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Bibliography of the Publications of S.S. Chern

Note: **Boldface** numbers at the end of each entry denote the volume in which the entry appears.

I. *Books and Monographs*

1. *Topics in Differential Geometry* (mimeographed), Institute for Advanced Study, Princeton (1951), 106 pp. **(IV)**
2. *Differentiable Manifolds* (mimeographed), University of Chicago, Chicago (1953), 166 pp.
3. *Complex Manifolds*
   a. University of Chicago, Chicago (1956), 195 pp.
   b. University of Recife, Recife, Brazil (1959), 181 pp.
   c. Russian translation, Moscow (1961), 239 pp.
4. *Studies in Global Geometry and Analysis* (Editor), Mathematical Association of America (1967), 200 pp.
5. *Complex Manifolds without Potential Theory,* van Nostrand (1968), 92 pp. Second edition, revised, Springer-Verlag (1979) 152 pp.
6. *Minimal Submanifolds in a Riemannian Manifold* (mimeographed), University of Kansas, Lawrence (1968), 55 pp. **(IV)**
7. (with Wei-huan Chen) *Differential Geometry Notes,* in Chinese, Beijing University Press (1983), 321 pp.
8. *Studies in Global Differential Geometry* (Editor), Mathematical Association of America (1988), 350 pp.

II. *Papers*

1932

[1] Pairs of plane curves with points in one-to-one correspondence. *Science Reports Nat. Tsing Hua Univ.* **1** (1932) 145–153. **(II)**

1935

*[2] Triads of rectilinear congruences with generators in correspondence. *Tohoku Math. J.* **40** (1935) 179–188.

[3] Associate quadratic complexes of a rectilinear congruence. *Tohoku Math. J.* **40** (1935) 293–316. **(II)**

[4] Abzählungen für Gewebe. *Abh. Math. Sem. Univ. Hamburg* **11** (1935) 163–170. **(I)**

1936

[5] Eine Invariantentheorie der Dreigewebe aus r-dimensionalen Mannigfaltigkeiten im R_{2r}. *Abh. Math. Sem. Univ. Hamburg* **11** (1936) 333–358. **(I)**

* Does not appear in these volumes.

1937

[6] Sur la géométrie d'une équation différentielle du troisième ordre. *C. R. Acad. Sci. Paris* **204** (1937) 1227–1229. **(II)**

[7] Sur la possibilitè de plonger un espace à connexion projective donné dans un espace projectif. *Bull. Sci. Math.* **61** (1937) 234–243. **(I)**

1938

[8] On projective normal coördinates. *Ann. of Math.* **39** (1938) 165–171. **(II)**

[9] On two affine connections. *J. Univ. Yunnan* **1** (1938) 1–18. **(II)**

1939

[10] Sur la géométrie d'un système d'équations différentielles du second ordre. *Bull. Sci. Math* **63** (1939) 206–212. **(II)**

1940

*[11] The geometry of higher path-spaces. *J. Chin. Math. Soc.* **2** (1940) 247–276.

[12] Sur les invariants intégraux en géométrie. *Science Reports Nat. Tsing Hua Univ.* **4** (1940) 85–95. **(II)**

[13] The geometry of the differential equation $y''' = F(x, y', y'')$. *Science Reports Nat. Tsing Hua Univ.* **4** (1940) 97–111. **(I)**

[14] Sur une généralisation d'une formule de Crofton. *C.R. Acad. Sci. Paris* **210** (1940) 757–758. **(II)**

[15] (with C.T. Yen) Sulla formula principale cinematica dello spazio ad n dimensioni. *Boll. Un. Mat. Ital.* **2** (1940) 434–437. **(II)**

*[16] Generalization of a formula of Crofton. *Wuhan Univ. J. Sci.* **7** (1940) 1–16.

1941

[17] Sur les invariants de contact en géométrie projective différentielle. *Acta Pontif. Acad. Sci.* **5** (1941) 123–140. **(II)**

1942

[18] On integral geometry in Klein spaces. *Ann. of Math.* **43** (1942) 178–189. **(I)**

[19] On the invariants of contact of curves in a projective space of N dimensions and their geometrical interpretation. *Acad. Sinica Sci. Record* **1** (1942) 11–15. **(I)**

[20] The geometry of isotropic surfaces. *Ann. of Math.* **43** (1942) 545–559. **(II)**

[21] On a Weyl geometry defined from an $(n-1)$-parameter family of hypersurfaces in a space of n dimensions. *Acad. Sinica Sci. Record* **1** (1942) 7–10. **(I)**

1943

[22] On the Euclidean connections in a Finsler space. *Proc. Nat. Acad. Sci. USA*, **29** (1943) 33–37. **(II)**

[23] A generalization of the projective geometry of linear spaces. *Proc. Nat. Acad. Sci. USA*, **29** (1943) 38–43. **(I)**

1944

[24] Laplace transforms of a class of higher dimensional varieties in a projective space of n dimensions. *Proc. Nat. Acad. Sci. USA*, **30** (1944) 95–97. **(II)**

[25] A simple intrinsic proof of the Gauss–Bonnet formula for closed Riemannian manifolds. *Ann of Math.* **45** (1944) 747–752. **(I)**

[26] Integral formulas for the characteristic classes of sphere bundles. *Proc. Nat. Acad. Sci. USA* **30** (1944) 269–273. **(II)**

*[27] On a theorem of algebra and its geometrical application. *J. Indian Math. Soc.* **8** (1944) 29–36.

BIBLIOGRAPHY OF THE PUBLICATIONS OF S.S. CHERN

1945

*[28] On Grassmann and differential rings and their relations to the theory of multiple integrals. *Sankhya* **7** (1945) 2–8.

[29] Some new characterizations of the Euclidean sphere. *Duke Math. J.* **12** (1945) 279–290. **(II)**

[30] On the curvature integra in a Riemannian manifold. *Ann. of Math.* **46** (1945) 674–684. **(I)**

*[31] On Riemannian manifolds of four dimensions. *Bull. Amer. Math. Soc.* **51** (1945) 964–971.

1946

[32] Some new viewpoints in the differential geometry in the large. *Bull. Amer. Math. Soc.* **52** (1946) 1–30. **(II)**

[33] Characteristic classes of Hermitian manifolds. *Ann. of Math.* **47** (1946) 85–121. (I)

1947

[34] (with H.C. Wang). Differential geometry in symplectic space I. *Science Report Nat. Tsing Hua Univ.* **4** (1947) 453–477. **(II)**

[35] Sur une classe remarquable de variétés dans l'espace projectif à *N* dimensions. *Science Reports Nat. Tsing Hua Univ.* **4** (1947) 328–336. **(I)**

*[36] On the characteristic classes of Riemannian manifolds. *Proc. Nat. Acad. Sci USA*, **33** (1947) 78–82.

*[37] Note of affinely connected manifolds. *Bull. Amer. Math. Soc.* **53** (1947) 820–823; correction ibid **54** (1948) 985–986.

*[38] On the characteristic ring of a differentiable manifold. *Acad. Sinica. Sci. Record* **2** (1947) 1–5.

1948

[39] On the multiplication in the characteristic ring of a sphere bundle. *Ann. of Math.* **49** (1948) 362–372. **(I)**

[40] Note on projective differential line geometry. *Acad. Sinica Sci. Record* **2** (1948) 137–139. **(II)**

*[41] (with Y.L. Jou) On the orientability of differentiable manifolds. *Science Reports Nat. Tsing Hua Univ.* **5** (1948) 13–17.

[42] Local equivalence and Euclidean connections in Finsler spaces. *Science Reports Nat. Tsing Hua Univ.* **5** (1948) 95–121. **(II)**

1949

[43] (with Y.F. Sun). The imbedding theorem for fibre bundles. *Trans. Amer. Math. Soc* **67** (1949) 286–303. **(II)**

*[44] (with S.T. Hu) Parallelisability of principal fibre bundles. *Trans. Amer. Math. Soc.* **67** (1949) 304–309.

1950

[45] (with E. Spanier). The homology structure of sphere bundles. *Proc. Nat. Acad. Sci. USA*, **36** (1950) 248–255. **(II)**

[46] Differential geometry of fiber bundles. *Proc. Int. Congr. Math.* (1950) II 397–411. **(II)**

1951

[47] (with E. Spanier). A theorem on orientable surfaces in four-dimensional space. *Comm. Math. Helv.* **25** (1951) 205–209. **(I)**

BIBLIOGRAPHY OF THE PUBLICATIONS OF S.S. CHERN

1952

[48] On the kinematic formula in the Euclidean space of N dimensions. *Amer. J. Math* **74** (1952) 227–236. **(II)**

[49] (with C. Chevalley). Elie Cartan and his mathematical work. *Bull. Amer. Math. Soc.* **58** (1952) 217–250. **(II)**

[50] (with N.H. Kuiper) Some theorems on the isometric imbedding of compact Riemann manifolds in Euclidean space. *Ann. of Math.* **56** (1952) 422–430. **(II)**

1953

[51] On the characteristic classes of complex sphere bundles and algebraic varieties. *Amer. J. of Math.*, **75** (1953) 565–597. **(I)**

*[52] Some formulas in the theory of surfaces. *Boletin de la Sociedad Matematica Mexicana*, **10** (1953) 30–40.

[53] Relations between Riemannian and Hermitian geometries. *Duke Math. J.*, **20** (1953) 575–587. **(II)**

1954

[54] Pseudo-groupes continus infinis *Colloque de Geom. Diff.* Strasbourg (1954) 119–136. **(I)**

[55] (with P. Hartman and A. Wintner) On isothermic coordinates. *Comm. Math. Helv.* **28** (1954) 301–309. **(I)**

1955

[56] La géométrie des sous-variétés d'un espace euclidien à plusieurs dimensions. *l'Ens. Math.*, **40** (1955) 26–46. **(II)**

[57] An elementary proof of the existence of isothermal parameters on a surface. *Proc. Amer. Math. Soc.*, **6** (1955) 771–782. **(II)**

[58] On special W-surfaces. *Proc. Amer. Math. Soc.*, **6** (1955) 783–786. **(II)**

[59] On curvature and characteristic classes of a Riemann manifold. *Abh. Math. Sem. Univ. Hamburg* **20** (1955) 117–126. **(II)**

1956

*[60] Topology and differential geometry of complex manifolds. *Bull. Amer. Math. Soc.*, **62** (1956) 102–117.

1957

[61] On a generalization of Kähler geometry. *Lefschetz jubilee volume*. Princeton Univ. Press (1957) 103–121. **(I)**

[62] (with R. Lashof) On the total curvature of immersed manifolds. *Amer. J. of Math.* **79** (1957) 306–318. **(I)**

[63] (with F. Hirzebruch and J-P. Serre) On the index of a fibered manifold. *Proc. Amer. Math. Soc.*, **8** (1957) 587–596. **(I)**

[64] A proof of the uniqueness of Minkowski's problem for convex surfaces. *Amer. J. of Math.*, **79** (1957) 949–950. **(II)**

1958

*[65] Geometry of submanifolds in complex projective space. *Symposium International de Topologia Algebraica* (1958) 87–96.

[66] (with R.K. Lashof) On the total curvature of immersed manifolds, II. *Michigan Math. J.* **5** (1958) 5–12. **(II)**

[67] Differential geometry and integral geometry. *Proc. Int. Congr. Math. Edinburgh* (1958) 441–449. **(II)**

1959

[68] Integral formulas for hypersurfaces in Euclidean space and their applications to uniqueness theorems. *J. of Math. and Mech.* **8** (1959) 947–956. **(I)**

1960

[69] (with J. Hano and C.C. Hsiung) A uniqueness theorem on closed convex hypersurfaces in Euclidean space. *J. of Math. and Mech.* **9** (1960) 85–88. **(I)**

[70] Complex analytic mappings of Riemann surfaces I. *Amer. J. of Math.* **82** (1960) 323–337. **(I)**

[71] The integrated form of the first main theorem for complex analytic mappings in several complex variables. *Ann. of Math.* **71** (1960) 536–551. **(I)**

*[72] Geometrical structures on manifolds. *Amer. Math. Soc. Pub.* (1960) 1–31.

*[73] La géométrie des hypersurfaces dans l'espace euclidean. *Seminaire Bourbaki*, **193** (1959–1960).

*[74] Sur les métriques Riemanniens compatibles avec une reduction du groupe structural. *Séminaire Ehresmann*, January 1960.

1961

[75] Holomorphic mappings of complex manifolds. *L'Ens. Math.* **7** (1961) 179–187. **(I)**

1962

*[76] Geometry of quadratic differential form. *J. of SIAM* **10** (1962) 751–755.

1963

[77] (with C.C. Hsiung) On the isometry of compact submanifolds in Euclidean space. *Math. Annalen* **149** (1963) 278–285. **(II)**

[78] Pseudo-Riemannian geometry and Gauss–Bonnet formula. *Academia Brasileira de Ciencias* **35** (1963) 17–26. **(I)**

1965

[79] Minimal surfaces in an Euclidean space of *N* dimensions. *Differential and Combinatorial Topology*, Princeton Univ. Press (1965) 187–198. **(I)**

[80] (with R. Bott) Hermitian vector bundles and the equidistribution of the zeroes of their holomorphic sections. *Acta. Math.* **114** (1965) 71–112. **(II)**

[81] On the curvatures of a piece of hypersurface in Euclidean space. *Abh. Math. Sem. Univ. Hamburg* **29** (1965) 77–91. **(III)**

[82] On the differential geometry of a piece of submanifold in Euclidean space. *Proc. of U.S.–Japan Seminar in Diff. Geom.* (1965) 17–21. **(III)**

1966

[83] Geometry of *G*-structures. *Bull. Amer. Math. Soc.* **72** (1966) 167–219. **(III)**

[84] On the kinematic formula in integral geometry. *J. of Math. and Mech.* **16** (1966) 101–118. **(III)**

*[85] Geometrical structures on manifolds and submanifolds. *Some Recent Advances in Basic Sciences*, Yeshiva Univ. Press (1966) 127–135.

1967

[86] (with R. Osserman) Complete minimal surfaces in Euclidean *n*-space. *J. de l'Analyse Math.* **19** (1967) 15–34. **(III)**

[87] Einstein hypersurfaces in a Kählerian manifold of constant holomorphic curvature. *J. Diff. Geom.* **1** (1967) 21–31. **(III)**

1968

[88] On holomorphic mappings of Hermitian manifolds of the same dimension. *Proc. Symp. Pure Math.* **11**. Entire Functions and Related Parts of Analysis (1968) 157–170. **(I)**

1969

[89] Simple proofs of two theorems on minimal surfaces. *L'Ens. Math.* **15** (1969) 53–61. **(I)**

1970

[90] (with H. Levine and L. Nirenberg) Intrinsic norms on a complex manifold. *Global analysis*, Princeton Univ. Press (1970) 119–139. **(I)**

[91] (with M. do Carmo and S. Kobayashi) Minimal submanifolds of a sphere with second fundamental form of constant length. *Functional Analysis and Related Fields*, Springer-Verlag (1970) 59–75. **(I)**

[92] (with R. Bott) Some formulas related to complex transgression. *Essays on Topology and Related Topics*, Springer-Verlag, (1970) 48–57. **(I)**

*[93] Holomorphic curves and minimal surfaces. *Carolina Conference Proceedings* (1970) 28 pp.

[94] On minimal spheres in the four-sphere, Studies and Essays Presented to Y. W. Chen, Taiwan, (1970) 137–150. **(I)**

[95] Differential geometry: Its past and its future. *Actes Congrès Intern. Math.* (1970) **1**, 41–53. **(III)**

[96] On the minimal immersions of the two-sphere in a space of constant curvature. *Problems in Analysis*, Princeton Univ. Press, (1970) 27–40. **(III)**

1971

[97] Brief survey of minimal submanifolds. *Differentialgeometrie im Grossen*. W. Klingenberg (ed.), **4** (1971) 43–60. **(III)**

*[98] (with J. Simons) Some cohomology classes in principal fibre bundles and their application to Riemannian geometry. *Proc. Nat. Acad. Sci. USA*, **68** (1971) 791–794.

1972

[99] Holomorphic curves in the plane. *Diff. Geom., in honor of K. Yano*, (1972) 73–94. **(III)**

*[100] Geometry of characteristic classes. *Proc. 13th Biennial Sem. Canadian Math. Congress*, (1972) 1–40. Also pub. in Russian translation.

1973

[101] Meromorphic vector fields and characteristic numbers. *Scripta Math.* **29** (1973) 243–251. **(I)**

*[102] The mathematical works of Wilhelm Blaschke. *Abh. Math. Sem. Univ. Hamburg* **39** (1973) 1–9.

1974

[103] (with. J. Simons) Characteristic forms and geometrical invariants. *Ann. of Math.* **99** (1974) 48–69. **(I)**

[104] (with M. Cowen, A. Vitter III) Frenet frames along holomorphic curves. *Proc. of Conf. on Value Distribution Theory*, Tulane Univ. (1974) 191–203. **(III)**

[105] (with J. Moser) Real hypersurfaces in complex manifolds. *Acta. Math.* **133** (1974) 219–271. **(III)**

1975

[106] (with S.I. Goldberg) On the volume decreasing property of a class of real harmonic mappings. *Amer. J. of Math.* **97** (1975) 133–147. **(III)**

[107] On the projective structure of a real hypersurface in C_{n+1}. *Math. Scand.* **36** (1975) 74–82. **(III)**

1976

[108] (with J. White) Duality properties of characteristic forms. *Inv. Math.* **35** (1976) 285–297. **(III)**

1977

*[109] Circle bundles. *Geometry and topology, III.* Latin Amer. School of Math, Lecture Notes in Math. Springer-Verlag, **597** (1977) 114–131.

*[110] (with P.A. Griffiths) Linearization of webs of codimension one and maximum rank. *Proc. Int. Symp. on Algebraic Geometry, Kyoto* (1977) 85–91.

1978

[111] On projective connections and projective relativity. *Science of Matter,* dedicated to Ta-you Wu, (1978) 225–232. **(III)**

[112] (with P.A. Griffiths) Abel's theorem and webs. *Jber. d. Dt. Math. Verein.* **80** (1978) 13–110. **(III)**

[113] (with P.A. Griffiths) An inequality for the rank of a web and webs of maximum rank. *Annali Sc. Norm. Super.–Pisa, Serie IV,* **5** (1978) 539–557. (III)

[114] Affine minimal hypersurfaces. *Minimal Submanifolds and Geodesics.* Kaigai Publications, Ltd. (1978) 1–14. **(III)**

1979

*[115] Herglotz's work on geometry. *Ges. Schriften Gustav Herglotz,* Göttingen (1979) xx–xxi.

[116] (with C.L. Terng) An analogue of Bäcklund's theorem in affine geometry. *Rocky Mountain J. Math.* **10** (1979) 105–124. **(III)**

[117] From triangles to manifolds. *Amer. Math. Monthly* **86** (1979) 339–349. **(III)**

[118] (with C.K. Peng) Lie groups and KdV equations. *Manuscripta Math.* **28** (1979) 207–217. **(III)**

1980

[119] General relativity and differential geometry. *Some Strangeness in the Proportion: A Centennial Symp. to Celebrate the Achievements of Albert Einstein, Harry Woolf* (ed.), Addison-Wesley Publ. (1980) 271–287. **(III)**

[120] (with W.M. Boothby and S.P. Wang) The mathematical work of H.C. Wang. *Bull. Inst. of Math,* **8** (1980) xiii–xxiv. **(III)**

*[121] Geometry and physics. *Math. Medley,* Singapore, **8** (1980) 1–6.

*[122] (with R. Bryant and P.A. Griffiths) Exterior differential systems. *Proc. of 1980 Beijing DD-Symposium,* (1980) 219–338.

1981

[123] Geometrical interpretation of the sinh-Gordon equation. *Annales Polonici Mathematici* **39** (1981) 63–69. **(IV)**

[124] (with P.A. Griffiths) Corrections and addenda to our paper: "Abel's theorem and webs." *Jber. d. Dt. Math.–Verein.* **83** (1981) 78–83. **(III)**

[125] (with R. Osserman) Remarks on the Riemannian metric of a minimal submanifold. *Geometry Symposium Utrecht 1980,* Lecture Notes in Math. Springer-Verlag **894** (1981) 49–90. **(IV)**

[126] (with J. Wolfson) A simple proof of Frobenius theorem. *Manifolds and Lie Groups, Papers in Honor of Y. Matsushima.* Birkhäuser (1981) 67–69. **(IV)**

[127] (with K. Tenenblat) Foliations on a surface of constant curvature and modified Korteweg–de Vries equations. *J. Diff. Geom.* **16** (1981) 347–349. **(IV)**

[128] (with C.K. Peng) On the Bäcklund transformations of KdV equations and modified KdV equations. *J. of China Univ. of Sci. and Tech.*, **11** (1981) 1–6. **(IV)**

1982

[129] Web geometry. *Proc. Symp. in Pure Math.* **39** (1983) 3–10. **(IV)**

[130] Projective geometry, contact transformations, and *CR*-structures. *Archiv der Math.* **38** (1982) 1–5. **(IV)**

1983

[131] (with J. Wolfson) Minimal surfaces by moving frames. *Amer. J. Math.* **105** (1983) 59–83. **(IV)**

[132] On surfaces of constant mean curvature in a three-dimensional space of constant curvature. *Geometric Dynamics*, Springer Lecture Notes **1007** (1983) 104–108. **(IV)**

1984

[133] Deformation of surfaces preserving principal curvatures, *Differential Geometry and Complex Analysis*, Volume in Memory of H. Rauch, Springer-Verlag (1984) 155–163. **(IV)**

1985

[134] (with R. Hamilton) On Riemannian metrics adapted to three-dimensional contact manifolds. *Arbeitstagung Bonn 1984* Springer Lecture Notes **1111** (1985) 279–308. **(IV)**

[135] (with J. Wolfson) Harmonic maps of S^2 into a complex Grassmann manifold. *Proc. Nat. Acad. Sci.* USA **82** (1985) 2217–2219. **(IV)**

[136] Moving frames, *Soc. Math. de France*, Astérisque, (1985) 67–77. **(IV)**

*[137] Wilhelm Blaschke and web geometry, Wilhelm Blaschke—Gesammelte Werke. **5**, Thales Verlag, (1985) 25–27.

*[138] The mathematical works of Wilhelm Blaschke—an update. Thales Verlag, (1985), 21–23.

1986

[139] (with K. Tenenblat) Pseudospherical surfaces and evolution equations. *Studies in Applied Math.* MIT **74** (1986) 55–83. **(IV)**

[140] On a conformal invariant of three-dimensional manifolds. *Aspects of Mathematics and Its Applications* Elsevier Science Publishers B.V. (1986) 245–252. **(IV)**

*[141] (with P.A. Griffiths) Pfaffian systems in involution. *Proceedings of 1982 Changchun Symposium on Differential Geometry and Differential Equations*, Science Press, China, (1986) 233–256.

1987

[142] (with J. Wolfson) Harmonic maps of the two-sphere into a complex Grassmann manifold II. *Ann. of Math.* **125** (1987) 301–335. **(IV)**

[143] (with T. Cecil) Tautness and Lie Sphere geometry *Math. Annalen*, Volume Dedicated to F. Hirzebruch **278** (1987) 381–399. **(IV)**

1988

[144] Vector bundles with a connection. *Studies in Global Differential Geometry*, MAA, no. 27 (1988), 1–26.

1989

[145] (with T. Cecil) Dupin submanifolds in Lie sphere geometry, to appear in *Differential Geometry and Topology*, Springer Lecture Notes 1989.

[146] Historical remarks on Gauss–Bonnet, to appear in Moser Volume, Academic Press.

[147] An introduction to Dupin submanifolds, to appear in Do Carmo Volume.

Contents

*Numbers in brackets refer to the Bibliography of the Publications of S.S. Chern (see pages vii–xiv).

ANNALES
POLONICI MATHEMATICI
XXXIX (1981)

Geometrical interpretation of the sinh-Gordon equation

by SHIING-SHEN CHERN* (Berkeley, Calif.)

Stefan Bergman in memoriam

Abstract. In a three-dimensional pseudo-Riemannian manifold of constant curvature consider a spacelike (resp. timelike) surface of constant negative (resp. positive) Gaussian curvature. Then the asymptotic curves are everywhere real and distinct, and the function 2ψ (= the angle between the asymptotic directions) satisfies, relative to the Tchebycheff coördinates, a sine-Gordon (resp. sinh-Gordon) equation. An example of such a manifold is SL (2; R) with the biinvariant metric.

1. Introduction. It is well known that the sine-Gordon equation (SGE)

$$u_{xx} - u_{tt} = \sin u \tag{1}$$

has a geometrical interpretation in terms of the surfaces of constant negative curvature in the three-dimensional euclidean space. We will show in this paper that by studying surfaces of constant Gaussian curvature in a three-dimensional pseudo-Riemannian manifold of constant curvature one is led to geometrical interpretations of (1) and of the sinh-Gordon equation (SHGE)

$$u_{xx} - u_{tt} = \sinh u. \tag{2}$$

2. Pseudo-Riemannian geometry. In this section we will give a review of local pseudo-Riemannian geometry, using moving frames. Let M be a smooth manifold of dimension m, with the local coordinates x^α. (In this section all small Greek indices run from 1 to m.) A pseudo-Riemannian metric in M is given by the non-degenerate quadratic differential form

$$ds^2 = \sum_{\alpha,\beta} G_{\alpha\beta}(x^1, \ldots, x^m)\, dx^\alpha dx^\beta, \quad G_{\alpha\beta} = G_{\beta\alpha}. \tag{3}$$

The metric is called *Riemannian* if the form is positive definite and Lorentzian if it is of signature $+ \ldots + -$.

Let $x \in M$ and let T_x, T_x^* be respectively the tangent and cotangent spaces

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of M at x. A frame at x is an ordered set of linearly independent vectors $e_\alpha \in T_x$. The essence of the method of moving frames is to free the frames from local coordinates, a freedom which gives handsome returns. To e_α is associated a dual coframe $\omega^\beta \in T_x^*$. When they are defined over a neighbourhood, ω^β can be identified with a linear differential form. Relative to ω^β we can write

(3a) $$ds^2 = \sum g_{\alpha\beta}\, \omega^\alpha \omega^\beta, \quad g_{\alpha\beta} = g_{\beta\alpha}.$$

The Levi-Civita connection is given by

(4) $$De_\alpha = \sum \omega_\alpha^\beta e_\beta,$$

where the connection forms ω_α^β are determined, uniquely, by the conditions

(5) $$d\omega^\alpha = \sum \omega^\beta \wedge \omega_\beta^\alpha,$$

(6) $$\omega_{\alpha\beta} + \omega_{\beta\alpha} = dg_{\alpha\beta}.$$

Geometrically the first condition (5) means the "absence of torsion". In the second condition (6) the $\omega_{\alpha\beta}$ are defined by

(7) $$\omega_{\alpha\beta} = \sum g_{\beta\gamma}\, \omega_\alpha^\gamma$$

and the condition means the preservation of the scalar product of vectors under parallelism. We use $g_{\alpha\beta}$ to lower indices, as in classical tensor analysis.

The curvature forms are defined by

(8) $$\Omega_\alpha^\beta = d\omega_\alpha^\beta - \sum_\gamma \omega_\alpha^\gamma \wedge \omega_\gamma^\beta,$$

(9) $$\Omega_{\alpha\beta} = \sum g_{\beta\gamma}\, \Omega_\alpha^\gamma.$$

It can be proved that

(10) $$\Omega_{\alpha\beta} + \Omega_{\beta\alpha} = 0.$$

The pseudo-Riemannian metric (3a) is said to be of *constant curvature* c if

(11) $$\Omega_{\alpha\beta} = -c\,\omega_\alpha \wedge \omega_\beta,$$

where

(12) $$\omega_\alpha = \sum g_{\alpha\beta}\, \omega^\beta.$$

In applications it will be advantageous to use frames, where $g_{\alpha\beta}$ = const, such as orthonormal frames in the Riemannian case. Then (6) becomes

(13) $$\omega_{\alpha\beta} + \omega_{\beta\alpha} = 0.$$

3. Surfaces in three-dimensional manifolds. Let M be a three-dimensional pseudo-Riemannian manifold and

(14) $$f\colon S \to M$$

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