ELECTRONS IN GASES

BY

Sir JOHN TOWNSEND, F.R.S.

Hon. Fellow New College, Oxford
Member of the Institute of France (Academy of Science)
Member of the Franklin Institute, Philadelphia

HUTCHINSON'S
SCIENTIFIC AND TECHNICAL PUBLICATIONS
London New York Melbourne Sydney Cape Town

PREFACE

In an earlier work on *Electricity in Gases* I gave a general account of the properties of currents, of ions and electrons in gases, which had been observed under widely different conditions. But at the time that work was published (1915) only a few experiments had been made with currents of electrons moving in a gas under the action of forces much less than those required to produce disruptive discharges between parallel plate electrodes.

The present work contains an account of more complete investigations of the motions of electrons in several gases.

The actions of electrons in collisions with molecules of the gases are deduced from the results of experiments with small currents in uniform fields of force, and the changes in the mean free paths of the electrons, and in the elasticity of molecules, are traced from the stages in which the electrons move under the action of small forces and lose a small proportion of their energy in collisions with molecules, to the stages in which the electrons acquire sufficient energy to ionise molecules and produce disruptive discharges.

I have also examined the objections to the theory of ionisation by the collisions of electrons and positive ions which I gave originally and described in my work on *Electricity in Gases*.

I find that the hypotheses on which these objections are based lead to results which are not in accordance with experimental investigations.

J. S. E. TOWNSEND.

May, 1947.

ELECTRONS IN GASES

CONTENTS

CHAPTER I.

THEORY OF THE MOTION OF ELECTRONS AND IONS IN GASES.

Sections. PAGE

CHAPTER II.

DETERMINATION OF THE ENERGY OF AGITATION AND MOBILITY OF ELECTRONS IN GASES.

Sections.

CHAPTER III.

THE MEAN FREE PATH AND MEAN LOSS OF ENERGY OF ELECTRONS IN ELASTIC COLLISIONS

Sections.

Appendix.

CHAPTER IV.

INEQUALITIES IN THE VELOCITIES OF ELECTRONS.

Sections.

CHAPTER V.

IONISATION BY COLLISION AND OTHER ACTIONS OF ELECTRONS.

Sections.

CHAPTER VI.

IONISATION OF MONATOMIC GASES.

Sections.

CHAPTER VII.

ACTIONS OF POSITIVE IONS IN DISCHARGES.

CHAPTER I.

THEORY OF THE MOTION OF ELECTRONS AND IONS IN GASES.

1. *Steady motion of electrons in a gas.*

The motion of electrons in a gas under the action of an electric force depends on the ratio of the force to the pressure of the gas.

With small forces and large pressures the electrons may become attached to molecules and ions are thus formed which have masses much greater than the mass of an electron, and move with velocities which are small compared with the velocities of free electrons.

The conditions under which ions are formed depend on the gas and on impurities such as water vapour which it may contain, but in all cases the tendency of electrons to become attached to molecules diminishes as the electric force increases and with sufficiently large forces the electrons move freely between the collisions with molecules.

In this state of motion the mean energy of the electrons increases with the force, and there are large ranges of forces and pressures where the electrons move freely between collisions, and the mean energy of the electrons is much greater than that of the molecules of the gas and less than the energy required to ionise the molecules.(1)

The mean loss of energy of an electron in one of these collisions is very small compared with the energy of the electron and the collisions may be said to be elastic and to resemble the collisions between elastic spheres. But this does not imply that the loss of energy is as small as in collisions between perfectly elastic spheres.

The experiments on the lateral diffusion of a stream of electrons show that under the action of a uniform force Z a steady motion is attained where the mean kinetic energy of the electrons is the same at all parts of the field of force.

In this state of motion the mean velocity W in the direction of the electric force is much less than the mean velocity of agitation $\overline{U}$ of the electrons, and both these velocities depend only on the ratio of the force Z to the pressure p of the gas.

The simplest explanation of this motion is that in which the motion of the electrons as found experimentally is compared with the motion of particles of small mass (m) equal to that of the electrons, which collide with elastic spheres of mass (M) equal to that of the molecules as in the kinetic theory of gases. The small particles have velocities of agitation which are very large compared with the mean velocity of agitation of the

---

(1) [*Proc. Roy. Soc.* A, 81, p. 464, 1908.]

spheres, so that the investigation of the motion is simplified since the free paths of the particles are independent of the velocities of the spheres. Also it may be assumed that after a collision with a sphere all directions of motion of a particle are equally probable.

The lengths of the mean free paths of electrons, and the mean losses of energy of electrons in collisions, which have thus been deduced from the results of the experiments, depend on the above specification of the motion of small particles.

The electrons may lose or gain energy in small amounts in the collisions with molecules, but the losses of energy exceed the gains when the mean energy of the electrons is greater than that of the molecules, and the mean effect of a large number of collisions is equivalent to a mean loss of energy in each collision. In the steady motion in a uniform field of force where the mean energy of the electrons is the same at all parts of the field the loss of energy of the electrons in the collisions with molecules is balanced by the gain of energy due to the displacement of the electrons in the direction of the force.

Some of the electrons have energies much greater than the mean energy and others have energies much less, and the theory of the motion involves calculations depending on the distribution of the energies of agitation about the mean. It is therefore necessary to consider the inequalities in the energies and to indicate to what extent the velocities of agitation are distributed about the mean velocity. It is also necessary to consider the inequalities in the free paths of electrons between collisions with molecules.

2. *Distribution of the velocities of agitation.*

As a basis of calculation the distribution of the velocities of agitation of electrons may be compared with the Maxwellian distribution of the velocities of molecules of gases. According to that distribution if N be the total number of small uncharged particles of mass m in unit volume of a gas, and V^2 the mean square of the velocity of agitation, the number dn with velocities greater than U and less than $U+dU$ is given by the following equation

$$dn = (4{\cdot}1\dot{4} \times NU^2/V^3)\,\epsilon^{-3U^2/2V^2}\,dU.$$

The mean velocity $\overline{U}$ is $(1/N)\int u\,dn = 0{\cdot}92V$ and the mean of the reciprocals of the velocities $(\overline{1/U})$ is $(1/N)\int dn/u = 1{\cdot}38/V$; the integrals being taken from $u = 0$ to $u = \infty$.

These expressions for the mean velocity and the mean of the reciprocals in terms of V are independent of the mass m of the small particles and of the number N per cubic centimetre. If M be the mass of the molecules of the gas and G^2 the mean square of the velocities of agitation of the molecules the mean kinetic energy of the particles $mV^2/2$ is equal to that of the molecules $MG^2/2$.

When the number of small particles per cubic centimetre is very small compared with the number of molecules of the gas, the number of collisions of one of the particles with other particles may be neglected in comparison with the number of collisions with molecules of the gas.

Under these conditions the distribution of the velocities of the particles of mass m about the velocity V and the kinetic energy $mV^2/2$ are established by the effects of the collisions of the particles with molecules of the gas.

In the experiments on the determinations of the velocities of electrons in gases, in a uniform field of force, the number of electrons per cubic centimetre of a current is very small compared with the number of molecules, and it appears from the measurements of the velocities that the effects of the collisions of electrons with molecules are in many respects similar to the effects of collisions of small uncharged particles, which tend to establish the Maxwellian distribution of the velocities.

It may therefore be assumed that the distribution of the velocities U of the electrons about the velocity V is nearly the same as in the Maxwellian distribution when the mean energy of agitation of the electrons is not much greater than that of the molecules of the gas. But the distribution changes as the energy of agitation increases.

When the electric force acting on the electrons is increased and the energy of the electrons becomes large compared with that of the molecules of the gas, it is necessary to take into consideration the energies acquired by an electron in moving under the action of the force along the free paths between the collisions with molecules.

It may be shown, by an approximate calculation, that in these cases the velocities tend to be distributed less widely than in the Maxwellian distribution. But it is impossible to obtain a general expression for the distribution of the velocities which would apply to a large range of the energies of agitation of the electrons. The proportion of the energy of the electrons that is lost in collisions and the lengths of the free paths depend on the energy of agitation, and these phenomena are different in different gases.

In general it may be assumed that, if $mV^2/2$ be the mean energy of agitation of the electrons, the ratio of the mean velocity $\overline{U}$ to V and the ratio of the mean of the reciprocals $(\overline{1/U})$ to $1/V$ depend on the mean energy.

For a general comparison of the theory with the results of experiments the distribution of the velocities of the electrons will be taken to be the same as in the Maxwellian distribution.

3. *Distribution of free paths.*

In order to take into consideration the inequalities in the lengths of the free paths of electrons between collisions with molecules of a gas, the distribution of the free paths about the mean free path may be estimated as follows(1)

Let a number n_0 of electrons start from a plane (yz) in the direction normal to the plane, and let n be the number that arrives at a distance x from the plane without any collisions with molecules of the gas.

The number of these that collide with molecules in the space between the two parallel planes at distances x and $x+dx$ from the plane (yz) is proportional to $n\,dx$ and may be taken to be $\theta n\,dx$ where θ is a constant.

The following equation therefore gives the rate at which n diminishes as x increases

$$dn = -\theta n\,dx,$$

and the number n is given by the integral $n = n_0\,\epsilon^{-\theta x}$.

Thus out of a large number n_0 of free paths beginning at any points in the gas, the number of paths greater than x and less than $x+dx$ is $\theta n\,dx$. The sum of the lengths of these paths is $x\,dn$ and the sum of the lengths of all the free paths is $\int_0^\infty x\,dn$,

On integrating by parts this integral becomes

$$\Big[xn\Big]_0^\infty - \int_0^\infty n\,dx,$$

the first term being zero since n is zero at the upper limit of x, and on substituting $-dn/\theta$ for $n\,dx$ the second term becomes n_0/θ.

If l be the mean free path, the sum of the paths is $n_0 l$ so that l is $1/\theta$. Thus out of a large number n_0 of free paths the number n that exceed the length x is given by the equation

$$n = n_0\,\epsilon^{-x/l}, \tag{1}$$

where l is the mean free path.

The mean of the squares of a large number of free paths, or the mean of the cubes may be found by similar calculations in terms of l^2 and l^3.

The sum of the squares of the free paths of lengths greater than x and less than $x+dx$ is x^2dn and the sum of the squares of all the paths is $\int_0^\infty x^2dn$. On integrating by parts the integral becomes $2n_0 l^2$ so that the mean square of a large number of free paths is $2l^2$. Thus if an electron traverses a large number c of free paths of lengths $l_1, l_2, \ldots, l_c$, the sum of the lengths of the paths is cl, l being the mean free path, and the sum of the squares $(l_1{}^2+l_2{}^2+\ \ l_c{}^2) = 2cl^2$. By a similar calculation it may be shown that the mean of the cubes of a large number of free paths is $6l^3$.

---

(1) [*Proc. Roy. Soc.* A, Vol. 86, p. 197 and p. 571, 1912].

4. *Diffusion of electrons.*

The motions of electrons which are observed experimentally are due either to diffusion or to the action of electric or magnetic forces. The process of diffusion depends on the velocity of agitation of the electrons and tends to equalise the distribution of the electrons throughout the gas.

✓ In the steady motion which is being considered the mean velocity of the electrons W in the direction of the electric forces is much smaller than the mean velocity of agitation, and the electric force has very little direct effect on the rate of diffusion. Under these conditions the changes in the distribution due to diffusion may be found by considering the case where there are no electric or magnetic forces acting on the electrons, and the mean energy of agitation $mV^2/2$ is the same throughout the gas.

In order to find equations to represent the effect of diffusion, it is necessary to consider the definition of the coefficient of diffusion in terms of the changes in the distribution of the electrons due to the motion of agitation.

If the number of electrons n_1 per cubic centimetre at any point P in the gas be greater than the number n_2 at another point Q, the number of electrons that flow per second from a small volume at P to a similar volume at Q is greater than the number that flow from Q to P, and the difference between the numbers that flow in opposite directions is proportional to (n_1-n_2).

In general the number of electrons n per cubic centimetre at a point varies continuously and n is a function of the distances x, y and z of the point from coordinate planes. The difference between the numbers flowing in opposite directions per second through unit area of a plane parallel to the coordinate plane (yz) is proportional to $-dn/dx$ and may be represented by the term $-K\,dn/dx$. The coefficient K in this expression is the coefficient of diffusion.

The electrons flow in the opposite directions with the same mean velocity $\overline{U}_x$ parallel to the axis of x normal to the plane. If U_x, U_y and U_z be the components of the velocity of agitation U of an electron in the directions of the axes, $U_x^2+U_y^2+U_z^2=U^2$. Thus the mean value of $U_x^2=V^2/3$ and the mean velocity $\overline{U}_x$ is approximately $V/\sqrt{3}$. If $N_1\overline{U}_x$ be the number flowing per second from left to right through unit area of the plane and $N_2\overline{U}_x$ the number flowing from right to left, the difference between the numbers N_1 and N_2 is small compared with sum (N_1+N_2) which is the total number n at the plane. The difference between the numbers N_1 and N_2 is given by the equation,

$$(N_1-N_2)\overline{U}_x=-K\,dn/dx.$$

In order to simplify the notation it is convenient in many calculations to indicate the flow $-K\,dn/dx$ as the product of the number n and a small velocity u, and the flows through unit areas of planes parallel to the coordinate planes (xz) and (yx) by the products nv and nw. The velocities

u, v and w are, in general, very small compared with the actual velocities of the electrons parallel to the axes.

In this notation the effect of diffusion in the directions of the axes is indicated by the three equations

$$\left.\begin{aligned} nu &= -K\, dn/dx, \\ nv &= -K\, dn/dy, \\ nw &= -K\, dn/dz. \end{aligned}\right\} \qquad (2)$$

It should be realised that the velocities u, v and w are introduced in the theory merely to simplify the notation, and in the actual process of diffusion there is no such motion as a general drift of all the electrons at a point with the velocities u, v and w due to inequalities in the number n at different points of the gas.

The velocities u, v and w are mathematical quantities which satisfy the condition that the actual changes in the distribution of the electrons, due to diffusion, are the same as the changes which would be obtained if all the electrons at a point had the small velocities u, v and w in the directions of the axes and no other motion.

This notation is generally adopted in the equation of continuity where the rate of increase in the number of electrons in any space is represented by the flow through the boundary of the space in terms of the velocities u, v and w.

5. *Equations of continuity.*

The simplest case is that in which $q\,dx$ is the total number of electrons in the space between the two planes A_1 and A_2 parallel to the coordinate plane (yz) and at distances x and $x+dx$ from the origin. If n be the number of electrons per cubic centimetre of the space, the number q is $\iint n\,dy\,dz$ and in accordance with the first of equations (1) the flow qu per second through the plane A_1 is $-K\,dq/dx$, q being a function of the time t and the distance x of the plane A_1 from the coordinate plane (yz). The flow through the plane A_2 is $qu+\left(d(qu)/dx\right)dx$ and the difference between the numbers flowing through the two planes $-\left(d(qu)/dx\right)dx$ is the rate of increase in the number of electrons between the planes which is $(dq/dt)dx$. The equation of continuity for the number q is therefore

$$\left.\begin{aligned} &\frac{dq}{dt}+\frac{d}{dx}\,(qu)=0 \\ \text{or} \qquad &\frac{dq}{dt}=K\,\frac{d^2 q}{dx^2}. \end{aligned}\right\} \qquad (3)$$

The equation of continuity for the number n is found by a similar method by considering the rate of change of the number of electrons in a small space bounded by three pairs of planes parallel to the coordinate planes as in figure 1. The two sides parallel to the plane (yz) of area $dy\,dz$ are at a distance apart dx and the number of electrons in the elementary volume is $n\,dx\,dy\,dz$. The number of electrons flowing per second through the side at a distance x from the plane (yz) is $nu\,dy\,dz$ and the number flowing through the opposite side at a distance $x+dx$ from the plane (yx) is $\left(nu+\left(d(nu)/dx\right)dx\right)dy\,dz$. The difference between these numbers $-\left(d(n\,u)/dx\right)dx\,dy\,dz$ is the rate of increase of the number of electrons in the elementary space due to diffusion through these two sides.

There is a similar rate of increase due to diffusion through the other pairs of opposite sides and the total rate of increase in the number of electrons in the elementary space $(dn/dt)\,dx\,dy\,dz$ is the rate of increase in the number n due to diffusion through the three pairs of sides. The equation of continuity for the number n is therefore

$$\frac{dn}{dt}+\frac{d}{dx}(nu)+\frac{d}{dy}(nv)+\frac{d}{dz}(nw)=0. \qquad (4)$$

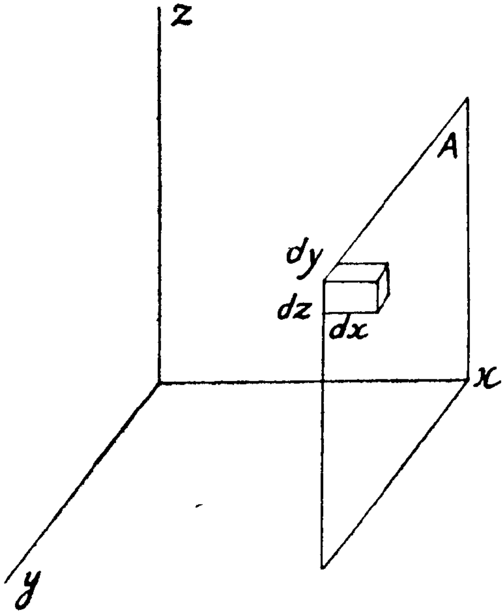

FIGURE 1.

In order to obtain an equation of continuity in a practical form which may be compared with the results of experiments on diffusion, it is necessary to eliminate the velocities u, v and w from equation (4) by substituting for nu, nv and nw the values in terms of K given by equations (2). The equation of continuity for n thus becomes

$$\frac{dn}{dt} = K\left(\frac{d^2 n}{dx^2} + \frac{d^2 u}{dq^2} + \frac{d^2 n}{dz^2}\right). \qquad (5)$$

6. *Rate of change of the mean square of the distances of electrons from any point.*

The rate at which a group of electrons tends to spread throughout a gas may be deduced from the equations of continuity. This is indicated by calculating the rate of increase of the mean square of the distance $\overline{R^2}$ of a group of electrons from a fixed point when the electrons move in a space remote from the boundary of the gas so that none of the electrons come into contact with the side of the vessel containing the gas or with electrodes.(1)

The change in the mean square of the distance $\overline{x^2}$ of a group of electrons from the coordinate plane (yz) may be found from the value of dq/dt given by equation (3). In this case $q\,dx$ is the number of electrons in the space between two planes A_1 and A_2 at distances x and $x+dx$ from the coordinate plane (yz), and the sum of the squares of the distances of these electrons from the plane (yz) is $x^2 q\,dx$. The total number of electrons N in the group is $\int q\,dx$ and the mean square of the distances $\overline{x^2}$ of all the electrons from the plane yz is $1/N\int x^2 q\,dx$. The rate of increase $d\overline{x^2}/dt$ of the mean square of the distance from the plane (yz) is therefore $1/N\int x^2(dq/dt)\,dx$ or $(K/N)\int x^2(d^2q/dx^2)\,dx$. It is assumed that during the time t none of the electrons come near to the boundary so that at the upper and lower limits of the integrals the numbers q and dq/dx are zero.

On integrating by parts the integral $\int x^2(d^2q/dx^2)\,dx$ becomes $[x^2\,dq/dx]-2\int x\,dq$, but the first term is zero since dq/dx is zero at the limits of integration. Also since q is zero at the limits the second term, on integrating again by parts, becomes $2\int q\,dx$ which is $2N$. Thus the rate of increase in the mean square of the distance $\overline{x^2}$ of any distribution from the coordinate plane (yz) is given by the equation

$$d\overline{x^2}/dt = 2K. \qquad (6)$$

The rate of increase of the mean squares of the distances $\overline{y^2}$ and $\overline{z^2}$ from the coordinate planes (xz) and (xy) are also equal to $2K$, and since the mean square $\overline{R^2}$ of the distance from the origin of coordinates is $(\overline{x^2}+\overline{y^2}+\overline{z^2})$

---

(1) [*Electricity in gases*, sections 80 and 81.]

the rate of increase of $\overline{R^2}$ is $6K$. Since the origin may be at any point the rate of change of the mean square of the distance of any group of electrons from any point is $6K$.

It also follows that the rate of increase of the mean square of the distance $\overline{r^2}$ from any axis is $4K$.

The rate of change of the mean square of the distance of a group of electrons from a point may be calculated in terms of the mean free path l and the mean velocity of agitation $\overline{U}$, and the expression for K in terms of l and $\overline{U}$ is thus obtained since K is given by the equation

$$d\overline{R^2}/dt = 6K. \tag{7}$$

7. *Coefficient of diffusion.*

For the purpose of this calculation it may be supposed that a large number of electrons N start uniformly in all directions at the time $t=0$ from a point P in the gas and traverse a large number c of free paths of different lengths $l_1, l_2, \ldots, lc$ in the same order with the same velocity U and in the same time cl/U. Also it is assumed that after a collision with a molecule all directions of motion of an electron are equally probable.

Under these conditions the electrons are distributed symmetrically about the point P at any time t, and since the centre of the distribution is at P the rate of change of the mean square of the distance from any point is the same as the rate of change of the mean square of the distance $\overline{R^2}$ from P.

Let t_1, t_2, etc., be the intervals l_1/U, l_2/U, etc., between the collisions of the electrons with molecules. After traversing the free paths of length l_1 the electrons are distributed uniformly over a sphere of radius l_1 with its centre at P and the mean square of the distances from P is $l_1{}^2$ at the time t_1. After traversing the free paths of length l_2 the mean square of the distance from P is $l_1{}^2+l_2{}^2$ at the time t_1+t_2. At the time cl/U when the electrons have traversed all the free paths the mean square of the distance $\overline{R^2}$ from P if $(l_1{}^2+l_2{}^2\ldots l_c{}^2)$ which is $2cl^2$.

Thus the change in the mean square of the distance in the time cl/U is $2cl^2$ and the rate of change of the mean square of the distance $d\overline{R^2}/dt$ is $2lU$. The coefficient of diffusion of electrons moving with the same velocity U is therefore $lU/3$ and the mean coefficient K of all the electrons when the velocities of agitation are distributed about the mean velocity $\overline{U}$ is

$$K = l\overline{U}/3. \tag{8}$$

In the Maxwellian distribution the mean velocity $\overline{U}$ is $0{\cdot}92\times V$, V^2 being the mean of the squares of the velocity of agitation, and it is convenient to express the coefficient of diffusion by the equation

$$K = 0{\cdot}92\times lV/3. \tag{9}$$