SIMULATION AND ANALYSIS OF GAS NETWORKS

A.J. OSIADACZ

SIMULATION AND ANALYSIS OF GAS NETWORKS

Andrzej J. Osiadacz
PhD DSc
Institute of Petroleum
and Gas Engineering,
Warsaw, Poland

and

The Control Systems Centre,
UMIST, Manchester, UK

GULF PUBLISHING COMPANY
HOUSTON, LONDON, PARIS, TOKYO

First published in 1987 by
E. & F.N. Spon Ltd
11 New Fetter Lane, London EC4P 4EE

This edition published 1987 by
Gulf Publishing Company
PO Box 2608
Houston, TX 77252, USA

Printed in Great Britain

ISBN 0-87201-844-X

Library of Congress Catalog No. 86-82883

PREFACE

In most countries gas transport and distribution systems consist of a large set of highly integrated pipe networks operating over a wide range of pressures. The ever growing demand for gas necessitates the development of gas transport systems and distribution networks and these create further problems associated with the rational operation of the systems to ensure, on the one hand, proper supply of gas to the consumers and, on the other hand, low system operating costs. Proper (optimum with regard to a certain criterion) development of a network, as well as its economically rational exploitation, are only possible if simulation procedures are applied.

Simulation is an experiment carried out on a model; since the model is by definition a faithful equivalent of the tested system and its input and output signals can be identified with the relevant signals of the system, the conclusions drawn from testing the model can be conveyed to the system under consideration. In most practical cases the model inputs play the role of independent variables. By testing the model we want to answer the question: 'how will the system behave in a given situation?', i.e. what will be its output signals if the input signals are known? We may also be interested in knowing the values or variations of input signals of the system that will yield the required values of the output signals. In view of the equivalence of the model and the tested system, the results of the experimental test carried out in this way on the model can be conveyed to the system. This sort of experimentation (simulation study) can, in fact, be treated as experimenting with the system with the use of a model. Simulation treated as an experiment with a model is used to provide an answer to a concrete question posed by an engineer who is analysing a given system – it is therefore a tool and not an end in itself.

Simulation of gas networks makes use of models of gas flow in pipes that have been developed based on the physical laws controlling the processes of flow. However, in view of the complexity of the problem (formulation of the equations, dimension of the problem, complexity of the model) in most cases we cannot do without a computer.

As has already been mentioned, simulation allows us to predict the behaviour of gas network systems under different conditions. Such predictions can then be used to guide decisions regarding the design and operation of the real system. At the stage of designing a network, simulation helps us to select a structure for the network, as well as the geometric parameters of the pipes in the case of given parameters of gas supply and demand. Simulation also facilitates the selection of sites where non-pipe elements should be installed. The control of a gas system also requires simulation in order to obtain information about the pressures and flow rates at given points of the network. Depending on the character of gas flow in the system we distinguish steady and unsteady states. The steady states in gas networks are described by systems of algebraic – in general non-linear – equations. Steady-state simulation is relatively simple to deal with and is far easier to understand; in some cases, however, the dynamics of the flow cannot be neglected without committing a serious error; it is then necessary to use a dynamic model which leads to simulation which is computationally much more complicated.

The aim of this book is to present in ten chapters the problems involved in gas network simulation both in steady and unsteady states.

Chapter 1 gives the fundamentals of fluid mechanics and thermodynamics necessary in order to understand the problems discussed further in the book; while Chapter 2 deals with such non-pipe elements of gas networks as compressors, pressure regulators and valves.

Simulation of a network of arbitrary topology requires that a mapping be found such that with its use calculations could be carried out in a fairly simple way. Such requirements are fulfilled by the graph theory which allows simple representation of the network structure in terms of the properties of incidence of its elements, the uniqueness of this representation being ensured. The relevant problems are discussed in Chapter 3.

Chapter 4 deals with mathematical models used for the simulation of networks in the steady state. The fundamental equation describing steady gas flow is derived based on Bernoulli's equation, and the equations commonly used in practice are presented. The loop and node models are formulated with the help of Kirchhoff's laws. In mathematical terms the steady-state simulation problem of gas networks consists of solving a given system of non-linear equations. The Newton multi-dimensional method is commonly used for this purpose. It requires the solution of a system of linear algebraic equations in every iteration, which, in the case of gas networks, has a sparse coefficient matrix.

The analysis of the Newton method, characteristics of selected methods of solving systems of linear algebraic equations with sparse matrices are given in Chapter 5.

Chapter 6 discusses the methods and algorithms for simulating steady-state gas networks, both simple ones (consisting of adequately connected pipes) and composite ones including non-pipe elements. Examples of computations illustrating the operation of the discussed algorithms are included.

Chapters 7, 8 and 9 deal with problems of simulating unsteady-state networks. In Chapter 7 mathematical models – accurate and simplified – describing unsteady gas flow in the network are discussed. Since the unsteady states in a pipe are described by means of partial differential equations, the necessary information on this subject is also provided.

Chapter 8 gives information on numerical methods of solving partial differential equations. These methods are put to use in Chapter 9 for solving concrete equations describing unsteady gas flow in networks. In this chapter algorithms of network simulation are also discussed and examples of computation are given to illustrate the operation of the algorithms.

Chapter 10 provides fundamental information on the interactive computer program, the Interactive Gas Flow Analysis (IGFA) program, developed at the UMIST, being used as an example.

In 1979 I was a UN research fellow at the University of Manchester Institute of Science and Technology. I studied simulation and control of gas networks in the Control Systems Centre and the Electrical Engineering and Electronics Department. This research motivated me to write this book. I gratefully acknowledge initial contributions by Dr Steven Aylmer and the encouragement of Dr Alfred Brameller at UMIST. My appreciation also goes to colleagues from the Institute of Petroleum and Gas Engineering, Warsaw, especially to Mr Krzysztof Rudowski. Special thanks are due to Mr Tony Fincham from the London Research Station, British Gas Corporation, who has given much advice and many suggestions for improvements. Whether the final effect of my long-lasting efforts is a success will be for the readers to judge.

A.J. Osiadacz
Manchester
April, 1986

CONTENTS

1

SELECTED CONCEPTS OF THERMODYNAMICS AND FLUID MECHANICS

1.1 FUNDAMENTALS OF THE THERMODYNAMICS OF GASES

1.1.1 Terms and definitions

In order to determine the concrete physical conditions under which a gas is considered and thereby to determine unambiguously the state of the gas, convenient parameters of the state of gas are introduced. The most convenient, and therefore the most widely used, parameters characterizing the state are the temperature, pressure and specific volume (or density) of the gas.

Consider the temperature characterizing the thermal state of the gas: let us introduce the following notation.

$$T = 273.15 + t\,°\mathrm{C} \tag{1.1}$$

The dimension of T is temperature and it should be regarded as the temperature read off a scale differing from the Celsius (Centigrade) scale in that its zero lies at a temperature of -273.15 °C. The temperature read off this scale is denoted as K (Kelvin).

Pressure p is defined as the normal component of force F per unit area exerted by a gas on a boundary. The pressure is defined for an area element sufficiently large to allow the gas to be treated as a continuum. Thus

$$p = \lim_{\Delta A \to \Delta A'} \frac{\Delta F_n}{\Delta A}\,[\mathrm{Nm}^{-2}] \tag{1.2}$$

where: $\Delta A'$ is the minimum area for which the gas behaves as a continuum. In general, the continuum behaviour is observed as long as the average distance that a gas molecule travels between collisions is small compared with the boundary dimension; in our case, to a side of the area element $\Delta A'$.

The total pressure exerted on the boundary wall is called the *absolute pressure*. The pressure exerted on a wall by the atmosphere is called *atmospheric pressure*; this pressure varies with latitude and elevation above the earth's surface. Atmospheric pressure is the result of the weight of air at a particular location.

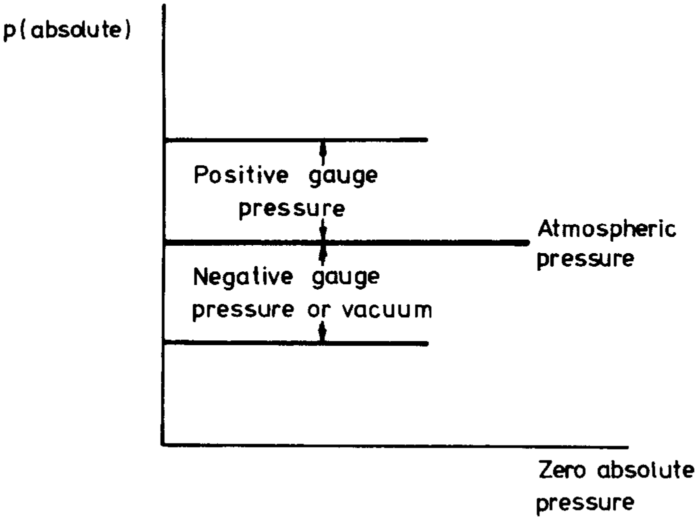

Fig. 1.1 Relationship between pressure terms.

Gauge pressure is the difference between the absolute and atmospheric pressures in a particular system and is usually measured with an instrument which treats atmospheric pressure as a reference.

Vacuum represents the amount by which the atmospheric pressure exceeds the absolute pressure of a system. The various pressures are shown in Fig. 1.1.

By *specific volume* of a gas we understand the volume occupied by its unit mass. The specific volume v is related to the mass G of a gas and its volume V as follows:

$$v=\frac{V}{G}[\mathrm{m^3\,kg^{-1}}] \tag{1.3}$$

Density of gas is defined as the inverse of the specific volume, i.e.

$$\rho=\frac{G}{V}=\frac{1}{v} \tag{1.4}$$

Specific weight denoted by γ is the weight of gas per unit volume. In accordance with Newton's Second Law, the density and specific weight of a gas are related by the equation

$$\gamma=\rho\cdot g=g/v[\mathrm{Nm^{-3}}] \tag{1.5}$$

where g is the acceleration of gravity ($\mathrm{m\,s^{-2}}$).

1.1.2 Gas laws and equations for gases (see Callen, 1960)

(a) Boyle's law

If the temperature of a given quantity of gas is held constant, the volume of gas varies inversely to the absolute pressure. This relationship, written as an equation, is

$$\frac{p_1}{p_2}=\frac{v_2}{v_1}, \text{ or } p_1v_1=p_2v_2, \text{ or } pv=\text{constant} \qquad (1.6)$$

A graphic representation of this relationship is given in Fig. 1.2.

(b) Charles' law

This law consists of two parts:

(i) If the pressure exerted on a particular quantity of gas is held constant, then with any change of state the volume will vary directly as the absolute temperature, which can be expressed by the equation:

$$\frac{v_1}{v_2}=\frac{T_1}{T_2} \text{ or } \frac{T_1}{v_1}=\frac{T_2}{v_2}, \text{ or } \frac{T}{v}=\text{constant} \qquad (1.7)$$

This is represented graphically in Fig. 1.3.

(ii) If the volume of a particular quantity of gas is held constant, then with any

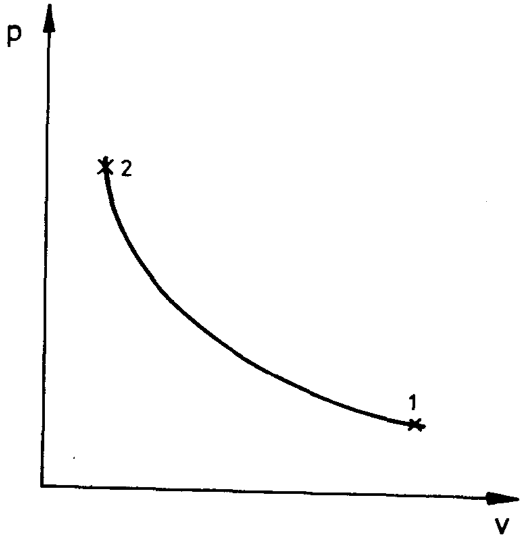

Fig. 1.2 Pressure–volume relationship according to Boyle's law, temperature constant.

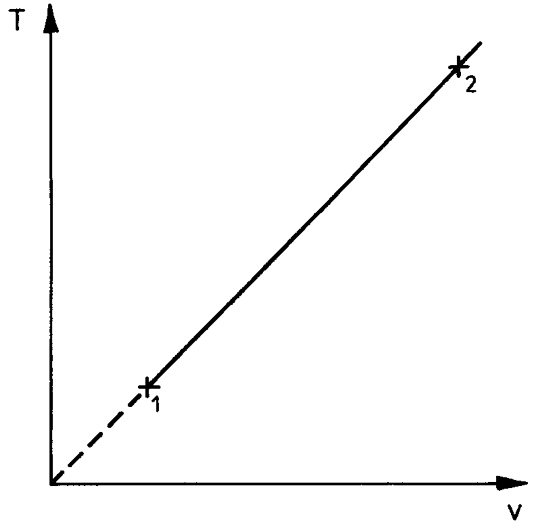

Fig. 1.3 Temperature–volume relationship according to Charles' law, pressure constant.

change of state, the absolute pressure will vary directly as the absolute temperature:

$$\frac{p_1}{p_2}=\frac{T_1}{T_2} \text{ or } \frac{T_1}{p_1}=\frac{T_2}{p_2} \text{ or } \frac{T}{p}=\text{constant} \tag{1.8}$$

Fig. 1.4 shows this relationship graphically.

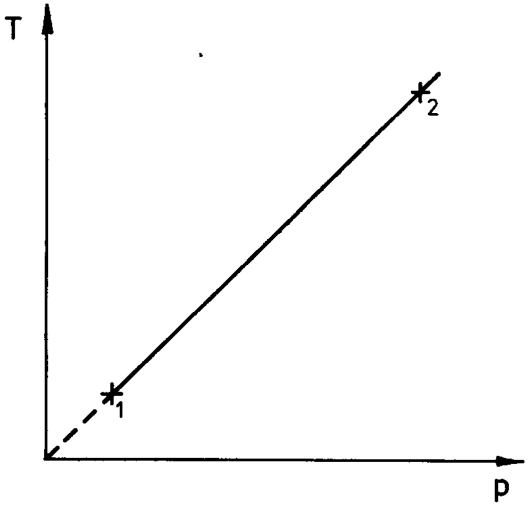

Fig. 1.4 Temperature–pressure relationship according to Charles' law, volume constant.

(c) Boyle's and Charles' Laws

The relations of Boyle's and Charles' laws may be combined to yield

$$\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2},\ \text{or}\ \frac{pv}{T}=\text{constant} \tag{1.9}$$

The constant in equation (1.9) does not depend on the state of the gas. It only depends on the properties of the gas and is specific for each gas. It is known as the gas constant. Denoting the gas constant by $R(\mathrm{J\,kg^{-1}\,K^{-1}})$, we can write equation (1.9) in the form

$$pv=RT \tag{1.10}$$

In this way we have obtained the equation which uniquely relates the parameters p, v, and T of a gas, i.e. the equation of state of an *ideal or perfect* gas.

An ideal gas is a gas whose molecules are considered to be material points, the interaction between them being restricted to collisions. The sense of introducing the concept of the ideal gas is that in practice equation (1.10) represents more or less approximately the behaviour of many gases under conditions close to normal atmospheric temperatures and pressures. Various gas processes can be calculated with sufficient accuracy with the aid of equation (1.10). Let us list the special processes:

(1) the constant temperature or *isothermal* process;
(2) the constant pressure or *isobaric* process;
(3) the constant volume or *isometric* or *isochoric* process;
(4) the zero heat transfer or *adiabatic* process.

A polytropic process is one which may be represented by the relationship

$$pv^n=\text{const}=C \tag{1.11}$$

or, alternatively, in the logarithmic form

$$\ln p=-n\ln v+\ln C \tag{1.12}$$

where n is called the *polytropic exponent.* We may compare equations (1.11) and (1.12) with (1.10) and show that the four special processes mentioned above can be fitted to the polytropic relation if we use the following values of n:

$$\begin{aligned} T&=\text{const} & n&=1.0\\ p&=\text{const} & n&=0\\ v&=\text{const} & n&=\pm\infty\\ q&=0 & n&=\delta=\frac{c_p}{c_v}\end{aligned}$$

where: q is the heat transferred, c_p the heat capacity at constant pressure, c_v the

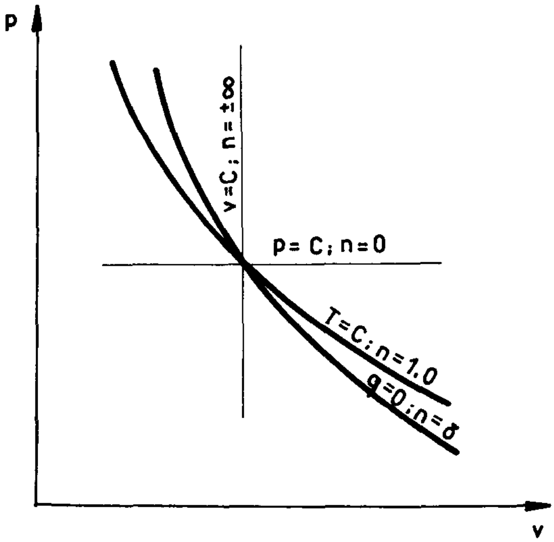

Fig. 1.5 Polytropic process according to $pv^n = \text{const.}$

heat capacity at constant volume and δ is the isentropic exponent.

These processes are plotted in the p-v diagram in Fig. 1.5. Between the two end states 1 and 2 equation (1.11) can be written as

$$p_1 v_1^n = p_2 v_2^n = C \tag{1.13}$$

By manipulating this relationship along with the ideal-gas law (equation (1.10)) we can obtain several alternate expressions for the end states in a polytropic process:

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/n} \tag{1.14}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \tag{1.15}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \tag{1.16}$$

Note that equations (1.15) and (1.16) hold for ideal gases only.

A real gas differs more from an ideal gas the greater its density. From the molecular–kinetic point of view the 'non-ideality' of a gas is due to the fact that the molecules have a certain volume and also to the fact that intermolecular interactions are of an intricate nature.

Since the ideal-gas equation of state is so simple, it is not unnatural to seek a means of modifying it in order to match it with non-ideal-gas behaviour. The technique employed consists in defining a factor Z, called the compressibility factor, such that

$$Z = \frac{pv}{RT} \tag{1.17}$$

Obviously, $Z = 1.0$ for an ideal gas, and the various virial coefficients (see Holman, 1980) simply provide a series of corrections to the ideal-gas behaviour. It is possible to give some generalized information for Z.

Let us define new variables called the reduced pressure, reduced specific volume and reduced temperature:

$$p_r = \frac{p}{p_c}$$

$$v_r = \frac{v}{v_c}$$

$$T_r = \frac{T}{T_c}$$

where p_c is the critical pressure, i.e. pressure which a gas will exert when in equilibrium with the liquid phase and at the critical temperature, T_c is the critical temperature, i.e. temperature above which a gas cannot be liquefied by the application of pressure alone regardless of the amount of pressure, and v_c is the critical specific volume, i.e. the specific volume of a gas at the critical temperature and pressure.

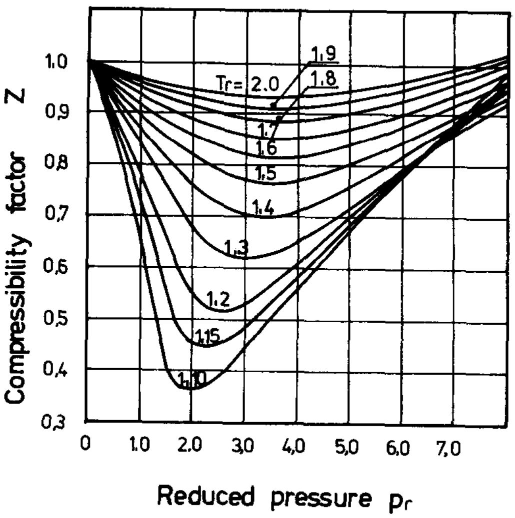

Fig. 1.6 Compressibility relation for methane.

Algebraic manipulation of the van der Waals equation (see Holman, 1980) gives

$$Z^3-\left(\frac{p_r}{8T_r}+1\right)Z^2+\left(\frac{27p_r}{64T_r^2}\right)Z-\frac{27p_r^2}{512T_r^3}=0 \tag{1.18}$$

where Z is the compressibility factor defined in equation (1.17). Obviously, equation (1.18) can be solved for Z explicitly in terms of p_r and T_r and, consequently, we postulate the existence of a relationship of the form

$$Z=f(p_r, T_r) \tag{1.19}$$

Equation (1.19) is called the *law of corresponding states*. Ideal-gas behaviour occurs when:

(i) p_r is small compared to 1.0, or
(ii) T_r is large compared to 1.0.

Fig. 1.6 shows graphically equation (1.19) for methane.

1.2 FUNDAMENTALS OF GAS FLOW IN PIPES

1.2.1 Viscosity

An important property of the gas is its *viscosity* (also referred to as dynamic or absolute viscosity). Viscosity is the gas resistance to flow, which reveals itself as a shearing stress within a flowing gas and between a flowing gas and its container. The viscosity is given the symbol μ and is defined as the ratio of the shearing stress τ to the rate of change in velocity w or, mathematically, $\mathrm{d}w/\mathrm{d}x$. This definition results in the following important equation for gas shear

$$\tau=\mu\frac{\mathrm{d}w}{\mathrm{d}x} \tag{1.20}$$

where: x = the distance.

Equation (1.20) is valid for laminar flow, but not for turbulent flow where much of the apparent shear stress is due to exchange of momentum between adjacent layers of flow. From equation (1.20) it can be found that the dimension of viscosity is $[\mathrm{N\,s\,m^{-2}}]$. Occasionally the viscosity is given in poise, one poise equalling $0.1\,\mathrm{N\,s\,m^{-2}}$. Because of its frequent occurrence, the absolute viscosity divided by the gas density is separately defined and called the kinematic viscosity ν: Thus

$$\nu=\mu/\rho$$

The dimension of ν is $[\mathrm{m^2\,s^{-1}}]$.
The variation of dynamic viscosity of methane with temperature is shown in Fig. 1.7.