ELEMENTS OF VIBRATION ANALYSIS

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To my wife and
to the memory of my parents

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PREFACE

The most important single factor affecting recent trends in the field of vibrations has been the electronic computer revolution. Indeed, the ability to perform routine computations with incredible speed has been behind the development of relatively new methods of analysis, such as the finite element method, as well as of new computational algorithms for efficient handling of matrices of high dimensions. On the other hand, it has rendered obsolete many methods, including certain graphical procedures.

The ability to solve increasingly sophisticated problems has led to new demands for rigor in analysis. This text addresses itself to this task by adopting an approach that is as mathematically rigorous as possible, while attempting to provide a large degree of physical insight into the behavior of systems. The text covers a broad spectrum of subjects, including matrix methods for discrete systems, various discretization procedures for continuous systems, rigorous qualitative and quantitative treatment of nonlinear oscillations, and statistical analysis of random vibrations. Notwithstanding this breadth, an attempt has been made not to sacrifice depth. Special emphasis has been placed on formulations and methods of solution suitable for automatic computation.

In recognition of the trend toward adopting the metric system in the United States, this text uses both the English and the standard international (SI) systems of units wherever appropriate.

This book is intended as an up-to-date text for a one-year course in vibrations beginning at the junior or senior level. A certain amount of more advanced material has also been included, making the book suitable for a senior elective or a beginning graduate course on dynamics of structures, nonlinear oscillations, and random vibrations. The position of courses in vibrations in engineering curriculums has never been defined clearly. Whereas some curriculums require an elementary course at the junior level and offer an elective course at the senior level, others offer the first course on vibrations as a dual-level course (open to seniors and graduate students), and still others regard vibrations as strictly graduate material. In recognition of this, the text has been designed to cover material from the very elementary to the more advanced in increasing order of difficulty. Moreover, relatively advanced material has been placed at the end of certain chapters and can be omitted on a first reading.

To help the instructor in tailoring the material to his needs, the book is reviewed briefly. In the process, the various levels of difficulty are pointed out.

Chapter 1 is devoted to the free vibration of single-degree-of-freedom linear systems. This is standard material for a first course in vibrations.

Chapter 2 discusses the response of single-degree-of-freedom linear systems to external excitation in the form of harmonic, periodic, and nonperiodic forcing functions. The response is obtained by classical and integral transform methods. A large number of applications is presented. If the response by integral transform methods is not to be included in a first course in vibrations, then Sections 2.11 to 2.14 can be omitted.

Chapter 3 is concerned with the vibration of two-degree-of-freedom systems. The material is presented in a way that makes the transition to multi-degree-of-freedom systems relatively easy. The subjects of beat phenomenon and vibration absorbers are discussed. The material is standard for a first course in vibrations.

Chapter 4 presents a matrix approach to the vibration of multi-degree-of-freedom systems, placing heavy emphasis on modal analysis. The methods for obtaining the system response are ideally suited for automatic computation. The material is suitable for a senior level course. Sections 4.8 to 4.13 can be omitted on a first reading.

Chapter 5 is devoted to exact solutions to response problems associated with continuous systems, such as strings, rods, shafts, and bars. Again the emphasis is on modal analysis. The intimate connection between discrete and continuous mathematical models receives special attention. The material is suitable for seniors.

Chapter 6 provides an introduction to analytical dynamics. Its main purpose is to present Lagrange's equations of motion. The material is a prerequisite for later chapters, where efficient ways of deriving the equations of motion are necessary.

Chapter 7 discusses approximate methods for treating the vibration of continua for which exact solutions are not feasible. Discretization methods based on series solutions, such as the Rayleigh-Ritz method, and lumped methods are presented. The material is suitable for seniors.

Chapter 8 is concerned with the finite element method, a relatively new method for structural dynamics. The material is presented in a manner that can be easily understood by seniors. Moreover, the same concepts can be used to apply the method to two- and three-dimensional structures.

Chapter 9 is the first of two chapters on nonlinear systems. It is devoted to such qualitative questions as stability of equilibrium. The emphasis is on geometric description of the motion by means of phase plane techniques. The material is suitable for seniors or first-year graduate students, but Sections 9.6 and 9.7 can be omitted on a first reading.

Chapter 10 uses perturbation techniques to seek quantitative solutions to response problems of nonlinear systems. Several methods are presented, and phenomena typical of nonlinear systems are discussed. The material can be taught in a senior or a first-year graduate course.

Chapter 11 is devoted to random vibrations. Various statistical tools are introduced, with no prior knowledge of statistics assumed. The material in Sections 11.1 to 11.11 can be included in a senior level course. In fact, its only prerequisites are Chapters 1 and 2, as it considers only the response of single-degree-of-freedom linear systems to random excitation. On the other hand, Sections 11.12 to 11.17 consider multi-degree-of-freedom and continuous systems and are recommended only for more advanced students.

Appendix A presents basic concepts involved in Fourier series expansions, Appendix B is devoted to elements of Laplace transformation, and Appendix C presents certain concepts of linear algebra, with emphasis on matrix algebra. The appendixes can be used for acquiring an elementary working knowledge of the subjects, or for review if the material was studied previously.

It is expected that the material in Chapters 1 to 3 and some of that in Chapter 4 will be used for a one-quarter, first level course, whether at the junior or senior level. For a first course lasting one semester, additional material from Chapter 4 and most of Chapter 5 can be included. A second-level course in vibrations has many options. Independent of these options, however, Chapter 6 must be regarded as a prerequisite for further study. The choice among the remaining chapters depends on the nature of the intended course. In particular, Chapters 7 and 8 are suitable for a course whose main emphasis is deterministic structural dynamics. On the other hand, Chapters 9 and 10 can form the core for a course in nonlinear oscillations. Finally, Chapter 11 can be used for a course on random vibrations.

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LEONARD MEIROVITCH

INTRODUCTION

The study of the relation between the motion of physical systems and the forces causing the motion is a subject that has fascinated the human mind since ancient times. For example, philosophers such as Aristotle tried in vain to find the relation; the correct laws of motion eluded him. It was not until Galileo and Newton that the laws of motion were formulated correctly, within certain limitations. These limitations are of no concern unless the velocities of the bodies under consideration approach the speed of light. The study relating the forces to the motion is generally referred to as *dynamics*, and the laws governing the motion are the well-known Newton's laws.

An important part of modern engineering is the analysis and prediction of the dynamic behavior of physical systems. An omnipresent type of dynamic behavior is *vibratory motion*, or simply *vibration*, in which the system oscillates about a certain equilibrium position. This text is concerned with the oscillation of various types of systems, and in particular with the vibration of mechanical systems.

Physical systems are in general very complex and difficult to analyze. More often than not they consist of a large number of components that act as a single entity. To analyze such systems, the various components must first be identified and then have physical properties ascribed to them. These properties, which govern their dynamic behavior, are generally determined by experimental means. When the characteristics of every individual component are known, the analyst is in a position to construct a mathematical model, which represents an idealization of the actual physical system. For the same physical system it is possible to construct a number of mathematical models. The most desirable is the simplest model which still retains the essential features of the actual physical system.

The physical properties, or characteristics, of a system are referred to as *parameters*. Generally real systems are continuous and their parameters distributed. However, in many cases it is possible to simplify the analysis by replacing the distributed characteristics of the system by discrete ones. This is accomplished by a suitable "lumping" of the continuous system. Hence, mathematical models can be divided into two major types: (1) *discrete-parameter systems*, or *lumped systems*, and (2) *distributed-parameter systems*, or *continuous systems*.

The type of mathematical model considered is of fundamental importance in analysis because it dictates the mathematical formulation. Specifically, the behavior of discrete-parameter systems is described by ordinary differential equations, whereas that of distributed-parameter systems is generally governed by partial differential equations. For the most part, discrete systems are considerably simpler to analyze than distributed ones. In this text we shall discuss both discrete and distributed systems.

Although there is an appreciable difference in the treatment of discrete and distributed systems, there is an intimate relation between the two types of mathematical models when the models represent the same general physical system. Hence, the difference is more apparent than real, a fact often overlooked. Throughout this text special emphasis is placed on the intimate relation between discrete and distributed models by pointing out common physical features and parallel mathematical concepts.

Vibrating systems can also be classified according to their behavior. Again the systems can be divided into two major types, namely, *linear* and *nonlinear*. There seems to be at times some confusion as to what separates a nonlinear system from a linear one, although the classification can be made by merely inspecting the system differential equations. Indeed, if the *dependent variables* appear to the first power only, and there are no cross-products thereof, then the system is linear. On the other hand, if there are powers higher than one, or fractional powers, then the system is nonlinear. Note that systems containing terms in which the *independent variables* appear to powers higher than one, or to fractional powers, are merely *systems with variable coefficients* and not necessarily nonlinear systems.

Quite frequently the distinction between linear and nonlinear systems depends on the range of operation, rather than being an inherent property of the system. For example, the restoring torque in a simple pendulum is proportional to $\sin \theta$, where θ denotes the amplitude. For large amplitudes $\sin \theta$ is a nonlinear function of θ, but for small amplitudes $\sin \theta$ can be approximated by θ. Hence, the same pendulum can be classified as a linear system for small amplitudes and as a nonlinear one for large amplitudes. Nonlinear systems require different mathematical techniques, as we shall have the opportunity to discover for ourselves.

At times the approach to the response problem is dictated not by the system itself but by the excitation. Indeed, the excitation produced by an earthquake on a building is random in nature, in the sense that its value at any given instant of time cannot be predicted. Such excitation is said to be *nondeterministic*. Perhaps if all the factors contributing to the excitation were known, the excitation could be regarded as deterministic. However, the complexity involved in handling irregular functions renders the deterministic approach unfeasible, and the excitation and response must be expressed in terms of *statistical averages*. This text is concerned with the response of systems to both deterministic and nondeterministic excitations.

CONTENTS

1

FREE VIBRATION OF SINGLE-DEGREE-OF-FREEDOM LINEAR SYSTEMS

1.1 GENERAL CONSIDERATIONS

As mentioned in the Introduction, vibrating systems can be classified according to two distinct types of mathematical models, namely, discrete and continuous. Discrete models possess a finite number of degrees of freedom, whereas continuous models possess an infinite number of degrees of freedom. The degree of freedom of a system is defined as the number of independent coordinates required to describe its motion completely (see also Sec. 4.2). Of the discrete mathematical models, the simplest one is the single-degree-of-freedom linear system, described by a second-order ordinary differential equation with constant coefficients. Such a model is often used as a very crude approximation for a generally more complex system, so that one may be tempted to regard its importance as being only marginal. This would be a premature judgment, however, because in cases in which a technique known as modal analysis can be employed, the mathematical formulation associated with many linear multi-degree-of-freedom discrete systems and continuous systems can be reduced to sets of *independent* second-order differential equations, each similar to the equation of the single-degree-of-freedom system. Hence, a thorough study of single-degree-of-freedom linear systems is amply justified. Unfortunately, the same technique cannot be used for nonlinear multi-degree-of-freedom discrete and continuous systems. The reason is that the above

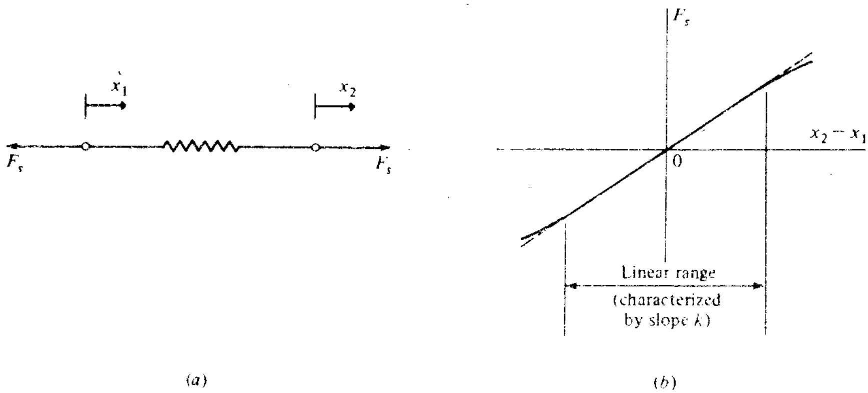

FIGURE 1.1

reduction is based on the principle of superposition, which applies only to linear systems (see Sec. 2.7). Nonlinear systems are treated in Chaps. 9 and 10 of this text and require different methods of analysis than do linear systems.

The primary objective of this text is to study the behavior of systems subjected to given excitations. The behavior of a system is characterized by the motion caused by these excitations and is commonly referred to as the system *response*. The motion is generally described by displacements, and less frequently by velocities or accelerations. The excitations can be in the form of initial displacements and velocities, or in the form of externally applied forces. The response of systems to initial excitations is generally known as *free vibration*, whereas the response to externally applied forces is known as *forced vibration*.

In this chapter we discuss the free vibration of single-degree-of-freedom linear systems, whereas in Chap. 2 we present a relatively extensive treatment of forced vibration. No particular distinction is made in this text between damped and undamped systems, because the latter can be regarded merely as an idealized limiting case of the first. The response of both undamped and damped systems to initial excitations is presented and the energy concept introduced.

1.2 CHARACTERISTICS OF DISCRETE SYSTEM COMPONENTS

The elements constituting a discrete mechanical system are of three types, namely, those relating forces to displacements, velocities, and accelerations, respectively.

The most common example of a component relating forces to displacements is the *spring* shown in Fig. 1.1*a*. Springs are generally assumed to be massless, so