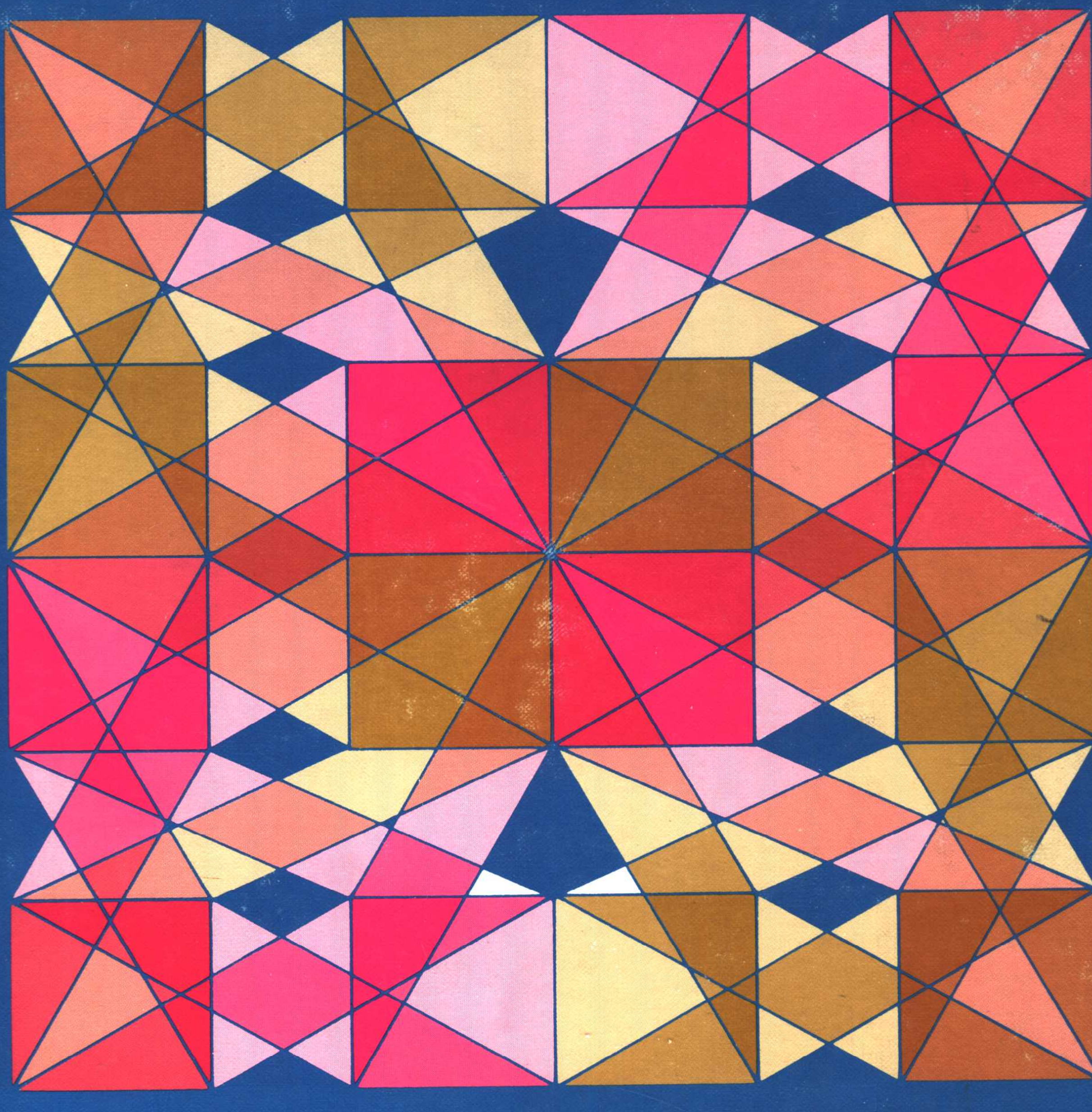
Engineering Mechanics

2nd edition

STATICS

R.C. Hibbeler



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Preface

The purpose of this book is to provide the student with a clear and thorough presentation of the theory and application of the principles of engineering mechanics. Emphasis is placed on developing the student's ability to analyze problems—a most important skill for any engineer. Furthermore, the Système International or SI system of units is used for numerical work since this system is intended in time to become the worldwide standard for measurement.

The contents of each chapter are organized into well-defined sections. Selected groups of sections contain the development and explanation of specific topics, illustrative example problems, and a set of problems designed to test the student's ability to apply the theory. Many of the problems depict realistic situations encountered in engineering practice. It is hoped that this realism will both stimulate the student's interest in engineering mechanics and provide a means for developing the skill to reduce any such problem from its physical description to a model or symbolic representation to which the principles of mechanics may be applied. In any set, the problems are arranged in order of increasing difficulty. Furthermore, the answers to all but every fourth problem, which is indicated by an asterisk, are listed in the back of the book. SI units are used in all the numerical examples and problems; however, for the convenience of some instructors, every fifth problem is stated *twice,* once in SI units and again in FPS units.

Besides a change from FPS to SI units and the addition of many new problems, this book differs from the author's first edition, *Engineering Mechanics: Statics,* in many respects. Most of the text material has been completely rewritten so that topics within each section are categorized into subgroups, defined by boldface titles. The purpose of this is to present a structured method for introducing each new definition or concept, and to provide a convenient means for later reference or review of the material.

Another unique feature used throughout this book is the "Procedure for Analysis." This guide to problem solving, which was initially presented in Sec. 9-3 of the first edition, is essentially a step-by-step set of instructions which provide the student with a logical and orderly method to follow when applying the theory. As in the first edition, the example problems are solved using this outlined method in order to clarify application of the steps.

Since mathematics provides a systematic means of applying the principles of mechanics, the student is expected to have prior knowledge of algebra, geometry, trigonometry, and, for complete coverage, some calculus. Vector analysis is introduced at points where it is most applicable. Its use often provides a convenient means for presenting concise derivations of the theory, and it makes possible a simple and systematic solution of many complicated three-dimensional problems. Occasionally, the example problems are solved using several different methods of analysis so that the student develops the ability to use mathematics as a tool whereby the solution of any problem may be carried out in the most direct and effective manner.

The book is divided into 11 chapters, in which the principles introduced are first applied to simple situations. Specifically, each principle is applied first to a particle, then to a rigid body subjected to a coplanar system of forces, and finally to the most general case of spatial force systems acting on a rigid body.

In particular, an introduction to mechanics and a discussion of units is outlined in Chapter 1. The notion of a vector and the properties of a concurrent force system are introduced in Chapter 2. This theory is then applied to the equilibrium of particles in Chapter 3. Chapter 4 contains a general discussion of both concentrated and distributed force systems and the methods used to simplify them. The principles of rigid-body equilibrium are developed in Chapter 5. These principles are applied to specific problems involving the equilibrium of trusses, frames, and machines in Chapter 6, and to the analysis of internal forces in beams and cables in Chapter 7. Applications to problems involving frictional forces are discussed in Chapter 8; and topics related to the centroid and the center of gravity are given in Chapter 9. If time permits, sections concerning more advanced topics, indicated by stars, may be covered. Some topics in Chapter 10 ("Moments of Inertia for an Area") and Chapter 11 ("Virtual Work") may be omitted from the basic course. Note, however, that this more advanced material provides a suitable reference for basic principles when it is discussed in more advanced courses.

At the discretion of the instructor, some of the material may be presented in a different sequence with no loss in continuity. For example, it is possible to introduce the concept of a force and all the necessary methods of vector analysis by first covering Chapter 2 and Secs. 4-2 and 4-11. Then, after covering the rest of Chapter 4 (force and moment systems), the equilibrium methods in Chapters 3 and 5 can be discussed. Furthermore, Chapter 9 may be covered after Sec. 4-10 (distributed force systems), since understanding of this material does not depend upon the methods of equilibrium.

The author has endeavored to write this book so that it will appeal to both the student and the instructor. In doing so, it must be admitted that many people helped in its development. In this regard, I wish to acknowledge the valuable suggestions and comments made by M. H. Clayton, North Carolina State University; D. Krajcinovic, University of Illinois at Chicago Circle; W. Lee, United States Naval Academy; G. Mavrigian, Youngstown State University; W. C. Van Buskirk, Tulane University; and P. K. Mallick, Illinois Institute of Technology. Many thanks are also extended to all of the author's students and to the professionals who have provided suggestions and comments. Although the list is too long to mention, it is hoped that those who have given help will accept this anonymous recognition. Lastly, I should like to acknowledge the able assistance of my wife, Cornelie, who has furnished a great deal of her time and energy in helping to prepare the manuscript for publication.

Russell C. Hibbeler

Contents

1

General Principles

1–1. Mechanics

In general, *mechanics* is that branch of the physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces. A thorough knowledge of this subject is required for further study of the physical sciences involving structural engineering, machine design, fluid flow, electrical instrumentation, and even the molecular and atomic behavior of elements.

Divisions of Mechanics. Depending upon the nature of the problem being studied, mechanics is generally subdivided into three branches: *rigid-body or classical mechanics, deformable-body mechanics, and fluid mechanics.* In this book only rigid-body mechanics will be studied, since this subject forms a suitable basis for the design and analysis of many engineering problems, and provides the necessary background for a study of the mechanics of deformable bodies and the mechanics of fluids.

Rigid-body mechanics is generally divided into two areas: statics and dynamics. *Statics* deals with the equilibrium of bodies, that is, those which are either at rest or move with a constant velocity; whereas *dynamics* is concerned with the accelerated motion of bodies. Although statics can be considered as a special case of dynamics in which the acceleration is zero, statics deserves separate treatment in engineering education, since most structures are designed with the intention that they remain in equilibrium.

Historical Development. The subject of statics developed very early in history, because the principles involved could be formulated simply from measurements of geometry and force. For example, the writings of Archimedes (287–212 B.C.) provide an explanation of the equilibrium of the lever. Studies of the pulley, inclined plane, and wrench are also recorded in ancient writings—at times when the requirements of engineering were limited primarily to building construction.

Since the principles of dynamics depend upon an accurate measurement of time, this subject developed much later. Galileo Galilei (1564–1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642–1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D'Alembert, Lagrange, and others.

Newton's Three Laws of Motion. The entire structure of rigid-body or classical mechanics is formulated on the basis of Newton's three laws of motion. These laws, which apply to the motion of a particle, may be briefly stated as follows.

First Law. A particle originally at rest, or moving in a straight line at a constant velocity, will continue to remain in this state provided the particle is not subjected to an unbalanced force.

Second Law. A particle acted upon by an unbalanced force **F** receives an acceleration **a** in the direction of the force. The acceleration is directly proportional to the force and inversely proportional to the particle mass m. This law is commonly expressed in mathematical terms as

$$\mathbf{F} = m\mathbf{a} \tag{1-1}$$

Third Law. For every force acting on a particle, the particle exerts an equal, opposite, and collinear reactive force.

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion for a particle, Newton postulated a law governing the mutual attraction between any two particles. This law can be expressed mathematically as

$$F = G\frac{m_1 m_2}{r^2} \tag{1-2}$$

where
F = force of attraction between the two particles
G = universal constant of gravitation; according to experimental evidence, $G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$
m_1, m_2 = masses of each of the two particles
r = distance between the centers of both particles

Any two particles or bodies have a mutual attractive (gravitational) force acting between them. In the case of a particle located at or near the surface of the earth, however, the only attractive force having any sizable magnitude is that of the earth's gravitation. Consequently, this force, termed the *weight,* will be the only gravitational force considered.

1–2. Units of Measurement

Four fundamental quantities commonly used in mechanics are length, time, force, and mass. In general, the magnitude of each of these quantities is defined by an arbitrarily chosen *unit* or "standard."

Length. The concept of *length* is needed to locate the position of a point in space and thereby to describe the size of a physical system. The standard unit of length measurement is the *metre* (m), which is represented by 1 650 763.73 wavelengths of light produced from the orange-red line of the spectrum of krypton 86. All other units of length are defined in terms of this standard. For example, 1 foot (ft) is equal to 0.3048 m.

Time. The concept of *time* is conceived by a succession of events. The standard unit used for its measurement is the second (s), which is based on the duration of 9 192 631 770 cycles of vibration of an isotope cesium 133.

Force. In general, *force* is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is either direct contact between bodies, such as a person pushing on a wall, or it can occur through a distance by which the bodies are physically separated. Examples of the latter type include gravitational, electrical, and magnetic forces. In any case, a force is completely characterized by its magnitude, direction, and point of application. Most often, engineers define the standard unit of force using either the newton (N) or the pound (lb). Each of these units can be *measured* with a spring balance to determine the amount of gravitational pull exerted by the earth upon an object. Since this force, defined as the *weight* of a body, *changes* with respect to the distance r from the center of the earth, Eq. 1–2, it is important to make measurements of weight at a specified latitude and height above sea level.

Mass. The *mass* of a body is regarded as a quantitative property of matter used to measure the resistance of matter to a change in velocity. Unlike weight, the mass of a body is *constant* regardless of its location. For this reason, comparison of masses is usually made by means of a lever-arm balance. The standard unit of mass is the *kilogram* (kg), defined by a bar of platinum–iridium alloy kept at the International Bureau of Weights and Measures in Sèvres, France.

Systems of Units. The four fundamental quantities—length, time, force, and mass—are not all independent from one another; instead, they are related by Newton's second law of motion—force is proportional to the product of mass and acceleration; i.e., $\mathbf{F} = m\mathbf{a}$. Hence, the units used to define the magnitude of force, mass, length, and time cannot *all* be selected arbitrarily. The equality $\mathbf{F} = m\mathbf{a}$ is maintained only if three of the four units, called *base units,* are *arbitrarily defined* and the fourth unit is *derived* from the equation.

Absolute System. A system of units defined on the basis of length, time, and mass is referred to as an *absolute system,* since the measurements of all these quantities can be made at *any location.* As shown in Table 1-1, the International System of Units (SI) is absolute, since it specifies length in metres (m), time in seconds (s), and mass in kilograms (kg). The unit of force, called a newton (N), is derived from $\mathbf{F} = m\mathbf{a}$. Thus, 1 newton is equal to a force required to give 1 kilogram of mass an acceleration of 1 m/s^2 ($N = kg \cdot m/s^2$).

Gravitational System. A system of units defined on the basis of length, time, and force is referred to as a *gravitational system.* This is because force is measured in a gravitational field, and hence its magnitude depends upon where the measurement is made. In the FPS system of units, Table 1-1, length is in feet (ft), time is in seconds (s), and force is in pounds (lb). The unit of mass, called a *slug,* is derived from $\mathbf{F} = m\mathbf{a}$. Hence, 1 slug is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1 lb ($slug = lb \cdot s^2/ft$).

Table 1-1 System of Units

Type of System	*Name of System*	*Length*	*Time*	*Mass*	*Force*
Absolute	International System of Units (SI)	metre (m)	second (s)	kilogram (kg)	newton* (N) $\left(\frac{kg \cdot m}{s^2}\right)$
Gravitational	British Gravitational (FPS)	foot (ft)	second (s)	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound (lb)

*Derived unit.

1-3. The International System of Units

The International System of units, abbreviated SI after the French "Système International d'Unités," is a modern version of the metric system which has received worldwide recognition at the 11th International Conference of Weights and Measures in 1960. This absolute system of units is used throughout this book since it is intended to become the worldwide standard for measurement.

Base and Derived Units. Only the seven arbitrarily defined *base units* listed in Table 1-2 exist in the SI system. All other units are *derived* from these. For example, as previously stated, the unit of force, the newton (N), is derived from Newton's law of motion ($N = kg \cdot m/s^2$). Another derived unit used in statics is the pascal (Pa), defined as the pressure caused by a force of 1 newton acting over an area of 1 square metre ($Pa = N/m^2$).

Table 1–2 Primary SI Units

Quantity	*Base Unit*	*SI Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electrical current	ampere	A
Amount of substance	mole	mol
Temperature	kelvin	K
Luminous intensity	candela	cd

Prefixes. Since units often measure quantities that may vary considerably in magnitude, prefixes representing multiples and submultiples often are used to modify units.* In the SI system, prefixes are increments of three digits, such as those shown in Table 1–3. Attaching a prefix to a unit in effect creates a new unit; thus if a multiple or submultiple unit is raised to a power, the power applies to this new unit, not just to the original unit *without* the multiple or submultiple. For example, $(2\ kN)^2 = (2000\ N)^2 = 4(10^6)\ N^2$. Also, $1\ mm^2 = 1\ (mm)^2$ *not* $1\ m(m^2)$. Note that the SI system does not include the multiple deca (10) or the submultiple centi (0.01), which form part of the old metric system. Except for some volume or area measurements, the use of these prefixes is to be avoided in science and engineering.

*The kilogram is the only *base unit* that is defined with a prefix.

Table 1–3 Prefixes

Multiple	*Exponential Form*	*Prefix*	*SI Symbol*
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Rules for Use. The following rules are given for the proper use of the various SI symbols:

1. A symbol is *never* written with a plural "s," since it may be confused with the unit for second (s).
2. Symbols are always written in lowercase letters, with two exceptions: symbols for the two largest prefixes shown in Table 1–3, giga and mega, are capitalized as G and M, respectively; and symbols named after an individual are capitalized, e.g., N and Pa.
3. Quantities defined by several units which are multiples of one another are separated by a *dot* to avoid confusion with prefix notation, as illustrated by $N = kg \cdot m/s^2 = kg \cdot m \cdot s^{-2}$. Also, $m \cdot s$ (metre-second); whereas ms (milli-second).
4. Physical constants or numbers having several digits on either side of the decimal point should be reported with a *space* between every three digits rather than with a comma, e.g., 73 569.213 427. In the case of four digits on either side of the decimal, the spacing is optional, e.g., 8537 or 8 537. Furthermore, always try to use decimals and avoid fractions; that is, write 15.25, *not* $15\frac{1}{4}$.
5. Compound prefixes should not be used, e.g., kμs (kilo-micro-second) should be expressed as ms (milli-second). It is also best to keep numerical values between 0.1 and 1000; otherwise, a suitable prefix should be chosen. For example, a force of 50 000 N is written as 50 kN.
6. With the exception of the base unit, the kilogram, in general avoid the use of a prefix in the denominator of a composite unit. For example, do not write N/mm, but rather kN/m.
7. Although not expressed in multiples of 10, the minute, hour, etc., are retained for practical purposes as multiples of the second. Furthermore, plane angular measurement is made using radians (rad). In this book, degrees will sometimes be used, where $360° = 2\pi$ rad. Fractions of a degree, however, should be expressed in decimal form rather than in minutes, as in 10.4°, not 10°24′.
8. When performing calculations, represent the numbers in terms of their *base or derived units* by converting any prefixes to powers of 10. The final result should then be expressed using a *single prefix*. For example,

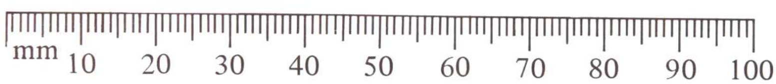
$$(50 \text{ kN})(60 \text{ nm}) = [50(10^3) \text{ N}][60(10^{-9}) \text{ m}]$$
$$= 3000(10^{-6}) \text{ N} \cdot \text{m} = 3 \text{ mN} \cdot \text{m}.$$

Weight. Two terms often confused in the SI system are weight and mass. Specifically, *weight* is the force due to gravity acting on the mass of a body. When the body is allowed to fall freely, the weight acts as an unbalanced force which gives the body an acceleration. Hence, if a body has a mass of m kg and is located at a point where the acceleration of the body due to gravity is $a = g$ m/s², then, since $F = ma$, the weight W measured in newtons ($N = kg \cdot m/s^2$) is

$$W = mg \tag{1–3}$$

In particular, when a freely falling body is located at sea level and at a latitude of 45° (considered the standard location), the acceleration due to gravity is $g = 9.806\ 65$ m/s². For calculations, the value $g = 9.81$ m/s² will be used; so that from Eq. 1–3, 1 kg mass exerts a force or has a weight of 9.81 N, 2 kg weighs 19.62 N, and so on.

When learning to use SI units, it is generally agreed that one should *not* think in terms of conversion factors between systems. Instead, it is better to think *only* in terms of SI units. A "feeling" for these units can only be gained through experience. Study, for example, the geometry and loads acting on the structures and machines illustrated in the problems throughout this book. As a memory aid, it might be helpful to recall that a standard flashlight battery or a small apple weighs about 1 newton. Your body is a suitable reference for small distances. For example, the millimetre scale in Fig. 1–1 can be used to measure, say, the width of three or four fingers pressed together, about 50 mm, or the width of the small fingernail, about 10 mm. For most people, a stretched walking pace is about 1 metre long.



Millimetre scale
Fig. 1–1

1–4. Dimensional Quantities

Dimensional Homogeneity. The terms of any equation used to describe a physical process must be *dimensionally homogeneous;* that is, each term must be expressed in the same units. Provided that this is the case, all the terms of an equation can then be combined if numerical values are substituted for the variables. Consider, for example, the equation $s = vt + \frac{1}{2}at^2$, where in SI units, s is the position in metres (m), t is time in seconds (s), v is velocity in m/s, and a is acceleration in m/s². Regardless of how this equation is evaluated, it maintains its dimensional homogeneity. In the form stated, each term can be expressed in metres [m, $(\text{m}/\cancel{\text{s}})\cancel{\text{s}}$, $(\text{m}/\cancel{\text{s}^2})\cancel{\text{s}^2}$], or solving for a, $a = 2s/t^2 - 2v/t$, the terms are each expressed in units of m/s² [m/s², m/s², (m/s)1/s].

Since mechanics problems involve the solution of dimensionally homogeneous equations, the fact that all terms of an equation are represented by a consistent set of units can be used as a partial check for algebraic manipulations of an equation.