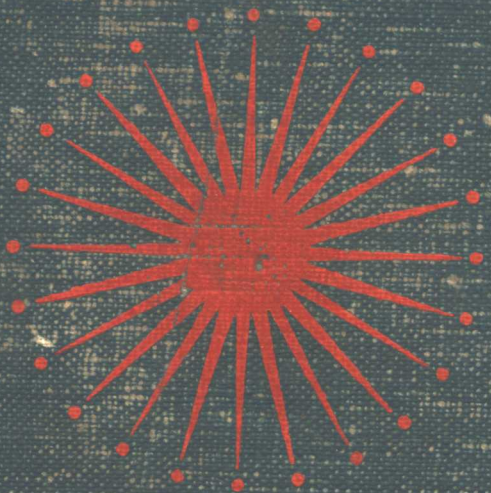
REACTOR ANALYSIS



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McGRAW-HILL BOOK COMPANY, INC.
New York Toronto London
1960

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 Library of Congress Catalog Card Number: 59-15469

THE MAPLE PRESS COMPANY, YORK, PA.

41328

PREFACE

The purpose of the present work is to provide a complete and consistent mathematical development of what is commonly known as reactor analysis, that is, the mathematical study of the nuclear behavior of reactors based on certain approximate physical models. The subject of reactor analysis differs somewhat from that of reactor physics in both viewpoint and content. Whereas reactor analysis deals primarily with the mathematical tools for treating the physical behavior of reactors, reactor physics places much more emphasis on the physical aspects themselves of these systems. The tone of this book is therefore much closer to that of advanced treatments in engineering analysis rather than to that of books on physics.

The formal level of the presentation is directed primarily toward the first- and second-year graduate student in engineering science although it is expected that students in physics will also find it useful. Considerable pains have been taken to provide a textbook which could also be used in a first course on reactor analysis. The introductory sections of each of the principal chapters have been organized and written with this thought in mind. In these sections the treatment is initiated with the aid of elementary mathematical models and emphasis is placed on a discussion of the principal physical concepts to be developed. The more sophisticated mathematical considerations and the development of the broader theory are left in each case to later sections. Thus it is expected that this work will serve as an elementary text which can also be used in an intermediate course by simply including the complete treatment.

The material presented in this book was developed from a course organized and presented by the authors over a period of five years at the Oak Ridge School of Reactor Technology at the Oak Ridge National Laboratory. The development of the subject matter in the ORSORT course constitutes one-third of this book.

It is presumed that the reader who desires a complete understanding of the contents of this book has had at least a course in advanced calculus and preferably a general course also in partial differential equations and boundary-value problems, or a first course in the methods of mathematical physics. It is also assumed that he is acquainted with the fundamental concepts involved in modern physics and has been introduced to

the use of analytical methods in the solution of engineering problems. For those who desire only an introductory knowledge of the subject and therefore limit their study to the elementary sections, the usual undergraduate course in differential equations will suffice.

In this treatment the subject matter of reactor analysis has been developed with the aid of various mathematical models. The models selected for this purpose are those which have proved to be useful in describing the various neutron phenomena peculiar to nuclear reactors. Emphasis is placed upon detailed presentations of each method in order that the reader becomes sufficiently well equipped to treat new and different situations. In nearly every instance the mathematical treatment has been extended to include the derivation of working formulas, and these are usually followed by numerical examples which display the computational techniques which may be used in application. In a few instances only a formal presentation is supplied, and in these cases the intent is merely to exhibit the principal physical ideas involved.

The authors have attempted to present a discussion of all the principal topics of reactor analysis, with an entire chapter devoted to each. In many instances several analytical methods are presented in order to provide as wide a treatment as possible. These include Chap. 2 on probability concepts; Chap. 3 on the neutron flux; Chap. 4 on slowing down; Chap. 5 on diffusion theory; Chap. 6 on the Fermi age model; Chap. 7 on transport theory; Chap. 8 on reflected reactors; Chap. 9 on reactor kinetics; and Chap. 10 on heterogeneity. It is important to mention that the remaining chapters represent in main part extensions and applications of these general topics.

The material in Chaps. 1, 2, and 3 and in the first section or so of Chaps. 4, 5, 6, 8, 9, and 10 constitutes a comprehensive first course in reactor analysis. A complete coverage of this text would constitute a second or intermediate course.

The authors wish to express their appreciation for the assistance and encouragement given by their friends and colleagues. To L. Nelson they are especially indebted for his penetrating criticism and gentle tolerance. To R. R. Coveyou, L. Dresner, R. K. Osborn, and H. Schweinler they are grateful for many suggestions and hours of stimulating discussion; to R. A. Charpie, W. K. Ergen, E. Guth, G. Leibfried, L. W. Nordheim, A. Simon, A. M. Weinberg, and T. A. Welton for reviews and comments; and to H. Honek, D. H. Platus, and D. L. Platus for help with the numerical examples.

Finally, the authors thank Mrs. Yvonne Lovely for her expert assistance in the preparation of the manuscript.

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CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Nuclear Chain Reactions

An understanding of the properties and behavior of nuclear chain reactors is achieved through a study of the neutron population which supports the chain. Information about the neutron population is conveniently expressed in terms of the neutron-density-distribution function.

The detailed features of the chain reaction are determined by the various nuclear processes which can occur between the free neutrons and the materials of the reactor system. As in chemical chain reactions, the rates of the reactions involved in the chain are directly dependent upon the density of the chain carrier, in the present case the neutrons. Thus in order to determine the various properties of a reactor, such as the power-production rate and the radiation-shielding requirements, it is necessary to obtain the fission reaction rate throughout the system and, therefore, the neutron-density distribution. In fact, all the basic nuclear and engineering features of a reactor may be traced back ultimately to a knowledge of these distribution functions.

The subject of reactor analysis is the study of the analytical methods and models used to obtain neutron-density-distribution functions. Since these functions are intimately related to various neutron-induced nuclear reactions, a knowledge of at least the basic concepts of nuclear physics is essential to a thorough understanding of reactor analysis.

The first section of this chapter is a brief discussion of those aspects of nuclear reactions which are of principal interest to reactor physics. This presentation assumes that the reader is equipped with an introductory course in nuclear physics. The second section is an outline of the basic nuclear components of reactors and of the various types of reactors, and the last section is a summary of the principal problems of reactor physics and the analytical methods of attack. It is intended that the last section be used primarily for purposes of review and to aid the reader in orienting the various topics with regard to the over-all structure and scope of the subject.

a. Fission Reaction. In introducing the general subject of chain reactions it will be helpful to begin with a review of some elementary but basic notions about nuclear reactions, in particular the fission reaction.

This information is included here primarily as a convenient reference for explaining the physical features of neutron phenomena relevant to reactors.

The bulk of the neutrons participating in the chain reaction within a reactor possess kinetic energies which range from thermal energy[1] (hundredths of an electron volt[2]) up to fission neutron energies (several million electron volts). Even though this range extends over some eight orders of magnitude, it is nevertheless possible (and convenient) to describe nearly all the important neutron-induced reactions within a reactor by means of a single conceptual model, namely, the compound-nucleus idea of Bohr.[3] This model is especially useful in studying the fission process.

The formation of the compound nucleus constitutes the first step in the reaction between the neutron and a nucleus. It may be represented symbolically by

$$X^A + n^1 \rightarrow \{X^{A+1}\}^* \tag{1.1}$$

where X^A denotes some nucleus of mass number A which has captured (absorbed) a neutron n^1. The symbol $\{\ \}^*$ indicates that the resultant compound nuclear structure X^{A+1} is in an excited state. The excitation energy of this nucleus is the combined kinetic and binding energies of the captured neutron (in the compound nucleus). If the captured neutron had exactly zero velocity relative to the nucleus, then the excitation energy would be precisely the binding energy E_b. This point is easily demonstrated with the aid of the inverse to the complete reaction implied in (1.1). The complete reaction is accomplished when the compound nucleus achieves one of several possible stable states by simply ejecting the excess energy E_γ in the form of electromagnetic radiation (gamma rays); thus,

$$X^A + n^1 \rightarrow \{X^{A+1}\}^* \rightarrow X^{A+1} + E_\gamma \tag{1.2}$$

Now consider the situation in which the nucleus X^{A+1} in the unexcited (ground) state acquires an energy E_b just large enough to separate a neutron. The appropriate reaction would be

$$X^{A+1} + E_b \rightarrow \{X^{A+1}\}^* \rightarrow X^A + n^1 \tag{1.3}$$

This reaction is called the photoelectric liberation of a neutron. Now, by the theorem of detailed balance,[4,5] this reaction is just the

[1] Energy comparable to the thermal motion of the nuclei in the medium supporting the neutron population.

[2] One electron volt (ev) $= 1.6023 \times 10^{-12}$ erg $= 1.6023 \times 10^{-19}$ watt sec.

[3] N. Bohr, *Nature*, **137**, 344 (1936); J. M. Blatt and V. F. Weisskopf, "Theoretical Nuclear Physics," pp. 340–342, John Wiley & Sons, Inc., New York, 1952.

[4] Blatt and Weisskopf, *op. cit.*, pp. 601–602.

[5] E. Fermi, "Nuclear Physics," rev. ed., pp. 145–146, course notes compiled by Jay Orear, A. H. Rosenfeld, and R. A. Schluter, University of Chicago Press, Chicago, 1950.

inverse of (1.2); a comparison of Eqs. (1.2) and (1.3) therefore reveals that

$$E_\gamma = E_b \tag{1.4}$$

The binding-energy concept is essential to an understanding of the nuclear-fission process. A short summary of the idea is therefore in order. By definition, the *total binding energy* E_b^{tot} of a nucleus is given by the difference between the mass of the nucleus and the sum of the masses of its constituent nucleons (protons and neutrons). If M is the nuclear mass, A the mass number (number of nucleons), N_n the number of neutrons, m_n and m_p the masses[1] of the neutron and proton, respectively, and c the velocity of light, then

$$\begin{aligned} E_b^{\text{tot}} &\equiv c^2[m_n N_n + m_p(A - N_n) - M] = c^2[N_n(m_n - m_p) + m_p A - M] \\ &\simeq c^2(Am_n - M) \end{aligned} \tag{1.5}$$

where the approximation follows from the fact that $m_n \simeq m_p$. In order to obtain the *average binding energy per nucleon* $\bar{E}_b$, we divide through by A:

$$\bar{E}_b \equiv \frac{E_b^{\text{tot}}}{A} \simeq c^2\left(m_n - \frac{M}{A}\right) \tag{1.6}$$

This is the average energy required per nucleon to separate the nucleus into its constituent particles. The value of $\bar{E}_b$ varies from about 1 to 9 Mev over the entire mass scale.[2] In the mass range of interest to reactor physics ($A \gtrsim 70$), $\bar{E}_b$ decreases monotonically from 8.6 Mev at $A = 70$ to 7.5 Mev at $A = 238$. It is this variation which determines two fundamental features of the fission reaction. It is shown later that this indicates (1) that there is a positive energy release if a nucleus with $A > 85$ is caused to disintegrate and (2) that nuclei in this range are theoretically unstable with regard to the fission process.

Consider first the question of the energy released when a nucleus disintegrates. As an example, let us take the case of the neutron-induced fission of the nucleus X^A which results in the formation of two fragments Y^{A_1} and Z^{A_2} of masses M_1 and M_2, respectively. On the basis of the compound-nucleus concept, this reaction may be written

$$n^1 + X^A \rightarrow \{X^{A+1}\}^* \rightarrow Y^{A_1} + Z^{A_2} \tag{1.7}$$

In general, the binding energies of the fragments will differ from the binding energy of the original nucleus; that is, the combined masses of the fragments will not equal the mass of the fissioned nucleus X^{A+1}. The difference appears as an energy release E_f which may be determined

[1] These are: $m_n = 1.00893$ amu (atomic mass units) and $m_p = 1.00758$ amu, where 1 amu $= 1.66 \times 10^{-24}$ g.

[2] See, for example, C. F. Bonilla, "Nuclear Engineering," Fig. 3-5, p. 63, McGraw-Hill Book Company, Inc., New York, 1957.

from the Einstein mass-energy relation. In the present case

$$E_f \equiv c^2[M_c - (M_1 + M_2)] \tag{1.8}$$

where the subscript c refers to the compound nucleus. This expression may be written in terms of the various binding energies if we use the relation (1.6); thus, in general, for nucleus i

$$M_i \simeq A_i\left[m_n - \frac{\bar{E}_b^{(i)}}{c^2}\right] \tag{1.9}$$

The substitution of this equation into (1.8) yields

$$E_f \simeq (A + 1)[\bar{E}_b^{(2)} - \bar{E}_b^{(c)}] + A_1[\bar{E}_b^{(1)} - \bar{E}_b^{(2)}] \tag{1.10}$$

where we have used $A_c \equiv A + 1 = A_1 + A_2$. The variation of this function with mass number has the general shape shown in Fig. 1.1.

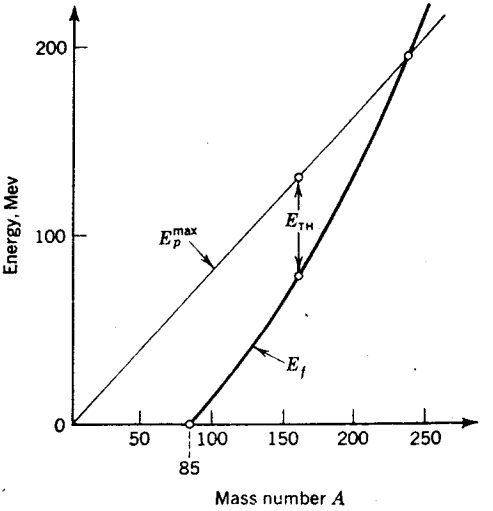

FIG. 1.1 Fission energy and electrostatic repulsion energy of a nucleus as a function of the mass number.

Precisely at what value of A, E_f becomes positive depends, of course, upon the mode of disintegration. In the case of symmetric division ($A_1 \equiv A_2$), the critical value of A is about 85. For mass numbers substantially greater than 85, there are many modes of disintegration which result in a positive energy release.

The principal observation to be drawn from the fission-energy relation (1.10) is that, in the case of the heavier nuclei, the fission fragments represent a lower energy state than the original nucleus. This would imply that the heavier nuclei are inherently unstable toward fission and could conceivably undergo spontaneous disintegration. Experience shows, however, that spontaneous fission does not occur at anything

like the rate one might be led to expect on the basis of the fission energetics alone. The explanation rests with the fact that a nucleus cannot disintegrate until it has first acquired a certain activation or threshold energy. Thus there exists an energy barrier between the state of the whole nucleus and the fragmented state. If one were to take the potential energy of the fragments at infinite separation to be zero, then the merged state of the fragments (forming the original nucleus) would have energy E_f. The variation of the potential-energy curve between these limits would exhibit a maximum, as shown in Fig. 1.2. If the maximum value of the potential energy is $E_p^{\max}$, then $E_p^{\max} - E_f \equiv E_{TH}$ would be the threshold energy for fission.

A nucleus can acquire the necessary excitation energy to overcome this barrier by absorbing either a nuclear particle or electromagnetic

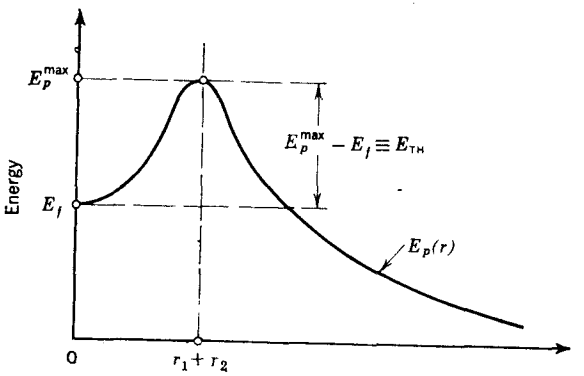

FIG. 1.2 Potential energy of fission fragments as a function of separation.

radiation. Of the two possibilities, the former leads more easily to fission since the absorption of a particle makes available not only its kinetic energy but its binding energy as well. This point was previously noted in connection with the neutron-capture reaction. If the energy acquired in this way is sufficiently large, the nuclear structure experiences increasingly violent oscillations which eventually rupture it, forming the various fragments.

An estimate of the fission threshold can be obtained from the energy required to distort the nucleus into an extreme shape which results in complete separation into fragments. It has been shown[1,2] that this calculation can be based on the *liquid-drop model* of the nucleus. The two principal contributions to the distortion energy of the nucleus are the "surface-tension" effect from the nuclear forces between the constituent

[1] N. Bohr and J. A. Wheeler, *Phys. Rev.*, **56**, 426 (1939); J. Frenkel, *J. Phys. (U.S.S.R.)*, **1**, 125 (1939).

[2] An elementary treatment is given by D. Halliday, "Introductory Nuclear Physics," pp. 417–421, John Wiley & Sons, Inc., New York, 1950.

nucleons and the electrostatic repulsion due to the charge on the protons. When the nucleus is set into oscillation, any departure from its original shape results in an increase in its potential energy because of the "surface tension." Such distortions tend, however, to separate the proton population and thereby create centers of electrostatic repulsion; these forces decrease the potential energy of the system and increase the distortions further. If the distortional oscillations lead to a "dumbbell-like" nuclear configuration, the electrostatic repulsive forces may eventually overcome the nuclear attractive forces and the nucleus will divide.

When the separation results in two major fragments, the threshold energy for fission is given by the potential energy of the fragments at the instant of separation. If one assumes that separation yields two spherical collections of nucleons with Z_i protons and of radius r_i, then the maximum value of the potential energy $E_p^{\max}$ due to the electrostatic field is proportional to $Z_1Z_2(r_1 + r_2)^{-1}$. When the separation r exceeds $r_1 + r_2$, the coulomb forces predominate, and when $r < r_1 + r_2$, nuclear forces predominate. A representative curve of the potential energy E_p of this system is shown in Fig. 1.2.

The variation of the potential energy $E_p^{\max}$ with mass number has the general shape shown in Fig. 1.1. It is seen that for all $A \lesssim 250$ the threshold energy $E_p^{\max} - E_f > 0$; energy must be added in order to produce fission in all nuclei. The figure reveals, however, that the threshold energy becomes progressively smaller with increasing mass number. As the mass number approaches 250, there is a rapid increase in the probability for spontaneous fission through the mechanism of barrier penetration.[1] When $A \gtrsim 250$, the probability is essentially 1, and the nucleons do not remain together long enough to be described as a nuclear structure. It is not surprising, therefore, that nuclei with such large A do not exist in nature.

b. Nuclear Fuels. Nuclei with mass numbers in the range $230 < A < 240$ have fission thresholds of some several Mev. Thus in these cases fission can be brought about by the absorption of radiation or neutrons of only a few Mev kinetic energy. There are a few nuclei that can even be caused to fission by thermal (very slow) neutrons. The specific energy requirements for a particular nucleus depend strongly upon the excitation energy which the captured particle can impart to it. It was shown previously that, in the case of the neutron-induced reaction, the binding energy of the neutron represented a large part (if not all) of this excitation energy. However, even though the mass numbers of the more easily fissionable nuclei differ but little, the binding energies vary by as much as 50 per cent; hence, the variation in the fissionability of the various nuclei. This relatively large variation is due primarily to the influence of the even-odd term in the nuclear-mass formula. If $M(A,Z)$ is the mass of a

[1] Blatt and Weisskopf, *op. cit.*, p. 567.

nucleus containing A nucleons of which Z are protons, then, in atomic mass units,[1]

$$M(A,Z) = 0.99395A - 0.00084Z + 0.0141A^{\frac{2}{3}} + 0.000627\frac{Z^2}{A^{\frac{1}{3}}} + 0.083\frac{(A/2 - Z)^2}{A} + \delta \quad (1.11)$$

The term of interest to the present discussion is the quantity δ, the so-called even-odd term. It is defined

$$\begin{aligned} \delta &\equiv 0 && \text{when } A \text{ is odd} \\ &\equiv -\frac{0.036}{A^{\frac{1}{3}}} && \text{when } N_n \text{ is even, } Z \text{ is even} \\ &\equiv +\frac{0.036}{A^{\frac{1}{3}}} && \text{when } N_n \text{ is odd, } Z \text{ is odd} \end{aligned} \quad (1.12)$$

These relations indicate the dependence of the mass of a nucleus, and therefore its binding energy, upon the number and type of nucleons it contains. An accurate computation of the binding energy of a neutron E_b in a compound nucleus is obtained from the equation

$$E_b = c^2(M + m_n - M_c) \quad (1.13)$$

which may be compared to the approximation (1.6).

Equations (1.11) through (1.13) reveal that neutron absorptions which result in compound structures with even numbers of protons and neutrons acquire the largest excitation energies since the δ term is negative for these nuclei. Compound nuclei with an odd number of nucleons acquire the next largest excitation energies, and odd-odd nuclei, the least. It is on this basis that the isotopes U^{233}, U^{235}, and Pu^{239} can be made to fission by neutron captures of any energy, whereas Th^{232} and U^{238} will fission only with very fast neutrons. In the case of the first three nuclei, a neutron capture leads to an even-even compound structure, and the excitation energy due to the neutron binding energy alone ($\sim$6.8 Mev) is equal to the fission threshold. Thus these nuclei can fission by thermal (very slow) neutron capture, as well as by captures of fast neutrons. It is this characteristic which makes these nuclei especially important as nuclear fuels. As will be discussed later, these nuclei fission with such relative ease in the thermal-energy range that it is well worth while to provide means to moderate (slow down) fission neutrons to thermal energies so that this characteristic may be fully exploited. In fact, the problem of neutron moderation is a principal consideration in reactor analysis.

In the case of Th^{232} and U^{238}, the compound nucleus has an even-odd collection of nucleons and the binding energy for this state ($\sim$5.3 Mev)

[1] Bonilla, *op. cit.*, pp. 69–72.

is less than the threshold energy (~7.1 Mev). Evidently, neutron-induced fission is not possible with these nuclei when the neutron kinetic energy is less than 1.8 Mev. This limited fissionability makes these isotopes much less attractive as the primary fuel component for a steady chain reaction, although their presence in a chain supported by any of the "thermal fuels" can lead to significant augmentation.

The energy released from the fission reaction of any of the isotopes mentioned above may be computed from the equation for E_f (1.8). This expression, which applies for the two-fragment divisions, will suffice for most situations of interest since a three-fragment (or more complex) division of the compound nucleus has a very low probability. A rough estimate of the fission energies generally available from such nuclei may be obtained from a sample computation for U^{235}. The resulting value will be representative since the various mass numbers differ but little, and the principal dependence on binding energy occurs in the $\bar{E}_b^{(2)} - \bar{E}_b^{(c)}$ term [cf. Eq. (1.10)]. Consider, therefore, the reaction

$$n^1 + U^{235} \rightarrow \{U^{236}\}^* \rightarrow Kr^{94} + Ba^{142} \qquad (1.14)$$

The average binding energy per nucleon $\bar{E}_b$ for a nucleus of mass number 236 is about 7.5 Mev, and for nuclei of mass numbers 94 and 142, 8.6 and 8.3 Mev, respectively. The approximate relation (1.10) yields for E_f:

$$E_f = (236)(8.6 - 7.5) + (142)(8.3 - 8.6) = 217 \text{ Mev}$$

Thus, roughly 200 Mev is released in a typical fission reaction involving a heavy nucleus.

The energy from fission appears principally as the kinetic energy of the fission fragments. As these fragments speed outward from the point of reaction, they encounter the various nuclei of the surrounding environment. Such encounters are relatively frequent since the fragments are usually highly ionized and therefore experience strong coulomb interactions with the electron clouds of these nuclei. These interactions are primarily scattering collisions, and each collision results in the transfer of some of the kinetic energy of the fragment to the struck nucleus. A series of such collisions eventually slow the fragments to thermal equilibrium with the environment. Approximately 85 per cent of the fission energy is liberated in this way and must be removed by a suitable cooling system. The remaining 15 per cent appears either as radiation or as the kinetic energy of neutrinos[1] and neutrons evaporated from the fragments or released at the instant of fission. A detailed breakdown of the energy distribution for U^{235} is given in Table 1.1.

The reaction of Eq. (1.14) gives one mode of division of the U^{235} nucleus as a consequence of fission. Of course, this is not the only one

[1] Halliday, *op. cit.*, p. 94.

possible and, in fact, any division consistent with the conservation of mass and energy can occur. The fragments which appear cover the entire mass scale; however, certain divisions are favored, depending on the target nucleus and the energy of the captured neutron. The mass distribution of the various fragments resulting from the fission of U^{235}, U^{238}, and Pu^{239} is shown in Fig. 1.3. For a given nucleus, the shape of the yield curve is highly sensitive to the energy of the incident neutron. The figure shows clearly the characteristic grouping of the fission fragments into two distinct regions of the mass scale for the case of a thermal neutron-induced fission. It is seen that such reactions most frequently produce two fragments, one of mass number around 95 and the other of mass number 140. Although other subdivisions can occur, their likelihood is very improbable. Of these alternative schemes, the division of the compound nucleus into two fragments of equal mass occurs in about 0.01 per cent of the time.

TABLE 1.1 DISTRIBUTION OF FISSION ENERGY FOR U^{235}

Form	*Energy, Mev*
Kinetic energy of fission fragments	165 ± 15
Prompt gamma rays	5
Kinetic energy of fission neutrons	5
Fission product decay:	
Gamma	6
Beta	5
Neutrinos	11
Total energy per fission	197 ± 15

The fragments formed in fission are generally very rich in neutrons. This is a consequence of the fact that among the stable nuclei[1] the ratio N_n/Z increases with Z. Thus the fissionable nuclei with $Z \sim 90$ will possess far more neutrons than are required in the nuclear structures of the stable fragments.[2] Nearly all these excess neutrons[3] are released at the instant of fission. These are the *prompt* neutrons. The remaining small fraction of neutrons to be released are evaporated off from the fragments at various time intervals after fission; these constitute the *delayed* neutron groups. In the event that the emission of a neutron or two by a primary fragment does not leave the nucleus in a stable configuration, further nuclear readjustment may then occur through the emission of β radiation (electrons).

For the purpose of studying the neutron economy in a reactor, it will be convenient to rewrite Eq. (1.7) so as to include the neutrons which are emitted by the primary fragments. For the present, we can omit

[1] Bonilla, *op. cit.*, Fig. 3-6, p. 68.
[2] Halliday, *op. cit.*, pp. 408–412.
[3] 99.245 per cent in the case of U^{235}.