Proceedings of the 1985 SEM Fall
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TRANSDUCER TECHNOLOGY FOR PHYSICAL MEASUREMENTS

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CONTENTS

EVALUATION OF STRAIN-GAGE FORCE SENSORS BY COMPUTER

Albert E. Brendel, President; Sensor Developments Inc.

I. FORWARD

One of the most important aspects of transducer manufacture is the process of calibration. Calibration in this instance refers to the determination of the transducer's transfer functions to allow the user to relate the output of the sensor to the value of the force being measured. A methodically run calibration can also be used as a quality control tool and in addition, give indications as to the most likely source of problems as well.

To calibrate a force sensor, two principal techniques are applied; dead weight reference or force sensor reference. For "traceability", dead weights are considered primary standards, while reference load cells are considered secondary standards (even though they may be more accurate than dead weight). The subject of traceability will not be addressed in this paper.

In a typical calibration cycle for a sensor, known loads are applied, and the sensors responses measured and recorded. In order to maximize hysteresis losses, common practice is to cycle the sensor between zero load and the full scale rating of the sensor several times before recording data. After a stable hysteresis loop is established, data is taken for both ascending and descending loads, and the principal transfer functions and deviations computed. While this process can be performed manually, modern instrumentation systems allow automatic data gathering, data reduction and subsequent data print-out at low cost. This type of system all but eliminates transcription errors, forces the operator to perform the calibration cycle in consistent and repeatable steps, and even monitors the process alerting the operator of problems which may have damaging consequences if left unattended.

II. HISTORY

My first attempt at computer controlled calibration occurred while employed at a small transducer manufacturing operation in Michigan called Lebow Associates. In approximately 1972, the operation had grown to the point where our production capability was being severely limited by the time required to perform the calibration of a load cell. Using manual techniques, a typical calibration cycle required approximately 1/2 hour. This meant we had a maximum production rate of 16 load cells per day per calibration station. Rather than simply adding calibration stations, which required both space and the addition and duplication of expensive equipment, it was decided to attempt to speed up the process itself by adding a real time computer to assist.

The hardware chosen was a Data General Nova 1200, rack mounted bridge amplifiers (one per channel), an Analogic 8-channel 14 bit converter and a teletype terminal which was the lowest cost system available at the time. Software was developed in BASIC and algorithms developed which duplicated the manner in which an operator was used to taking data. Several years and $80,000 later, the system was finally completed but had mutated into a multi-user system with CRT terminals with hard disk drives and multi-tasking Fortran as the operational language. Nonetheless, the system was a success, reducing the time required for a calibration to approximately 2 minutes with most of the time spent in fixturing the sensor rather than running the calibration. As far as I know, this original system is still in use today, running the same program developed over 10 years ago or its direct descendent.

Tremendous changes have occurred in the computer field in the years since this first attempt. The purpose of this paper is to describe a computer calibration system that has resolutions approximately three orders of magnitude greater than the first system , but constructed at less than 1/10 the cost.

III. HARDWARE

The computer system was developed around the IEEE-696 bus, commonly known as the S-100 bus system. Originally intended as a hobby system, and joined today by many other competing bus and operating systems, it is still supported by many vendors and has high quality cards available at extremely low costs. Also affecting the decision was the availability of a 17 bit transducer signal conditioning card from a canadian firm which was designed for this bus system (If a STD bus card had been available, the described hardware might have been different).

The hardware for the system consists of a S-100 bus cabinet with 10 available card slots containing 4 cards: a single board Z-80 computer card containing 64K of ram, serial I/O ports, disk drive controller and a CP/M operating system; two SCALAR signal conditioning boards and a custom designed relay card for engaging shunt calibration resistors under computer control. Also part of the system are: two single sided double density 8" floppy disk drives, a CRT terminal and a high speed dot matrix printer.

IV. SOFTWARE

Software is written in Microsoft Basic (referred to as MBASIC) with assembly language calls to obtain data from the two SCALAR cards. MBASIC is an extended version of BASIC and allows calculations with 15 digit precision which is necessary for this application. The program is lengthy, with it and the MBASIC interpreter using the full 64k of system RAM (memory). However, if additional program extensions are desired, CHAINING or OVERLAYING programs or segments from the floppy disk drives allow programs of any length to be accommodated.

The software has been written in modular format as a series of subroutines which are menu selected by the operator. The choices presented allow the operator to view the raw outputs of both the load cell and the applied loads (either deadweight values or load values computed from look-up tables of the reference load cell being used); or guide the operator through a defined calibration sequence, giving the operator visual and audible cues telling him to apply or reduce loads to the sensor under calibration. Data is gathered at approximately even increments of the transducer's range from which best-fit equations are generated. Currently, only 2nd degree equations are used, which for the small degrees of non-linearity normally present, appear adequate. Once ascending and descending data curves are generated, the operator is given a screen printed summary of the data, along with a chart of deviations. At this point, the operator can decide if he has a valid calibration, to rerun the test and/or wishes to print a final data sheet.

V. TYPICAL DATA ACQUISITION RUN

Upon initially starting the system, the program asks to have reference bridges installed on each of the two channels. After installation, the program then proceeds to establish reference zero readings for each channel, engages shunt calibration resistors across these bridges producing known offsets, and stores these values for subsequent "gain" computations.

The operator is then queried as to the reference he will be using, either deadweights or reference load cell. If deadweight, the system asks for a list of the values he will be using or if using a reference load cell, asks for an identification of which one he will be using.

Assuming he is using a reference load cell, the operator is then asked for the full scale rating of the sensor and the loading direction. Alarms are automatically set for applied loads exceeding these values and/or sensor outputs exceeding 3 mv/v (corresponding to excessive stress levels in the sensor). The operator can change these alarm settings at any time if he so desires. If an alarm value is exceeded at any time during the test, the CRT bell is periodically sounded, alerting the operator.

The operator then enters a DISPLAY mode in which the sensor output is displayed in Mv/v, as well as the applied load in pounds. Also displayed are running averages of these parameters which is equivalent to digitally filtered data, a pseudo-analog meter display of applied load registering from Ø to 1ØØ% of full scale rating, and a predicted sensor output reading. The predicted output is computed by linear extrapolation of the sensor's output taken after the load exceeds 1Ø% of its rated value. The predicted output value is extremely useful in guarding against inadvertent overloading due to operator inputting the wrong full scale rating, mislabeling of the sensor, or design errors. The feature also is quite useful when "trimming" a sensor to known outputs without requiring the applied load to be held "steady".

In the DISPLAY mode, the operator cycles the sensor 3 times through its load range to establish a stable hysteresis loop, enters a FINAL CAL mode which asks the operator how many data points are desired, and then proceeds to obtain reference zero load readings and various shunt calibration readings before proceeding. A large Pseudo-analog display is then presented showing applied load from Ø to 1ØØ% with a DESIRED load point highlighted along with an indication of the currently applied load. The operator applies or reduces load to this desired load point and then pauses. The system continuously monitors load and sensor outputs. If it senses it is within an acceptable "window" for obtaining a load point, it then checks for a stable reading. If a stable reading is obtained, the system records it and displays a new desired load point for the operator. After all load points are obtained, the system uses the data obtained to compute 2nd degree equations which describe the data. The initial data is then discarded and sensor outputs at even load increments are computed and displayed for operator inspection on the CRT screen. If acceptable, the operator then enters a PRINT DATA SHEET mode, at which time he enters the sensors identification. The printing of the data sheet then occurs automatically while the operator fixtures the next sensor for test.

VI. SUMMARY AND PROLOG

The described sensor calibration system has proven to be remarkably efficient, producing repeatable calibrations of sensors at high speed, greatly easing the operator's tasks. The system is highly modular, both from a hardware and software standpoint, and the use of a high level language permits special testing routines to be quickly written and used. These programs, can be stored indefinitely on floppy disks without burdening the main resources of the system.

While the system described used the CP/M operating system, it has recently been modified to use the Turbodos operating system which may be considered to be a multi-user "superset" of CP/M. This recent modification, allowed the addition of two more SCALAR cards and an additional terminal to allow two independent calibration systems to co-exist in the same main frame cabinet at a very small cost addition. The new Turbodos system uses the same MBASIC program routines, but allows each user to share the disk drives and printer. Our main office computer, presently operating with 6 users, also utilizes the Turbodos system which will allow us in the future to NETWORK both of these systems together if we so desire.

ILLUSTRATIONS:

1. A typical calibration set up using a hydraulic loading frame and reference load cell.

2. A view of the calibration computer, CRT terminal and high speed printer.

3. An internal view of the computer showing the 4 "cards" in the system.

4. Initial screen display.

5. "Alarm set" screen display.

6. "Display mode" screen display.

7. "Final Cal mode" screen display.

8. Copy of typical data sheet.

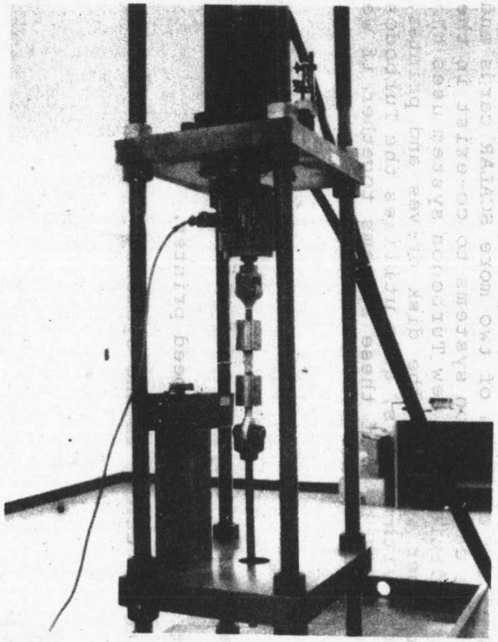

Fig. 1

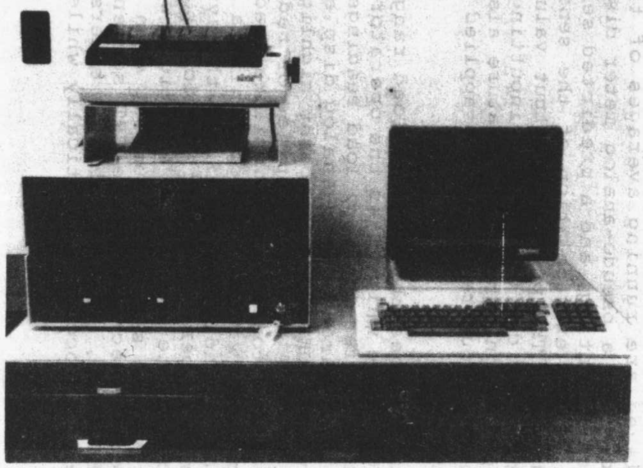

Fig. 2

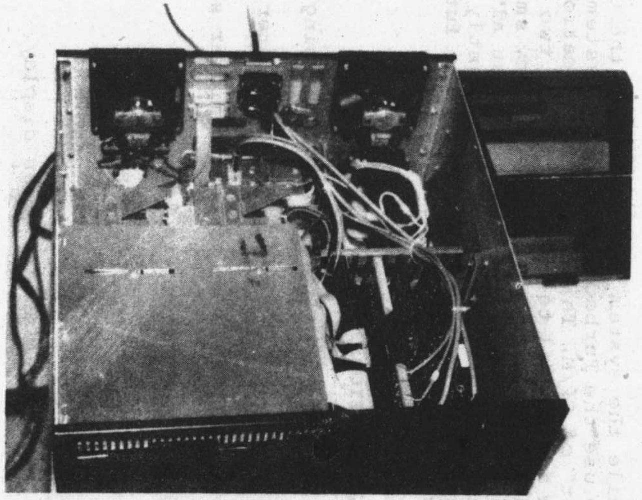

Fig. 3

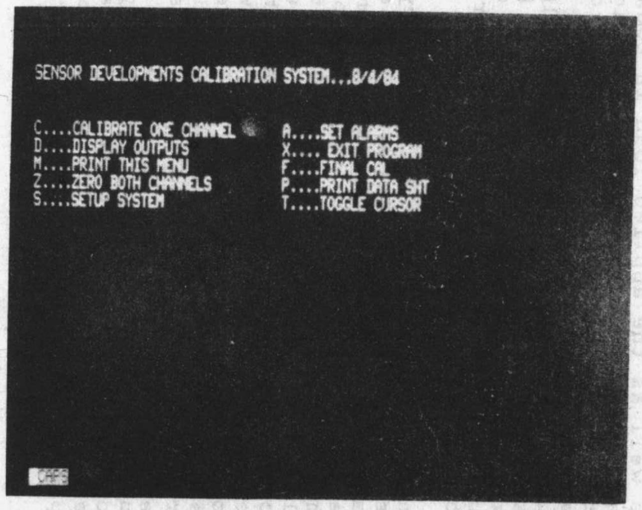

Fig. 4

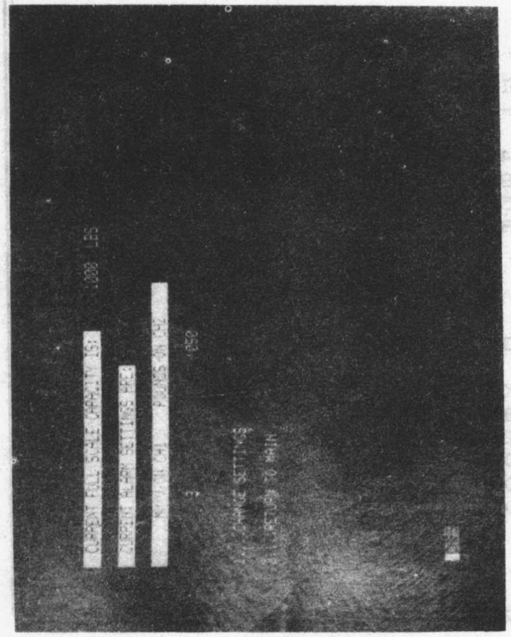

Fig 5

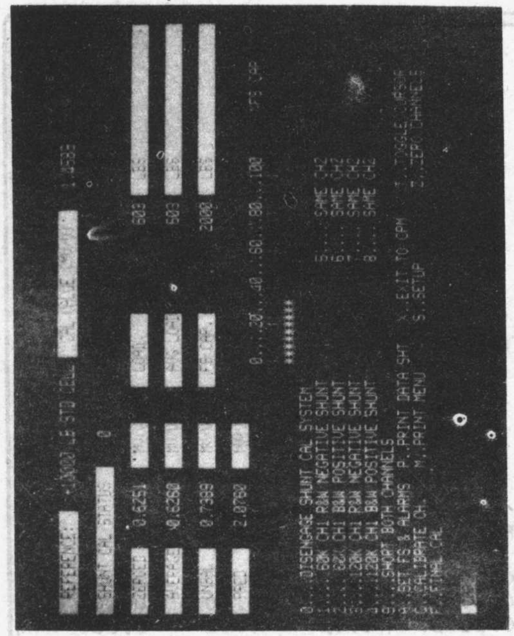

Fig. 6

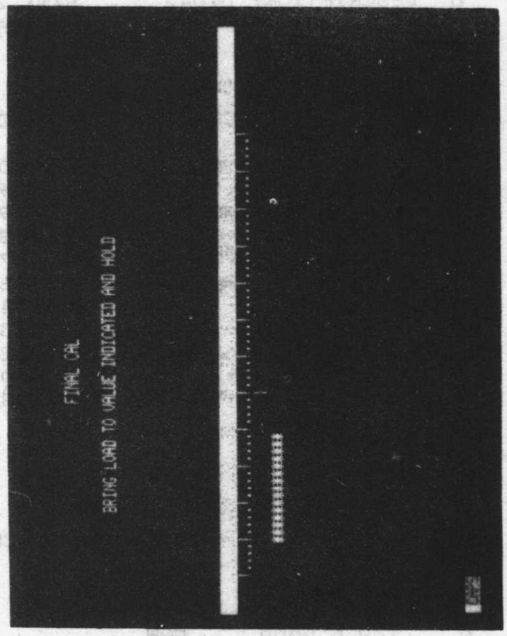

Fig. 7

CALIBRATION DATA SHEET

DESCRIPTION: 01042 S/N: 2219-CCW DATE: 9-27-85
OPERATOR: JB FS CAPACITY: -250 IN-LBS JOB NUMBER:

LOAD	OUTPUTS ASCENDING	DESCENDING	BOTH	BF/O	DEVIATIONS N/L	HYS	BF/O
0.000	0.0000	0.0000	0.0000	0.0000	0.00	0.00	0.00
-25.000	-0.1940	-0.1939	-0.1939	-0.1939	0.00	-0.00	0.00
-50.000	-0.3879	-0.3879	-0.3878	-0.3878	0.01	-0.00	0.00
-75.000	-0.5818	-0.5818	-0.5818	-0.5816	0.01	-0.00	0.01
-100.000	-0.7757	-0.7757	-0.7756	-0.7755	0.01	-0.00	0.01
-125.000	-0.9696	-0.9696	-0.9695	-0.9694	0.01	-0.00	0.01
-150.000	-1.1635	-1.1634	-1.1634	-1.1633	0.01	-0.00	0.00
-175.000	-1.3574	-1.3573	-1.3572	-1.3572	0.01	-0.00	0.00
-200.000	-1.5512	-1.5512	-1.5511	-1.5511	0.01	-0.00	0.00
-225.000	-1.7450	-1.7450	-1.7449	-1.7449	0.00	-0.00	-0.00
-250.000	-1.9388	-1.9388	-1.9387	-1.9388	0.00	0.00	-0.01

SHUNT CAL DATA:

VALUE (OHMS)	ACROSS	PRODUCES SIMULATED SIGNAL OF: MV/V	IN-LBS (TERM)	(BF/O)
60 K OHMS	B&W	2.9071	374.874	374.852
120 K OHMS	B&W	1.4582	188.037	188.026
60 K OHMS	R&W	-2.9117	-375.470	-375.448
120 K OHMS	R&W	-1.4603	-188.311	-188.300

LOAD MAY BE COMPUTED USING THE FOLLOWING EQUATIONS:
LOAD= (K1 + K2 * OUTPUT) * OUTPUT
WHERE LOAD IS IN IN-LBS AND OUTPUT IS IN MV/V
THE VALUES FOR K1 AND K2 ARE:

CURVE	K1	K2
ASCENDING	128.888771671082	-0.027847531553
DESCENDING	128.907322296076	-0.018279656132
AVERAGE	128.907713456366	-0.023068784779
TERMINAL	128.952436893423	0.000000000000
BEST FIT /0	128.944764016050	0.000000000000

RESISTANCES (OHMS): EXCITATION- 0
SIGNAL- 0

STRAIN GAGE REINFORCEMENT EFFECTS ON ORTHOTROPIC COMPOSITE MATERIALS

C. C. Perry*

There is both experimental and analytical evidence that stiffness of a strain gage can produce a significant reinforcement error when it is installed on a low-modulus material such as a plastic.[1,2,3,4] This being the case, it raises the question of errors due to the same effect when strain measurements are made on some types of orthotropic materials (e.g., unidirectionally reinforced plastics) which are characterized by a low elastic modulus in at least the minor principal material direction. Actually, as indicated by the goniometric distribution of mechanical properties plotted in Fig. 1, the elastic modulus of such a material is typically low in most directions, and not far from that of the plastic matrix, except for an angular range of about ±30 degrees from the major principal material axis.

A method has previously been described by which approximate compensation for reinforcement effects can be achieved when the material is isotropic in its elastic properties.[5] The procedure involves calibration of the material for its apparent elastic properties (E,ν), employing the identical type of strain gage intended for subsequent use in experimental stress analysis tests. Later, when indicated strains are converted to stresses with Hooke's law, based on the apparent elastic properties, the reinforcement errors (as well as those due to transverse sensitivity) are canceled in the data-reduction process. It is shown here that an extension of the same method can be applied, with certain restrictions, to some types of composite materials having directionally variable elastic properties.

For the purpose of this demonstration, a unidirectionally reinforced plastic has been selected as an example. The proposed method should be applicable, however, to other material types which conform to the same reinforcement model. An orthotropic material such as that considered here has four independent elastic constants, usually taken as E_1,E_2,ν_{12}, and G_{12}. These represent, respectively, the major and minor elastic moduli, the major Poisson's ratio, and the shear modulus. Since the normal-stress characteristics of the material (E_1,E_2,ν_{12}) are commonly measured in separate tests from that used to determine the shear modulus; and since, with respect to the principal material axes, normal and shear responses are uncoupled, this method employs separate compensation of the normal and shear components.

With a strain gage installed on a metal surface, where reinforcement by the gage is negligible, the output of the gage can be expressed in the following general form:[6]

$$\frac{\Delta R}{R} = F_a\epsilon_a+F_t\epsilon_t \tag{1}$$

where: F_a,F_t = axial and transverse gage factors of strain gage
ϵ_a,ϵ_t = axial and transverse surface strains

When, on the other hand, the test material is low enough in elastic modulus that it is significantly reinforced by the gage, the strain transmitted to the gage grid differs from the unperturbed surface strain, and the gage output is altered correspondingly. The effect is modeled here, and in the preceding study for isotropic plastics, by introducing two additional variables into the expression for gage output:

$$\frac{\Delta R}{R} = F_a\lambda_a\epsilon_a+F_t\lambda_t\epsilon_t \tag{2}$$

where: λ_a,λ_t = strain-transmission coefficients

The coefficients λ_a and λ_t represent, respectively, the fractions of the surface strains ϵ_a and ϵ_t that are transmitted to the gage grid under reinforcement conditions. Alternatively, the products $F_a\lambda_a$ and $F_t\lambda_t$ can be viewed as the <u>effective</u> axial and transverse gage factors applicable to the same conditions. It is assumed that λ_a and λ_t are independent of the strain level, and are functions only of gage proportions and the ratio

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E_i/E_g, where E_i is the relevant elastic modulus of the test material, and E_g the "equivalent modulus" of the gage. In the case of a metal test member, with negligible reinforcement, both coefficients must closely approach unity in order that Eq. (2) effectively revert to Eq. (1). Judging from the experimental data for gages installed on plastics, the coefficients tend to decrease as some function of E_i/E_g, reflecting a reduction in strain transmitted to the gage grid as the elastic modulus of the test material becomes lower.[1,2] Although both λ_a and λ_t may be characterized by the same function, they are treated here as separate coefficients for the sake of generality.

Assume that a calibration specimen has been fabricated from a unidirectionally reinforced plastic as indicated in Fig. 2. Identical strain gages, aligned in the 1 and 2 directions, are installed on the specimen, which is then subjected to a uniaxial stress, σ_1. Although not drawn to scale in the illustration, the specimen cross section should be great enough to assure only local reinforcement effects by the gages.[5] In other words, the gage stiffness should be small enough compared to the overall section stiffness that perturbation of the strain field is confined to the immediate vicinity of the gage.

Applying the model of Eq. (2) to this calibration specimen, the output of the gage aligned in the 1 direction can be expressed as:

$$\left(\frac{\Delta R}{R}\right)_1^1 = F_a\lambda_{1a}\epsilon_1^1 + F_t\lambda_{1t}\epsilon_2^1 \tag{3}$$

where: $\left(\frac{\Delta R}{R}\right)_1^1$ = output of gage aligned in the 1 direction (subscript) due to uniaxial stress applied in the 1 direction (superscript).

$\lambda_{1a},\lambda_{1t}$ = axial and transverse strain-transmission coefficients for a gage oriented in the 1 direction.

$\epsilon_1^1,\epsilon_2^1$ = actual surface strains in the 1 and 2 directions (subscripts) due to uniaxial stress in the 1 direction (superscripts).

The relationship in Eq. (3) can be rendered more convenient for the present purposes if re-expressed in terms of the erroneous strain indicated by the gage under reinforcement conditions. Introducing the standard gage factor definition:

$$GF = \frac{\frac{\Delta R}{R}}{\epsilon}$$

and substituting into Eq. (3)

$$\hat{\epsilon}_1^1 = (F_a\lambda_{1a}\epsilon_1^1 + F_t\lambda_{1t}\epsilon_2^1)/GF \tag{4}$$

where: $\hat{\epsilon}_1^1$ = indicated strain in the 1 direction due to uniaxial stress applied in the 1 direction.

In accordance with the normal practice of gage manufacturers, the transverse gage factor F_t is replaced by K_tF_a, where K_t is defined as the "transverse sensitivity". Noting also that $\epsilon_2^1 = -\nu_{12}\epsilon_1^1$, Eq. (4) can be rewritten as:

$$\hat{\epsilon}_1^1 = F_a\epsilon_1^1(\lambda_{1a} - \nu_{12}K_t\lambda_{1t})/GF \tag{5}$$

Similarly, for the gage in the 2 direction,

$$\hat{\epsilon}_2^1 = F_a\epsilon_1^1(-\nu_{12}\lambda_{2a} + K_t\lambda_{2t})/GF \tag{6}$$

where: $\lambda_{2a},\lambda_{2t}$ = axial and transverse strain transmission coefficients for a gage oriented in the 2 direction.

The apparent major elastic modulus is then:

$$E_1^* = \frac{\sigma_1}{\hat{\epsilon}_1^1} = \frac{\sigma_1\cdot GF}{F_a\epsilon_1^1(\lambda_{1a} - \nu_{12}K_t\lambda_{1t})}$$

But, since $\sigma_1/\epsilon_1^1 = E_1$,

$$E_1^* = \frac{E_1\cdot GF}{F_a(\lambda_{1a} - \nu_{12}K_t\lambda_{1t})} \tag{7}$$

From Eqs. (5) and (6), the apparent major Poisson's ratio becomes:

$$\nu_{12}^{*} = \frac{-\hat{\epsilon}_2^1}{\hat{\epsilon}_1^1} = \frac{\nu_{12}\lambda_{2a}-K_t\lambda_{2t}}{\lambda_{1a}-\nu_{12}K_t\lambda_{1t}} \tag{8}$$

The calibration procedure can then be repeated (using the identical gage type) for uniaxial stress applied in the 2 direction as shown in Fig. 3. In this case, the indicated strain for the gage oriented in the 1 direction is:

$$\hat{\epsilon}_1^2 = F_a\epsilon_2^2(-\nu_{21}\lambda_{1a}+K_t\lambda_{1t})/GF \tag{9}$$

And that for the gage in the 2 direction becomes:

$$\hat{\epsilon}_2^2 = F_a\epsilon_2^2(\lambda_{2a}-\nu_{21}K_t\lambda_{2t})/GF \tag{10}$$

The apparent minor elastic modulus is then:

$$E_2^{*} = \frac{E_2\cdot GF}{F_a(\lambda_{2a}-\nu_{21}K_t\lambda_{2t})} \tag{11}$$

From Eqs. (9) and (10), the apparent minor Poisson's ratio is:

$$\nu_{21}^{*} = \frac{\nu_{21}\lambda_{1a}-K_t\lambda_{1t}}{\lambda_{2a}-\nu_{21}K_t\lambda_{2t}} \tag{12}$$

When strain measurements are subsequently made on actual test objects in an arbitrary strain field, with gages of the identical type aligned along the principal material directions, the indicated strains can be expressed as:

$$\hat{\epsilon}_x = F_a(\lambda_{1a}\epsilon_x+K_t\lambda_{1t}\epsilon_y)/GF \tag{13}$$

$$\hat{\epsilon}_y = F_a(\lambda_{2a}\epsilon_y+K_t\lambda_{2t}\epsilon_x)/GF \tag{14}$$

Note that subscripts x and y are used in Eqs. (13) and (14) to designate strains in the 1 and 2 directions, respectively, to avoid confusion with the previously used notation for the calibration strains in the same directions.

Assuming linear-elastic behavior of the test material, and writing the usual orthotropic normal-stress/normal-strain relationships in terms of the indicated strains and the apparent elastic properties:

$$\hat{\sigma}_x = \frac{E_1^{*}}{1-\nu_{12}^{*}\nu_{21}^{*}}(\hat{\epsilon}_x+\nu_{21}^{*}\hat{\epsilon}_y) \tag{15}$$

$$\hat{\sigma}_y = \frac{E_2^{*}}{1-\nu_{12}^{*}\nu_{21}^{*}}(\hat{\epsilon}_y+\nu_{12}^{*}\hat{\epsilon}_x) \tag{16}$$

After substituting Eqs. (7), (8), (11), (12), (13), and (14) into Eqs. (15) and (16), and reducing,

$$\hat{\sigma}_x = \frac{E_1}{1-\nu_{12}\nu_{21}}(\epsilon_x+\nu_{21}\epsilon_y) = \sigma_x \tag{17}$$

$$\hat{\sigma}_y = \frac{E_2}{1-\nu_{12}\nu_{21}}(\epsilon_y+\nu_{12}\epsilon_x) = \sigma_y \tag{18}$$

This result demonstrates that the reinforcement and transverse-sensitivity errors in the indicated strains are cancelled by the corresponding errors in the apparent elastic properties when normal strains are converted to normal stresses using Eqs. (15) and (16). Although it is common practice in orthotropic mechanics to use the products $\nu_{21}E_1$ and $\nu_{12}E_2$ interchangeably, the same relationship evidently does not hold for the apparent elastic properties. Since the product of Eqs. (7) and (12) is not equal to that of Eqs. (8) and (11) in this model of the reinforcement effect, it is necessary that Eqs. (15) and (16) be applied in the form shown to achieve error cancellation.

The method of compensation for reinforcement and transverse-sensitivity effects proposed here is based on the model generally expressed in Eq. (2). It implicitly assumes that mechanical interaction effects between gages in the 1 and 2 directions, if present, are the same for the calibration conditions as they are for strain measurement on a test part. To satisfy this condition, a tee rosette (with two grids, 90° apart) represents a repeatably convenient means for implementing the method in the compensation of indicated normal strains.

To fully establish the state of stress on the principal material planes, it is also necessary to determine the shear stress, which is related to the shear strain through the shear modulus:

$$\tau_{12} = G_{12}\gamma_{12} \quad (19)$$

Equation (19) presents a similar opportunity for cancellation of reinforcement and transverse-sensitivity errors by combining indicated strains with an apparent shear modulus.

The American Society for Testing and Materials (ASTM) has established a standard practice for measuring the shear modulus of a unidirectionally reinforced plastic with strain gages.[7,8] The ASTM standard calls for a calibration specimen in the form of a balanced, symmetric, ±45° laminate, fabricated from layers of the test material. A tensile specimen is then made from the laminate, and two strain gages are installed, as indicated in Fig. 4.

With this construction, the shear stress on the principal material planes is the same for each lamina in the laminate, and is equal to $\sigma_3/2$. Similarly, the shear strain is the same in every lamina. Ignoring reinforcement effects for the moment (as the ASTM standard does), it can be demonstrated that the difference in indicated strains from two gages with their axes 90° apart is equal to the shear strain along the bisector of those axes.[9] For the specimen and gage arrangement of Fig. 4, the bisector of the gage axes is a principal material axis, and thus,

$$\gamma_{12} = \epsilon_3 - \epsilon_4 \quad (20)$$

where: ϵ_3, ϵ_4 = strains parallel and perpendicular, respectively, to the longitudinal axis of the calibration specimen in Fig. 4.

Applying the previously used reinforcement model to express the indicated strain in the 3 direction on the calibration specimen,

$$\hat{\epsilon}_3 = (F_a\lambda_{3a}\epsilon_3 + F_t\lambda_{3t}\epsilon_4)/GF$$

where: $\lambda_{3a}, \lambda_{3t}$ = axial and transverse strain-transmission coefficients for a gage oriented in the 3 direction.

And substituting $F_t = K_tF_a$,

$$\hat{\epsilon}_3 = F_a(\lambda_{3a}\epsilon_3 + K_t\lambda_{3t}\epsilon_4)/GF \quad (21)$$

Similarly, for the indicated strain in the 4 direction,

$$\hat{\epsilon}_4 = F_a(\lambda_{4a}\epsilon_4 + K_t\lambda_{4t}\epsilon_3)/GF \quad (22)$$

From the mechanical symmetry of the gage environments, it can be assumed that $\lambda_{3a} = \lambda_{4a}$ and $\lambda_{3t} = \lambda_{4t}$. Thus, the indicated shear strain becomes:

$$\hat{\gamma}_{12c} = \hat{\epsilon}_3 - \hat{\epsilon}_4 = F_a(\epsilon_3 - \epsilon_4)(\lambda_{3a} - K_t\lambda_{3t})/GF \quad (23)$$

where: $\hat{\gamma}_{12c}$ = indicated shear strain on the principal material axes under calibration conditions.

The apparent shear modulus is then:

$$G^*_{12} = \frac{\tau_{12}}{\hat{\gamma}_{12c}} = \frac{\sigma_3 \cdot GF}{2F_a(\epsilon_3 - \epsilon_4)(\lambda_{3a} - K_t\lambda_{3t})}$$

But, $\sigma_3/2(\epsilon_3-\epsilon_4) = G_{12}$, the actual shear modulus of the material. Therefore,

$$G^*_{12} = \frac{G_{12} \cdot GF}{F_a(\lambda_{3a} - K_t\lambda_{3t})} \quad (24)$$

Subsequently, the same gage arrangement, with the identical gage type, is used to determine the shear strain on an actual test part in an arbitrary stress state. If the strains in the 3 and 4 directions are labeled ϵ_x and ϵ_y, respectively, the indicated shear strain on the principal material axes is:

$$\hat{\gamma}_{12} = \hat{\epsilon}_x - \hat{\epsilon}_y = F_a(\epsilon_x - \epsilon_y)(\lambda_{3a} - K_t\lambda_{3t})/GF \quad (25)$$

The indicated shear stress is calculated from:

$$\hat{\tau}_{12} = G^{*}_{12}\hat{\gamma}_{12} \qquad (26)$$

Substituting Eqs. (24) and (25) into Eq. (26) demonstrates that:

$$\hat{\tau}_{12} = G_{12}(\epsilon_x - \epsilon_y) = G_{12}\gamma_{12} = \tau_{12} \qquad (27)$$

Thus, the errors due to reinforcement and transverse sensitivity are canceled when the shear stress is calculated from the indicated shear strain and the apparent shear modulus as previously measured with the same type of strain gage.

A method has been described here for achieving compensation of reinforcement and transverse-sensitivity errors when making strain measurements on an orthotropic material such as a unidirectionally reinforced plastic. The method is applied separately to normal and shear strains to obtain the complete state of stress on the principal material axes. Although not expressly noted in the foregoing, this compensation procedure will also cancel a constant error in gage factor, if present. As a result, the gage factor control of the instrumentation can be set at any convenient value, as long as it is the same during properties calibration and experimental stress analysis.

In the practical implementation of this method, when compensating for reinforcement effects in both normal and shear strains, four strain gage grids are required -- two along the principal material axes, and the other two at $\pm 45^{\circ}$ from one of the axes. Accurate gage alignment is, of course, critical to the procedure. To eliminate possible secondary reinforcement effects of adjacent gages, the gage configuration should be the same in the calibration tests as it is during experimental stress analysis. In other words, if an array of four gage grids is used to determine the complete state of stress for experimental stress analysis purposes, the same array should probably be present during all calibrations for elastic properties, whether or not strain measurements are made with the superfluous grids. A further restriction on the physical arrangement of the array, or rosette, is that the two grids used for shear measurement should lie in a mechanically symmetric environment, so that they have the same axial and transverse strain-transmission coefficients ($\lambda_{3a} = \lambda_{4a}$ and $\lambda_{3t} = \lambda_{4t}$). One such arrangement is indicated schematically in Fig. 5.

It is worth noting that much of the published data on the elastic properties of unidirectionally reinforced plastics was measured with strain gages. Such being the case, these properties may already include, in varying degrees, errors due to gage reinforcement effects. When the same properties are employed in the data reduction of strain measurement (also containing reinforcement errors) for stress analysis purposes, at least partial compensation for the errors must occur by default. Considering the variability in gage stiffness from type to type, however, and pending the quantitative characterization of strain gage reinforcement effects, the method proposed here seems to offer improved accuracy in the experimental stress analysis of materials conforming to the reinforcement model.

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