MULTIVARIABLE FEEDBACK DESIGN

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Preface

Useful techniques for the design of multivariable feedback systems have been known for at least fifteen years, yet these techniques have remained known to only a relatively small part of the community of control engineers. This book has been written in order to spread familiarity with the techniques more widely.

My objective has been to enable feedback engineers to design real systems, and the choice of material has been constantly, and I hope consistently, guided by that objective. The reader will therefore find that a complete theoretical understanding of the techniques discussed will sometimes require the consultation of other text-books or research journals, but I believe that I have given enough detail of techniques and algorithms to allow complete analyses and designs to be executed. However, this is a genuine text-book and not just a cook-book, so most of the theory required to understand each technique has been included. There is also enough theoretical development to allow the reader to go on to read the research literature relatively easily.

The basic view espoused is that sensible design of feedback systems is possible only if a frequency-domain point of view is adopted. Between about 1965 and 1980, the advantages of frequency-domain approaches were championed by Professors Rosenbrock and MacFarlane in the UK, and by Professor Horowitz in Israel, at a time when most of the academic community – particularly in the USA – regarded the frequency domain as obsolete and inherently unsuitable for the solution of multivariable problems. The instinct of those who kept faith with the frequency domain has been spectacularly vindicated during the past ten years. Not only have the rather *ad hoc* techniques of the 'British school' proved to be useful in practical design

(Chapter 4), but also the central bastion of time-domain approaches, namely 'LQG' optimal control theory, has been turned into an easily usable technique which yields sensible designs, by giving it a frequency-domain interpretation (Chapter 5). And the very latest technique, H_∞ optimal control theory, has arisen entirely as a result of frequency-domain thinking (Chapter 6).

Despite this shift towards the frequency domain, most of the 'time-domain' content of linear systems theory – state feedback, observers, minimal realizations, LQG controllers and so on – remains essential for multivariable design. Although it is necessary to think and analyse in frequency-domain terms, most computational algorithms, and many proofs of theorems, are based on state-space methods (Chapters 6 and 8).

I have omitted some material, such as multivariable root-loci and pole-placement methods, because it does not fit easily into my view of multivariable feedback design, apparently being more concerned with the shaping of transient responses than with obtaining the potential benefits of feedback. On the whole I have made such omissions without misgivings, but there are two topics which do not appear in the book and which I feel a little apprehensive about leaving out. One is the 'graph topology' introduced by Vidyasagar, which may well turn out to play a fundamental role in feedback theory; the other is the idea of 'internal-model control' introduced by Morari, which has had some impact on the process-control industries, and which is closely related to the Youla parametrization introduced in Chapter 6. But anyone wishing to learn about these topics should have no difficulty in reading the relevant literature, after reading this book, and internal-model control appears in one of the exercises.

I have also omitted material on discrete-time systems, because of lack of space. Almost everything in Chapters 1–4 and 7, and much of Chapter 8, holds for discrete-time systems defined by z-transform transfer functions or by state-space models, with some obvious modifications. The material on LQG design, in Chapter 5, can be given a nearly parallel development for discrete-time systems, but the details are considerably different. In principle, all the material on H_∞ design in Chapter 6 is applicable to discrete-time systems, since the bilinear transformation $z=1+s/1-s$ can be used to transform discrete-time problems into continuous-time ones. But an explicit development of H_∞ theory for discrete-time systems is not yet available.

Some feedback and control specialists hold the view that the teaching of feedback design should be radically revised in the light of recent advances in feedback theory. They would begin, for example, with the Youla parametrization of all stabilizing feedback controllers, which is indeed a logical starting point if the subject is viewed as a branch of mathematics. In contrast with this view, I have adopted a more conservative sequence of presentation, and have begun with a review of the 'classical' techniques developed for the analysis and design of single-input, single-output feedback loops. There is a pedagogical advantage in this since the field of multivariable feedback systems can be entered painlessly by extending the 'classical' ideas. But there is a deeper reason for adopting this approach: the 'classical' treatment gives appropriate emphasis to questions which are of real significance to engineering design, but which may not appear important in a purely mathematical account of the subject. For example, there are several results in linear system theory which rely on the non-existence of plant zeros in the right half-plane (of the complex plane). An acquaintance with the classical theory makes one suspect immediately that the *location* and not the *existence* of such zeros should be significant, particularly as the non-existence of such zeros cannot be confirmed by any experimental means.

In order to facilitate comparisons between alternative design methods, each major technique presented in this book is illustrated by being applied to the *same system* – a linearized model of an aircraft, defined in the Appendix – and the design specification is similar in each case.

Readership

This book is aimed at graduate students who have taken at least one elementary course on feedback and control systems, and who have some acquaintance with linear systems theory. It is possible, although difficult, to take a course in linear systems theory at the same time as a course based on this book. A more detailed discussion of prerequisites is given in Chapter 1. The book should also be accessible to practising feedback engineers who are familiar with classical servo design and have had some exposure to 'modern' (that is, post-1960) control theory.

Most of the material has been classroom-tested in a number of graduate courses given at Imperial College, Cambridge University and, especially, the University of California at Santa Barbara, where in 1986 I gave a 40-hour course covering much of Chapters 1 to 5 and 7. There are a number of difficulties associated with teaching this material. The first is that it is essential for both instructor and students to have access to suitable software which can be run interactively. All the examples and exercises in the book have been solved using PC-Matlab (or Pro-Matlab), together with its associated *Control System Toolbox* and *Multivariable Frequency Domain Toolbox* (and, if I were solving them again, I would use the *Robust Control Toolbox* for Chapters 5 and 6). The second difficulty to be aware of is that the design exercises, such as Exercises 4.1 and 4.2, take much longer to solve than traditional homework exercises, particularly when students are familiarizing themselves with new software or if there is a shortage of computer resources. A student cannot be expected to solve more than one, or perhaps at most two, such exercises per week (assuming that the course is given at a rate of 4 hours per week, say, and that the students are taking a typical mix of courses). It is also beneficial for students to solve these exercises working in pairs. The temptation to set lots of traditional paper-and-pencil problems, and avoid the design exercises, should be resisted, however. The third difficulty is that of examining the students on the material. It seems entirely unsatisfactory to emphasize design during the course, and set computer-based design exercises for homework, but then examine the students on points of theory and grossly simplified examples in order to comply with the requirements of the traditional three-hour examination paper. The solution I have successfully adopted, both at Cambridge and at Santa Barbara, is to set the class a realistic design exercise (possibly varying the details slightly for each student) and ask them to hand in solutions by some fixed time (typically 48 hours later).

Acknowledgements

I should like to acknowledge the help of various people with the writing of this book. In writing Sections 2.2 to 2.5 I was strongly influenced by some unpublished notes written at Cambridge by A.G.J. MacFarlane, Y.-S. Hung, D.J.N. Limebeer and M.C. Smith, and my overall view of the subject has been influenced by lecture notes made available to me by M.G. Safonov. Discussions with K. Glover have left their mark on Chapter 6, and W.-Y. Ng provided detailed criticisms of Chapter 7. Alistair MacFarlane has provided me with opportunities to develop this material through teaching and research over a number of years, and has constantly provided the encouragement required to complete it. Alan Laub arranged my stay at Santa Barbara, where several sets of half-completed and semi-connected lecture notes were first transmuted into book chapters. The whole manuscript has been typed and revised by Celia Sharpe. Finally, my wife Mara and daughters Kasia and Lucy have tolerated many weekends and evenings devoted to the book rather than to them.

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30 September 1989
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A note on language
For reasons of simplicity, the pronoun 'he' is used to relate to both male and female throughout the book.

Symbols and abbreviations

abs(X)	Matrix with (i, j) element $\lvert x_{ij} \rvert$.
arg z	Argument of the complex number z.
$\arg\max_z (.)$	That value of z which maximizes (.).
BD_δ	The set of block-diagonal perturbations $\mathrm{diag}\{\Delta_1, \ldots, \Delta_2, \ldots, \Delta_n\}$, with $\lVert \Delta_i \rVert_\infty \leqslant \delta$.
$\mathbb{C}^{m \times l}$	The set of complex matrices with m rows and l columns.
CLHP, CRHP	Closed left half-plane, closed right half-plane.
cond(G)	Condition number, $\bar{\sigma}(G)/\underline{\sigma}(G)$.
cond*(G)	$\min_{S,T} \mathrm{cond}(SGT)$, where S and T are diagonal matrices.
dB	Decibels: x dB represents a gain of $10^{x/20}$.
deg	Degree (of polynomial).
det	Determinant (of matrix).
diag$\{x_i\}$	Diagonal matrix with elements $x_1, x_2, \ldots$. If the matrix is not square, then $x_1, x_2, \ldots$ are the elements on the principal diagonal, and all other elements are zero. The x_i may themselves be matrices.
dim A	Dimension of square matrix A.
DNA	Direct Nyquist Array.
$E\{x\}$	Expected (mean) value of stochastic process $\{x\}$.

e_i	The ith standard basis vector $[0 \ldots 0\ 1\ 0 \ldots 0]^T$, with 1 occurring in the ith position.
$G(A, B, C, D)$	Denotes that (A, B, C, D) is a state-space realization of the transfer function (matrix) G.
$G(s)$	A transfer function (matrix), frequently abbreviated to G.
$G^*(s)$	Denotes $G^T(-s)$. (But $G^H(s)$ denotes $G^T(\bar{s})$.)
gcd$\{\cdot\}$	Greatest common divisor of the set $\{\cdot\}$.
H_∞	Set of asymptotically stable transfer functions G, with $\|G\|_\infty < \infty$.
I	Unit matrix of unspecified dimension.
I_n	Unit matrix of dimension n.
Im$\{x\}$	Imaginary part of x.
INA	Inverse Nyquist Array.
j	$\sqrt{-1}$; sometimes an index, as in x_{ij}.
LHS, RHS	Left-hand side, right-hand side (of equation or inequality).
ln	Natural logarithm.
log or $\log_{10}$	Logarithm to base 10.
LQG	Linear Quadratic Gaussian.
LTR	Loop Transfer Recovery
MFD	Matrix-fraction description.
MIMO	Multi-input, multi-output.
ms(G)	Measure of skewness of G.
norm(X)	$\begin{bmatrix} \bar{\sigma}(X_{11}) & \ldots & \bar{\sigma}(X_{1m}) \\ \vdots & & \vdots \\ \bar{\sigma}(X_{m1}) & \ldots & \bar{\sigma}(X_{mm}) \end{bmatrix}$ if $X = \begin{bmatrix} X_{11} & \ldots & X_{1m} \\ \vdots & & \vdots \\ X_{m1} & \ldots & X_{mm} \end{bmatrix}$
OLHP, ORHP	Open left half-plane, open right half-plane.
QFT	Quantitative Feedback Theory.
$R^{m \times l}$	The set of real matrices with m rows and l columns.
Re$\{x\}$	Real part of x.
RFN	Reversed-Frame Normalization.
SISO	Single-input, single-output.
tr(X)	Trace (spur) of matrix X, $\Sigma_i x_{ii}$.
$\Gamma(s)$	Relative gain array, with elements $\gamma_{ij}(s)$.
$\lambda_i(X)$	The ith eigenvalue of X.
$\lambda_{max}(X)$, $\lambda_{min}(X)$	Largest and smallest eigenvalues of X.

$\lambda_{\mathrm{P}}(X)$	Perron–Frobenius eigenvalue of X.
$\mu(G)$	Structured singular value of transfer function (matrix) G.
$\rho(X)$	Spectral radius of X, $\max_i \lvert\lambda_i(X)\rvert$.
$\sigma_i(X)$	The ith singular value of X.
$\sigma(G)$, $\sigma(\omega)$	Principal gain (singular value) of $G(\mathrm{j}\omega)$, the notation depending on whether dependence on the system G, or on the frequency ω, is being emphasized.
$\bar{\sigma}$, $\underline{\sigma}$	Largest and smallest singular values.
$\phi_{xx}(\tau)$	Autocovariance function: $E\{x(t)x^{\mathrm{T}}(t+\tau)\}$.
$\Phi_{xx}(\omega)$	Power spectral density of $\{x\}$, Fourier transform of $\phi_{xx}(\tau)$.
$0_{m,l}$	Zero matrix with m rows and l columns.
$\bar{x}$	Complex conjugate of x.
X^{H}	Transpose of complex conjugate of matrix X, $\bar{X}^{\mathrm{T}}$.
X^{T}	Transpose of matrix X.
$X^{\dagger}$	Pseudo-inverse of matrix X.
$\{x(t)\}$	A stochastic process.
$[X]_{ij}$	The (i, j) element of X, also denoted by x_{ij}.
$\lVert x \rVert$	Euclidean norm of vector, $\{x^{\mathrm{H}}x\}^{1/2}$.
$\lVert X \rVert_1$	1-norm, $\max_j \Sigma_i \lvert x_{ij}\rvert$.
$\lVert X \rVert_{\mathrm{F}}$	Frobenius norm, $\{\Sigma_{i,j}\, x_{ij}^{\mathrm{H}} x_{ij}\}^{1/2} = \{\mathrm{tr}(X^{\mathrm{H}}X)\}^{1/2}$.
$\lVert X \rVert_{\mathrm{S}}$	Spectral or Hilbert norm of matrix, $\bar{\sigma}(X)$.
$\lVert x \rVert_2$	$\{\int_{-\infty}^{\infty} x^{\mathrm{T}}(t)x(t)\,\mathrm{d}t\}^{1/2}$, if $x(t)$ is a (real, vector-valued) signal.
$\lVert G \rVert_2$	$\{(1/2\pi)\int_{-\infty}^{\infty} \mathrm{tr}[G(\mathrm{j}\omega)G^{\mathrm{T}}(-\mathrm{j}\omega)]\,\mathrm{d}\omega\}^{1/2}$, if G is a transfer function (matrix).
$\lVert G \rVert_H$	Hankel norm, if G is a transfer function (matrix).
$\lVert G \rVert_\infty$	$\sup_\omega \bar{\sigma}(G(\mathrm{j}\omega))$, if G is a transfer function (matrix).
$\lVert G \rVert_\mu$	$\sup_\omega \mu(G(\mathrm{j}\omega))$, if G is a transfer function (matrix).
$\in$	'Is an element of'.
$*$	Element-by-element multiplication (Schur or Hadamard product).
$\otimes$	Kronecker or tensor product of matrices.
$\cup$	Union (of sets).
$\cap$	Intersection (of sets).
$\subset$	'Is a subset of'.

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Contents

CHAPTER 1

Single-loop Feedback Design

1.1 Overview and prerequisites

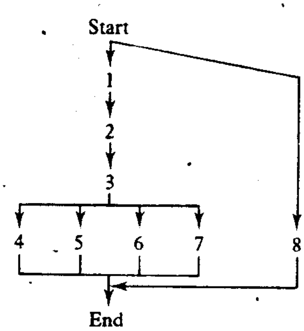
In this book it is assumed that the reader has taken a typical first course in the design of feedback systems. Such a course usually covers the Nyquist stability criterion, the use of Bode and root-locus plots, and the use of simple compensators to achieve reasonable stability margins, steady-state performance and transient response; we shall review the content of such a course very briefly (Section 1.2). We shall go on to discuss single-loop feedback design in more depth than is usual in first courses, and in the process of doing so we shall establish some nomenclature and relationships which will continue to hold when we come to examine multivariable problems in later chapters.

Chapters 2 and 3 extend the concepts and results of Chapter 1 to multivariable systems. Chapter 2 is concerned with establishing stability criteria which generalize the classical Nyquist criterion. In order to do this it is necessary to define poles and zeros of multivariable systems, and to tangle with some linear systems theory; most of the results obtained also find application in later chapters. Chapter 3 deals with the analysis of performance and stability margins of multivariable systems. In addition to straightforward extensions of the results of Chapter 1, Chapter 3 presents a considerable amount of recent material on analysing the robustness of feedback systems in the face of specific disturbances or parameter variations which goes beyond the classical notions of gain and phase margins.

Chapters 4 to 7 are concerned with design techniques for multivariable feedback systems. Chapter 4 deals mostly with direct extensions of classical methods to multivariable systems. These methods have become known as the 'British school' of multivariable design, and provide the simplest and most easily comprehensible design techniques. Chapter 5 describes the use of 'linear quadratic Gaussian' control theory in such a way that sensible feedback designs are obtained. This involves analysing the resulting designs in the frequency-domain terms developed in Chapters 2 and 3. Chapter 6 moves to the very new area of H_∞ optimal control. This approach is of great current interest, and has provided some new fundamental results about feedback systems, as well as a very powerful design technique. Chapter 7 describes some methods of design by parameter optimization; these are relatively 'brute-force' approaches, but sometimes they have to be resorted to when other techniques either fail or are inapplicable, and they can produce excellent designs.

Finally, Chapter 8 discusses the software which is needed to do any analysis or design in the realm of multivariable systems.

The logical interdependence of the chapters can be represented as in the diagram below. Thus the reader interested mainly in practical design techniques may concentrate initially on Chapters 1, 2, 3, 4 and 7, while a research student may prefer Chapters 1, 2, 3 and 6. All readers are urged to look at Chapter 8, even if they skip most of the details of Sections 8.2 and 8.3.



Since the basic representation of a multivariable system which we use is the transfer-function matrix – that is, a matrix whose elements are transfer functions – some familiarity with linear algebra is essential to the reader wishing to progress beyond Chapter 1. At the very least, he should be familiar