negative supply voltage
v_{NA}	amplifier noise voltage
v_{NB}	base noise voltage
v_{NO}	output noise voltage
v_{NR}	resistor noise voltage
v_S	signal voltage
W	watt, unit of power
X	reactance
Y	admittance
Z	impedance
(+)	non-inverting input terminal
(−)	inverting input terminal
α	alpha, collector to emitter current ratio
β	beta, feedback factor
β_0	d.c. current gain
δ	delta, secondary electron emission ratio
ε	epsilon, permittivity
ε_0	permittivity of free space, $8{\cdot}854 \times 10^{-12}$ F/m
ζ	zeta, damping factor

η	eta, quantum efficiency
λ	lambda, wavelength
μ	mu, permeability
μ_0	permeability of free space, $1{\cdot}257 \times 10^{-6}$ H/m
µ	prefix micro, 10^{-6}
τ	tau, time constant
ω	omega, angular frequency
ω_T	transition frequency

Mathematical symbols

$>$	greater than
$\gg$	much greater than
$<$	less than
$\ll$	much less than
$=$	equals
$\doteq$	approximately or very nearly equals
$\approx$	of the order of
$\Rightarrow$	tends to, becomes
$\langle\ \rangle$	the average value of
$\overline{\quad}$	negation, in logic signals
$\lvert\ \rvert$	modulus or absolute value of
∂	partial differential
δ	a small increment
Δ	a small change or increment
exp	exponential
j	square root of -1
$\mathscr{Im}$	imaginary part of a complex number
$\mathscr{Re}$	real part of a complex number

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1 Review of basic circuit theory

1.1 Ohm's law and all that

The basic circuit relation is Ohm's law which states that the current, I, in a conductor is proportional to the voltage, V, between its terminals:

$$V = IR \quad \text{(volts = amps} \times \text{ohms)} \tag{1.1.1}$$

with the proviso that the constant of proportionality R, the resistance, is not dependent on factors such as temperature, time or, more directly, voltage or current. These conditions are often not met, so that although a resistance can be determined for a particular V and I, a different value will be found if either or both are changed. In normal circumstances, most commonly used conductors and resistors are assumed to have constant resistance (i.e., to be linear) for the sake of circuit analysis, but in critical situations these factors must be considered. Notwithstanding its apparent simplicity, Ohm's law is often all that is needed to analyse the quiescent conditions in circuits.

The current flowing in a resistor dissipates energy, the rate of dissipation or power being given by:

$$P = IV = I^2R = V^2/R \text{ watt} \tag{1.1.2}$$

If there are reactive elements in the circuit and V and I are varying with time, then this relation must be modified (see Eq. (1.1.14)), or I and V more particularly defined. The energy dissipated in this way heats up the resistor, and is largely lost to the system since a resistor cannot store electrical energy as can a reactive element. There is a small usually undesirable return in that the noise power is increased by the increased temperature (see Sec. 1.8). The random signal due to this noise is not of significance in terms of the circuit analysis, although it is of course of considerable importance in other respects (chapter 3 and Sec. 4.1).

In measuring the efficiency or amplification of a system the ratio of two powers is taken. These ratios often span a large range and arise from several stages of gain. It is then convenient to use a logarithmic scale to compress the range, and to allow gains of individual stages to be added rather than multiplied to give the overall value. The logarithm to base 10 of the power ratio is measured in a unit called after Bell although, in practice, one-tenth of this, the decibel (dB), is used:

$$\text{Power gain or ratio} \quad G_P = 10 \log_{10}(P_1/P_2) \text{ dB} \tag{1.1.3}$$

so that if we have a series of amplifiers with power gains G_{P1}, G_{P2}, etc., then the overall gain is:

$$G = G_{P1} \times G_{P2} \times \cdots \times G_{Pn}$$

or

$$G_P = G_{P1}\,(\text{dB}) + G_{P2}\,(\text{dB}) + \cdots + G_{Pn}\,(\text{dB}) \qquad (1.1.4)$$

If the input and output resistances are specified then Eq. (1.1.3) can be written in terms of V and R using Eq. (1.1.2). For equal resistances:

$$G_P = 10 \log (V_1^2/V_2^2) = 20 \log (V_1/V_2) \qquad (1.1.5)$$

The convenience of the logarithmic scale has led to its use for voltage or current ratios, as in Eq. (1.1.5), which is legitimate enough, but the unit used is still the dB,

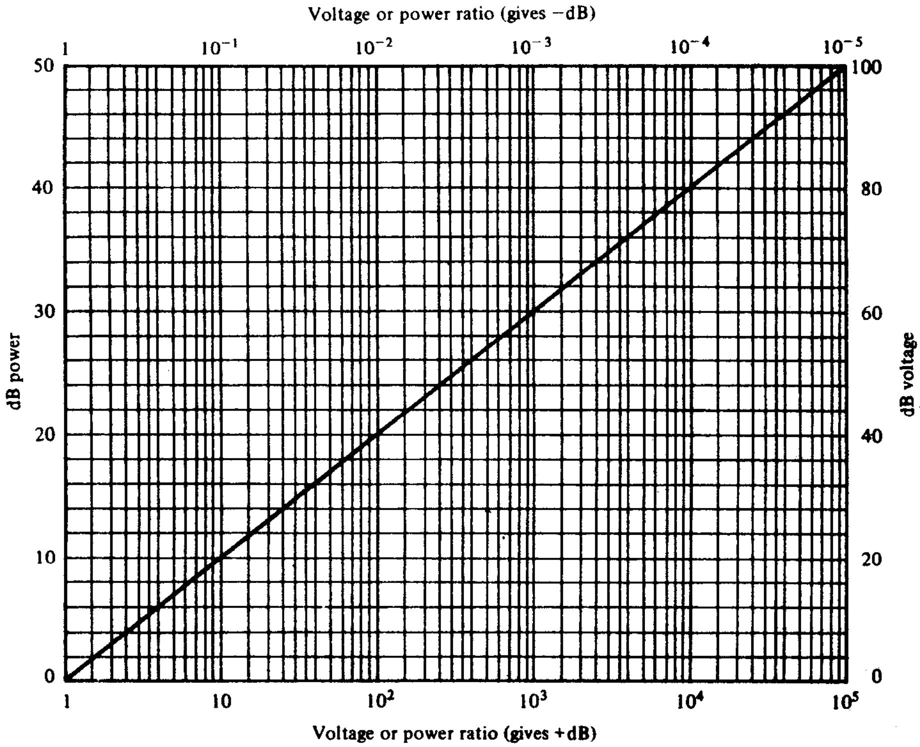

Fig. 1.1.1 Conversion of voltage and power ratios to dB

which is strictly not correct [n1, o1]. However, so long as we specify voltage or power gain there should be no confusion. For example a gain of 60 dB means a power ratio of 10^6 (1.1.3) but a voltage ratio of only 10^3 (1.1.5). Since $\log (P_1/P_2) = \log (P_1) - \log (P_2)$, then if $P_1 > P_2$ (i.e., gain) we get $+X$ dB say, while if $P_1 < P_2$ (i.e., loss or attenuation) we get $-Y$ dB say. The latter is stated as a 'gain' of $-Y$ dB, a loss of Y dB, or the output is said to be Y dB down on the input. For example, the most frequently quoted value is ± 3 dB which corresponds to a voltage gain of $\sqrt{2} = 1{\cdot}414$ or an attenuation of $1/\sqrt{2} = 0{\cdot}707$. Conversions may readily be made using Table 1.1.1 or Fig. 1.1.1.

Table 1.1.1 Conversion of power and voltage ratios to dB

Gives −dB		dB	Gives +dB		Gives −dB		dB	Gives +dB	
Voltage ratio	Power ratio		Voltage ratio	Power ratio	Voltage ratio	Power ratio		Voltage ratio	Power ratio
1	1	0	1	1	0·519	0·269	5·7	1·928	3·715
0·989	0·977	0·1	1·012	1·023	0·513	0·263	5·8	1·95	3·802
0·977	0·955	0·2	1·023	1·047	0·507	0·257	5·9	1·972	3·89
0·966	0·933	0·3	1·035	1·072	0·501	0·251	6	1·995	3·981
0·955	0·912	0·4	1·047	1·096	0·495	0·245	6·1	2·018	4·074
0·944	0·891	0·5	1·059	1·122	0·49	0·24	6·2	2·042	4·169
0·933	0·871	0·6	1·072	1·148	0·484	0·234	6·3	2·065	4·266
0·923	0·851	0·7	1·084	1·175	0·479	0·229	6·4	2·089	4·365
0·912	0·832	0·8	1·096	1·202	0·473	0·224	6·5	2·113	4·467
0·902	0·813	0·9	1·109	1·23	0·468	0·219	6·6	2·138	4·571
0·891	0·794	1	1·122	1·259	0·462	0·214	6·7	2·163	4·677
0·881	0·776	1·1	1·135	1·288	0·457	0·209	6·8	2·188	4·786
0·871	0·759	1·2	1·148	1·318	0·452	0·204	6·9	2·213	4·898
0·861	0·741	1·3	1·161	1·349	0·447	0·2	7	2·239	5·012
0·851	0·724	1·4	1·175	1·38	0·442	0·195	7·1	2·265	5·128
0·841	0·708	1·5	1·188	1·413	0·437	0·191	7·2	2·291	5·248
0·832	0·692	1·6	1·202	1·445	0·432	0·186	7·3	2·317	5·37
0·822	0·676	1·7	1·216	1·479	0·427	0·182	7·4	2·344	5·495
0·813	0·661	1·8	1·23	1·514	0·422	0·178	7·5	2·371	5·623
0·804	0·646	1·9	1·245	1·549	0·417	0·174	7·6	2·399	5·754
0·794	0·631	2	1·259	1·585	0·412	0·17	7·7	2·427	5·888
0·785	0·617	2·1	1·273	1·622	0·407	0·166	7·8	2·455	6·025
0·776	0·603	2·2	1·288	1·66	0·403	0·162	7·9	2·483	6·166
0·767	0·589	2·3	1·303	1·698	0·398	0·158	8	2·512	6·309
0·759	0·575	2·4	1·318	1·738	0·394	0·155	8·1	2·541	6·456
0·75	0·562	2·5	1·334	1·778	0·389	0·151	8·2	2·57	6·607
0·741	0·55	2·6	1·349	1·82	0·385	0·148	8·3	2·6	6·761
0·733	0·537	2·7	1·365	1·862	0·38	0·145	8·4	2·63	6·918
0·724	0·525	2·8	1·38	1·905	0·376	0·141	8·5	2·661	7·079
0·716	0·513	2·9	1·396	1·95	0·372	0·138	8·6	2·691	7·244
0·708	0·501	3	1·413	1·995	0·367	0·135	8·7	2·723	7·413
0·7	0·49	3·1	1·429	2·042	0·363	0·132	8·8	2·754	7·586
0·692	0·479	3·2	1·445	2·089	0·359	0·129	8·9	2·786	7·762
0·684	0·468	3·3	1·462	2·138	0·355	0·126	9	2·818	7·943
0·676	0·457	3·4	1·479	2·188	0·351	0·123	9·1	2·851	8·128
0·668	0·447	3·5	1·496	2·239	0·347	0·12	9·2	2·884	8·317
0·661	0·437	3·6	1·514	2·291	0·343	0·117	9·3	2·917	8·511
0·653	0·427	3·7	1·531	2·344	0·339	0·115	9·4	2·951	8·709
0·646	0·417	3·8	1·549	2·399	0·335	0·112	9·5	2·985	8·912
0·638	0·407	3·9	1·567	2·455	0·331	0·11	9·6	3·02	9·12
0·631	0·398	4	1·585	2·512	0·327	0·107	9·7	3·055	9·332
0·624	0·389	4·1	1·603	2·57	0·324	0·105	9·8	3·09	9·55
0·617	0·38	4·2	1·622	2·63	0·32	0·102	9·9	3·126	9·772
0·61	0·372	4·3	1·641	2·691	0·316	0·1	10	3·162	10
0·603	0·363	4·4	1·66	2·754	0·313	0·098	10·1	3·199	10·232
0·596	0·355	4·5	1·679	2·818	0·309	0·096	10·2	3·236	10·471
0·589	0·347	4·6	1·698	2·884	0·305	0·093	10·3	3·273	10·715
0·582	0·339	4·7	1·718	2·951	0·302	0·091	10·4	3·311	10·964
0·575	0·331	4·8	1·738	3·02	0·299	0·089	10·5	3·35	11·22
0·569	0·324	4·9	1·758	3·09	0·295	0·087	10·6	3·388	11·481
0·562	0·316	5	1·778	3·162	0·292	0·085	10·7	3·428	11·748
0·556	0·309	5·1	1·799	3·236	0·288	0·083	10·8	3·467	12·022
0·55	0·302	5·2	1·82	3·311	0·285	0·081	10·9	3·507	12·302
0·543	0·295	5·3	1·841	3·388	0·282	0·079	11	3·548	12·589
0·537	0·288	5·4	1·862	3·467	0·279	0·078	11·1	3·589	12·882
0·531	0·282	5·5	1·884	3·548	0·275	0·076	11·2	3·631	13·182
0·525	0·275	5·6	1·905	3·631	0·272	0·074	11·3	3·673	13·489

Table 1.1.1, continued

Gives −dB		dB	Gives +dB		Gives −dB		dB	Gives +dB	
Voltage ratio	Power ratio		Voltage ratio	Power ratio	Voltage ratio	Power ratio		Voltage ratio	Power ratio
0·269	0·072	11·4	3·715	13·803	0·162	0·026	15·8	6·166	38·016
0·266	0·071	11·5	3·758	14·125	0·16	0·026	15·9	6·237	38·901
0·263	0·069	11·6	3·802	14·454	0·158	0·025	16	6·309	39·807
0·26	0·068	11·7	3·846	14·79	0·157	0·025	16·1	6·382	40·735
0·257	0·066	11·8	3·89	15·135	0·155	0·024	16·2	6·456	41·683
0·254	0·065	11·9	3·935	15·487	0·153	0·023	16·3	6·531	42·654
0·251	0·063	12	3·981	15·848	0·151	0·023	16·4	6·607	43·648
0·248	0·062	12·1	4·027	16·217	0·15	0·022	16·5	6·683	44·665
0·245	0·06	12·2	4·074	16·595	0·148	0·022	16·6	6·761	45·705
0·243	0·059	12·3	4·121	16·981	0·146	0·021	16·7	6·839	46·769
0·24	0·058	12·4	4·169	17·377	0·145	0·021	16·8	6·918	47·859
0·237	0·056	12·5	4·217	17·782	0·143	0·02	16·9	6·998	48·974
0·234	0·055	12·6	4·266	18·196	0·141	0·02	17	7·079	50·114
0·232	0·054	12·7	4·315	18·62	0·14	0·02	17·1	7·161	51·282
0·229	0·052	12·8	4·365	19·053	0·138	0·019	17·2	7·244	52·476
0·226	0·051	12·9	4·416	19·497	0·136	0·019	17·3	7·328	53·698
0·224	0·05	13	4·467	19·951	0·135	0·018	17·4	7·413	54·949
0·221	0·049	13·1	4·518	20·416	0·133	0·018	17·5	7·499	56·229
0·219	0·048	13·2	4·571	20·892	0·132	0·017	17·6	7·585	57·539
0·216	0·047	13·3	4·624	21·378	0·13	0·017	17·7	7·673	58·879
0·214	0·046	13·4	4·677	21·876	0·129	0·017	17·8	7·762	60·25
0·211	0·045	13·5	4·731	22·386	0·127	0·016	17·9	7·852	61·654
0·209	0·044	13·6	4·786	22·907	0·126	0·016	18	7·943	63·09
0·207	0·043	13·7	4·842	23·441	0·124	0·015	18·1	8·035	64·559
0·204	0·042	13·8	4·898	23·987	0·123	0·015	18·2	8·128	66·063
0·202	0·041	13·9	4·954	24·545	0·122	0·015	18·3	8·222	67·602
0·2	0·04	14	5·012	25·117	0·12	0·014	18·4	8·317	69·176
0·197	0·039	14·1	5·07	25·702	0·119	0·014	18·5	8·414	70·788
0·195	0·038	14·2	5·128	26·301	0·117	0·014	18·6	8·511	72·436
0·193	0·037	14·3	5·188	26·913	0·116	0·013	18·7	8·61	74·124
0·191	0·036	14·4	5·248	27·54	0·115	0·013	18·8	8·709	75·85
0·188	0·035	14·5	5·309	28·182	0·114	0·013	18·9	8·81	77·617
0·186	0·035	14·6	5·37	28·838	0·112	0·013	19	8·912	79·425
0·184	0·034	14·7	5·432	29·51	0·111	0·012	19·1	9·015	81·275
0·182	0·033	14·8	5·495	30·197	0·11	0·012	19·2	9·12	83·168
0·18	0·032	14·9	5·559	30·901	0·108	0·012	19·3	9·225	85·105
0·178	0·032	15	5·623	31·62	0·107	0·011	19·4	9·332	87·087
0·176	0·031	15·1	5·688	32·357	0·106	0·011	19·5	9·44	89·116
0·174	0·03	15·2	5·754	33·111	0·105	0·011	19·6	9·549	91·191
0·172	0·03	15·3	5·821	33·882	0·104	0·011	19·7	9·66	93·316
0·17	0·029	15·4	5·888	34·671	0·102	0·01	19·8	9·772	95·489
0·168	0·028	15·5	5·956	35·479	0·101	0·01	19·9	9·885	97·713
0·166	0·028	15·6	6·025	36·305	0·100	0·01	20·0	10·0	100·0
0·164	0·027	15·7	6·095	37·151					

If voltage and current are varying with time then Ohm's law must be interpreted more carefully. The instantaneous ratio of voltage to current is not constant but varies over an infinite range, unless the circuit contains purely resistive elements. If inductance or capacity is included energy may be stored, so that the current depends not only on the instantaneous voltage but also on all that happened before. In terms of alternating or sinusoidal voltages there will be a phase difference between the voltage and current, and it is only the averages over a cycle (r.m.s. values), or the peak values that will satisfy Ohm's law, although the effective 'resistance' may not be a purely real number [a1].