OPTIMUM SYSTEMS CONTROL

second edition

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Preface

In the last several years, strong interest has continued in the study of optimization theory as applied to the control of systems. The purpose of this text is to provide a reasonably comprehensive treatment of this optimum systems control field at a level comparable to that of a beginning graduate student. In this regard, the book does not require prior background in state space techniques, calculus of variations, or probability theory, although some exposure, particularly to the first and third topics, would be of value. The text has been written strictly from the point of view of an engineer with interest in the study of systems. Consequently, we emphasize the basic concepts of various techniques and the relations, similarities, and limitations of these basic concepts at the expense of mathematical rigor. As befits an introductory text, the level of presentation is generally monotone increasing from chapter to chapter.

Structurally, the text is divided into four areas although overlap certainly exists. These are:

1. Optimal control with deterministic inputs (Chapters 2, 3, 4, 5, 6).
2. Systems concepts including controllability, observability, sensitivity, and stability (Chapter 7).
3. State estimation and combined estimation and control (Chapters 8, 9).
4. Computational techniques in systems control (Chapter 10).

There are several ways in which the text can be used. There is undoubtedly too much material covered for a one three-semester credit hour course, although the material can easily be covered in two three-quarter hour courses. For a single three-semester hour course, we suggest that the instructor consider eliminating either Chapters 8 and 9 or Chapter 10, as best fits following courses in mathematical system theory. If a course using this text has been preceded by a graduate level course in state space techniques, then Chapter 7 may be eliminated, and the more advanced backgrounds of the students may well allow completion of the remainder of the text in one semester.

This edition of *Optimum Systems Control* is considerably revised and we hope much improved over the first edition. Every chapter in the original text has been subject to this revision. Several new derivations and examples have been included as have developments in optimum systems control that were unknown during the writing of the first edition. The senior author considers it his personal good fortune that he was able to obtain the full and complete collaboration of an outstanding young professional who has contributed mightily to this updating. The authors wish to acknowledge the helpful assistance of many former students, including those mentioned in the first edition, who have offered many helpful comments when reading through earlier versions of the text.

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Introduction

1

In recent years much attention has been focused upon optimizing the behavior of systems. A particular problem may concern maximizing the range of a rocket, maximizing the profit of a business, minimizing the error in estimation of position of an object, minimizing the energy or cost required to achieve some required terminal state, or any of a vast variety of similar statements. The search for the control which attains the desired objective while minimizing (or maximizing) a defined system criterion constitutes the fundamental problem of optimization theory.

The fundamental problem of optimization theory may be subdivided into four interrelated parts:

1. Definition of a goal.
2. Knowledge of our current position with respect to the goal.
3. Knowledge of all environmental factors influencing the past, present, and future.
4. Determination of the best policy from the goal definition (1) and knowledge of our current state (2) and environment (3).

To solve an optimization problem, we must first define a goal or a cost function for the process we are attempting to optimize. This requires an adequate definition of the problem in physical terms and a translation of this physical description into mathematical terms. To effectively control a process, we must know the current state of the process. This we will call the problem of state estimation. Also, we must be able to characterize the process by an effective model which will depend upon various environmental factors. This we will call system identification. With a knowledge of the cost function, and the system states and parameters, we then determine the best control which minimizes (or maximizes) the cost function. Thus we may define five problems, which are again interrelated, and which we must solve in order to determine the best, or optimum, system:

1. The Control Problem. We are given a known system with relation between system states and input control. We desire to find the control which changes the state $x(t)$ so as to accomplish some desirable objective. Figure 1.1 illustrates the salient features of the control problem. This may be an open- or closed-loop problem, depending upon whether or not the control is a function of the state.

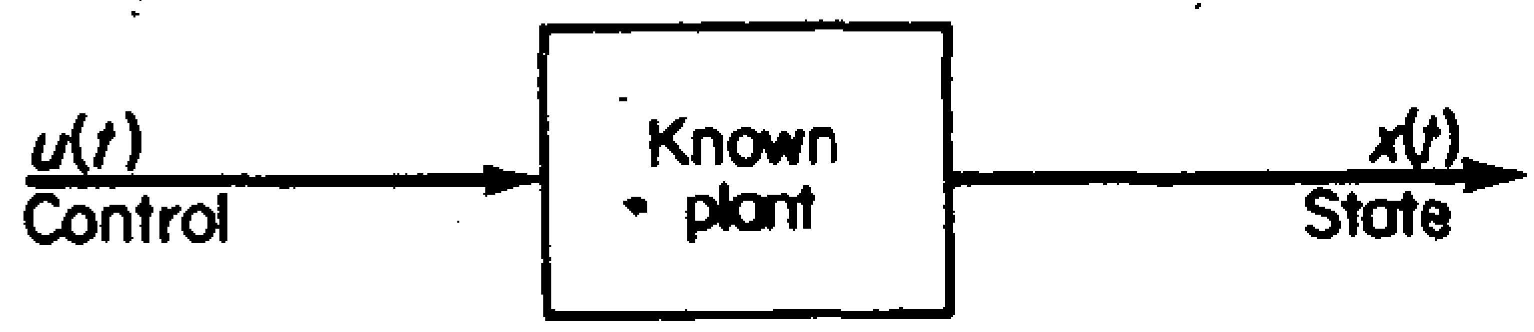

Fig. 1.1 Deterministic optimum control problem.

2. The State Estimation Problem. We are given a known system with a random input and measurement noise such that we measure an output $z(t)$ which is a corrupted version of $x(t)$ as indicated in Fig. 1.2. We know the statistics of the plant noise $w(t)$ and the measurement noise $v(t)$, and we desire to determine a "best" estimate $\hat{x}(t)$ of the true system state $x(t)$ from a knowledge of $z(t)$.

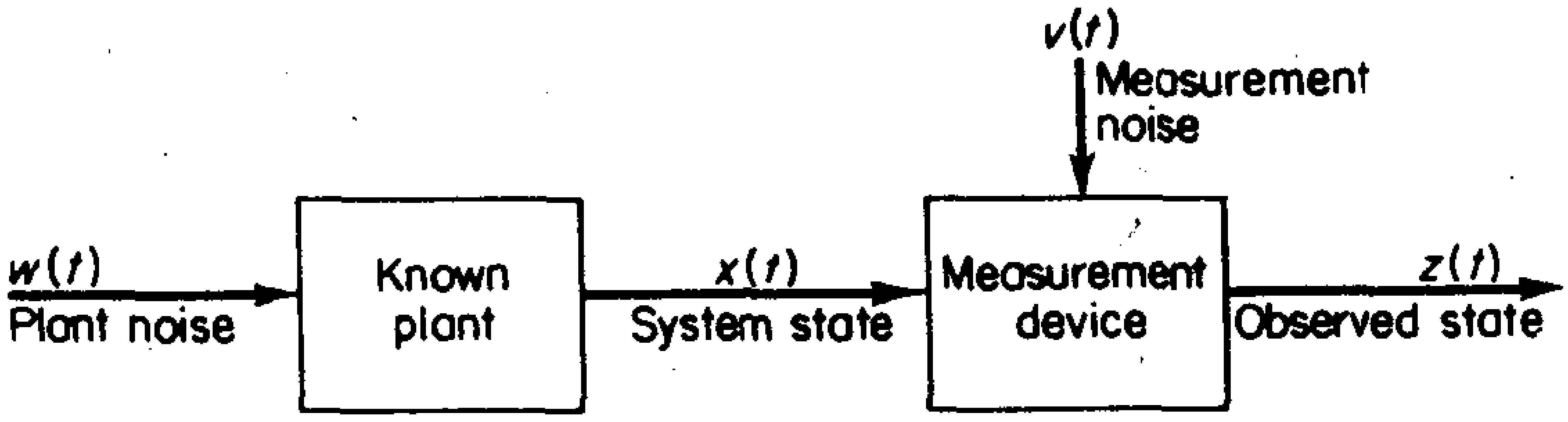

Fig. 1.2 State estimation problem.

3. The Stochastic Control Problem. We may combine problems 1 and 2 to form a stochastic control problem as depicted in Fig. 1.3. We desire to determine a control $u(t)$ such that the output state $x(t)$ is changed in accordance with some desired objective. Plant noise $w(t)$ and measurement noise $v(t)$ are present. We know the statistics of these noises and must of course determine a best estimate, $\hat{x}(t)$, of $x(t)$ from a knowledge of the output $z(t)$ before we may discern the "best" control which may be open- or closed-loop.

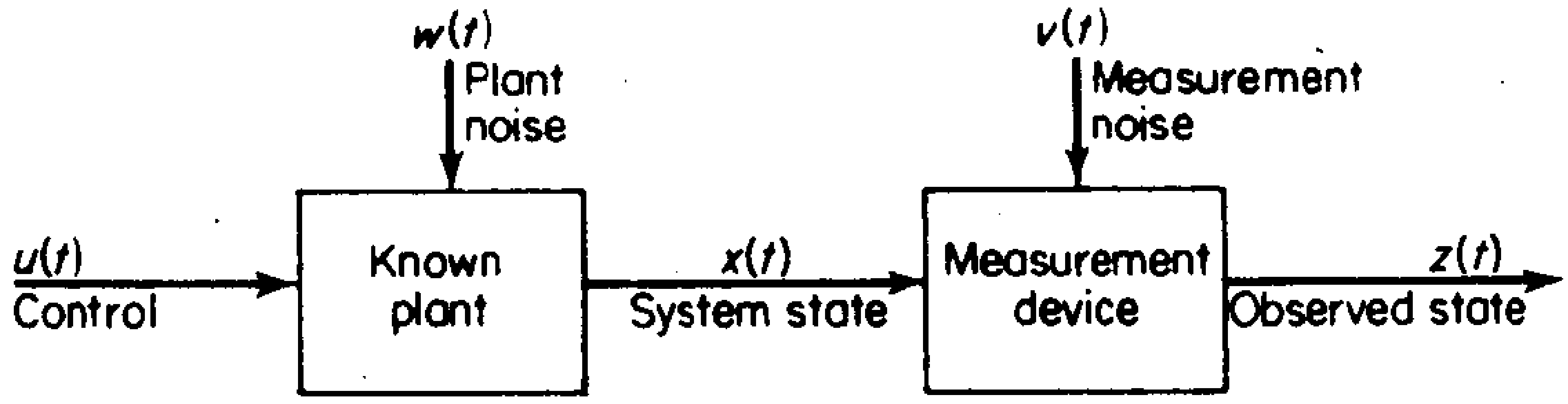

Fig. 1.3 Stochastic control problem.

4. The Parameter Estimation Problem. In many systems we must incorporate some method of identification of system parameters which may vary as a function of the environment. We are given a system such as that shown in Fig. 1.4, where we again know the statistical characteristics of the plant and the measurement noise, and we wish to determine the best estimate of certain plant parameters based upon a knowledge of the deterministic input $u(t)$, the measured output $z(t)$, and possibly some a priori knowledge of the system plant structure. As we shall see, we often must accomplish state estimation in order to obtain parameter estimation.

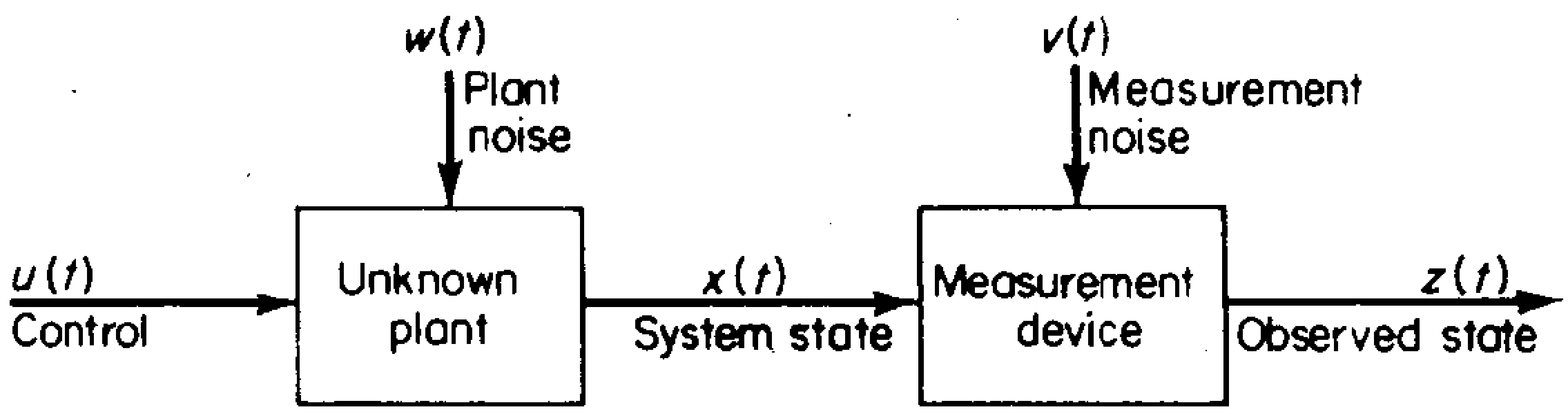

Fig. 1.4 Parameter estimation problem.

5. The Adaptive Control Problem. We may combine problems 1 through 4 to form an adaptive control problem. We are given the statistical characteristics of $w(t)$ and $v(t)$ or some method of determining these characteristics. Plant parameters are random. We desire to determine a control $u(t)$ to best accomplish some desired objective in terms of the measurement noise and plant noise as well as the uncertainty in system dynamics. If the control $u(t)$ is determined as a function of the measured output $z(t)$, we have a closed-loop adaptive system.

We will divide our efforts in optimum systems control into ten chapters. These chapters and their respective purposes and contents will now be described briefly. Each chapter will contain several examples to illustrate our developed theory. Many problems, of varying complexity, will be posed at the end of each chapter for the interested reader.

2
Calculus of extrema and single-stage decision processes

This chapter examines ordinary scalar maxima and minima and extrema of functions of two or more variables. Constrained extrema and the vector formulation of extrema problems are presented for single-stage decision processes.

3
Variational calculus and continuous optimal control

In this chapter, we introduce the subject of the variational calculus for continuous decision processes through a derivation of the Euler-Lagrange equations and associated transversality conditions. We discuss the use of Lagrange multipliers to treat equality constraints and briefly mention the inequality constraint problem. Several very simple optimal control problems are considered.

4
The maximum principle and Hamilton-Jacobi theory

In this chapter, the Bolza formulation of the variational calculus leads into a proof of the Pontryagin maximum principle and the development of the Hamilton canonic equations and the associated transversality conditions. We discuss at some length problems involving control and state, and state variable inequality constraints. The Hamilton-Jacobi equations are then developed and modified to produce Bellman's equations of continuous dynamic programming.

5
Optimum systems control examples

This chapter formulates and solves numerous optimal control problems of interest; among those solved are:

1. Minimum time problems.
2. Linear regulator problems.
3. Servomechanism problems.
4. Minimum fuel problems.
5. Minimum energy problems.

6. Singular solution problems.
7. Distributed parameter problems.

6
Discrete variational calculus and the discrete maximum principle

In this chapter, we develop a simplified discrete maximum principle for cases in which control and state variable inequality constraints are absent. We give a meaningful comparison of the discrete maximum principle and the discretized results of application of the continuous maximum principle for a rather general optimization problem. We conclude our discussion with a brief presentation of the relationship between discrete time optimal control and mathematical programming.

7
Systems concepts

After having established and solved many state estimation problems and optimal control problems, we now inquire into the conditions which must be established in order for many of these problems to have meaningful solutions. First we examine the manner in which the output of a system is constrained with respect to the ability to observe system states. Then we examine the dual requirement and find the characterization of the manner in which a system is constrained with respect to control of system states or system outputs.

Also presented are various methods for studying the parameter sensitivity problem in continuous systems. The use of sensitivity concepts in optimal and optimal adaptive systems are presented. A brief introduction to system stability concepts and a discussion of stability-optimality relations for linear systems concludes the chapter.

8
Optimum state estimation

Chapter 8 introduces the subject of optimum filtering. The state transition approach is used, which allows us to develop the celebrated Kalman-Wiener computational algorithms for nonstationary filtering. The dual relations between the filter and the regulator problems are observed, and the difference between optimum smoothing and optimum filtering is discussed.