

THE FUNDAMENTAL PRINCIPLES OF QUANTUM MECHANICS

With Elementary Applications

BY
EDWIN C. KEMBLE
Professor of Physics,
Harvard University

INTERNATIONAL SERIES IN PHYSICS

THE FUNDAMENTAL PRINCIPLES OF QUANTUM MECHANICS

With Elementary Applications

BY
EDWIN C. KEMBLE
Professor of Physics, Harvard University

FIRST EDITION

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK AND LONDON
1937

COPYRIGHT, 1937, BY THE
MCGRAW-HILL BOOK COMPANY, INC.

PRINTED IN THE UNITED STATES OF AMERICA

*All rights reserved. This book, or
parts thereof, may not be reproduced
in any form without permission of
the publishers.*

THE MAPLE PRESS COMPANY, YORK PA.

INTERNATIONAL SERIES IN PHYSICS

F. K. RICHTMYER, *Consulting Editor*

Bacher and Goudsmit—ATOMIC ENERGY STATES

Bitter—INTRODUCTION TO FERROMAGNETISM

Clark—APPLIED X-RAYS

Condon and Morse—QUANTUM MECHANICS

Curtis—ELECTRICAL MEASUREMENTS

Davey—CRYSTAL STRUCTURE AND ITS APPLICATIONS

Edwards—ANALYTIC AND VECTOR MECHANICS

Eldridge—THE PHYSICAL BASIS OF THINGS

Hardy and Perrin—THE PRINCIPLES OF OPTICS

Harnwell and Livingood—EXPERIMENTAL ATOMIC PHYSICS

Houston—PRINCIPLES OF MATHEMATICAL PHYSICS

Hughes and DuBridge—PHOTOELECTRIC PHENOMENA

Hund—HIGH-FREQUENCY MEASUREMENTS

PHENOMENA IN HIGH-FREQUENCY SYSTEMS

Kemble—THE FUNDAMENTAL PRINCIPLES OF QUANTUM MECHANICS

Koller—THE PHYSICS OF ELECTRON TUBES

Morse—VIBRATION AND SOUND

Pauling and Goudsmit—THE STRUCTURE OF LINE SPECTRA

Richtmyer—INTRODUCTION TO MODERN PHYSICS

Ruark and Urey—ATOMS, MOLECULES AND QUANTA

Slater and Frank—THEORETICAL PHYSICS

White—INTRODUCTION TO ATOMIC SPECTRA

Williams—MAGNETIC PHENOMENA

DEDICATED TO
DAYTON CLARENCE MILLER

PREFACE

This volume was originally intended to be an expansion of a summary of the elements of quantum mechanics written some years ago for the *Reviews of Modern Physics* by the author in collaboration with Professor E. L. Hill. The point of view is essentially the same as in the summary, but as the present work has grown in my hands it has lost most of its resemblance to the initial pattern.

The method of approach was dictated by the desire to meet the needs of graduate students of physics. For this reason the argument is inductive in form and applications of the theory have been interwoven with the development of the basic mathematical structure. In order to minimize the necessity for frequent consultation of mathematical reference books, a good deal of background mathematical material is included in Chap. IV.

In reading other treatises on quantum theory I have frequently been distressed by the tendency to gloss over the numerous mathematical uncertainties and pitfalls which abound in the subject. From the standpoint of the beginner there is much to be said for this practice of minimizing the defects of the theory in order to exhibit its main outlines in a compact and attractive form. Nevertheless it has seemed to me that a book which deliberately called attention to the weak spots in the argument would be of considerable value to teachers and to students of the more mature type. The work of the mathematician von Neumann provides a masterly antidote to the lack of rigor characteristic of the average physicist, but by common consent this work is too difficult for any but the most mathematical students of this subject. Hence I have been led to try my hand at bridging the gap between the exacting technique of von Neumann and the usual less rigorous formulations of the theory. In carrying this project through I have restricted the discussion to such elementary mathematical methods as are the common property of physicists today. The reader must judge my success in avoiding the Scylla of sloppy thinking and the Charybdis of tedious complexity. Fine print, starred sections, and appendices indicate portions of the material which may well be omitted or briefly scanned on first reading.

A feature of the present volume on the physical and philosophical side is its consistent emphasis on the operational point of view and on the fundamental importance of Gibbsian assemblages of independent systems in the physical interpretation of the mathematical formalism.

A considerable collection of references indicates the author's indebtedness to the ideas of others, but the list is by no means exhaustive. I have borrowed freely from other books and am particularly indebted to those of von Neumann, Dirac, and of Born and Jordan.

It is a pleasure to thank my colleagues and former colleagues, Dr. Eugene Feenberg, Dr. W. H. Furry, Professor J. C. Slater, and Professor J. H. Van Vleck for invaluable suggestions and generous assistance. I am particularly indebted to Professor Van Vleck for reading the entire manuscript and for his constant encouragement. Dr. Montgomery H. Johnson is responsible for much of the work on the continuous spectrum in Sec. 31, while Dr. Bela Lengyel and Dr. Charles H. Fay have at various times spent long hours in checking equations and other technical assistance. The author is very grateful to the librarian of the Harvard Physics Laboratory, Mrs. Miner T. Patton, for her cheerfulness and accuracy in the repeated typing of successive editions of the manuscript.

To the Milton Fund of Harvard University I am indebted for a generous grant for technical help in preparing the manuscript for the printer.

EDWIN C. KEMBLE.

PEACHAM, VERMONT,
August, 1937.

NOTATION

The number of different physical and mathematical quantities to be represented by separate symbols in this book is embarrassingly large in comparison with the available letters of the Roman and Greek alphabets. For this reason the establishment of a one-to-one correspondence of symbols and meanings has proved impracticable. The author has endeavored to keep the notation consistent within each chapter and, with a few exceptions which should not be confusing, has used only one symbol for each well-defined and recurrent meaning.

The following notes may be of use to the reader who attempts to dip into the middle of the book.

An asterisk * used as a superscript denotes the complex conjugate of the number or function in question.

Ordinarily the symbol Ψ denotes a time-dependent wave function, while ψ indicates the time-free space factor of a monochromatic or single-energy Ψ . At times ψ is also used for the instantaneous form of a general wave function.

Vectors are indicated by superior arrows.

Three-dimensional vector and scalar products are indicated by the conventional $\times$ and $\cdot$, e.g., $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$.

The scalar products of many-dimensional complex vectors and of functions are denoted by heavy parentheses, e.g.,

$$(\vec{A}, \vec{B}) = \sum_k A_k B_k^*,$$

$$(\psi(x), \varphi(x)) = \int \psi \varphi^* dx.$$

In Chap. IV the *norm* of a function f , viz., (f, f) , is indicated by Nf , while the *magnitude*, or square root, of the norm is indicated by $\|f\|$.

$\overline{\Sigma}\alpha'$ denotes a mixed process of summation and integration over all eigenvalue points in α' -space. Cf. p. 246.

Matrices are denoted by boldface type or by a typical element enclosed in double vertical rules. Thus,

$$\mathbf{H} = \|\mathbf{H}(m, n)\|.$$

The *first* of the two indices of the typical element of an ordinary two-dimensional matrix indicates the *row*, while the *second* denotes the *column*.

The Dirac notation for an eigenfunction of α in x' -space, viz., $(x'|\alpha')$, is introduced in Sec. 36*h*, while the Dirac notation for matrix elements, e.g., $(\beta''|\gamma|\beta')$, appears in Sec. 44*d*. The Dirac symbolism is employed only at points where it is particularly convenient.

REFERENCE ABBREVIATIONS

- D. P.* *Differentialgleichungen der Physik*, Riemann-Weber, Braunschweig, edition of 1927.
- E. Q.* *Elementare Quantenmechanik*, M. Born and P. Jordan, Berlin, 1930.
- M. G. Q.* *Mathematische Grundlagen der Quantenmechanik*, J. v. Neumann, Berlin, 1932.
- M. M. P.* *Methoden der Mathematischen Physik I*, R. Courant and D. Hilbert, Berlin, 2d ed., 1931.
- P. Q. M.* *The Principles of Quantum Mechanics*, P. A. M. Dirac, Oxford, 1st ed., 1930; 2d ed., 1935.
- Q. M.* *Quantum Mechanics*, E. U. Condon and P. M. Morse, New York, 1929.
- T. A. S.* *The Theory of Atomic Spectra*, E. U. Condon and G. H. Shortley, Oxford, 1935.

CONTENTS

	PAGE
PREFACE	vii
NOTATION	xvii
REFERENCE ABBREVIATIONS	xviii

CHAPTER I

INTRODUCTION TO DUALISTIC THEORY OF MATTER; DEVELOPMENT OF SCHRÖDINGER'S WAVE EQUATION.	1
1. Historical Introduction.	1
2. The Dualistic Theory of Radiation.	4
3. An Analogy between Geometrical Optics and Classical Mechanics.	7
4. Wave Packets and Group Velocity.	10
5. The Schrödinger Wave Equation for a Single Particle.	14
5a. The Time-free Equation	14
5b. The Second (General) Schrödinger Equation.	15
*6. The Application of the Restricted Relativity Principle.	19
7. The Wave Equation for a System of Many Particles.	21
7a. Formulation of Equation	21
7b. Relation of Schrödinger Equation to the Classical Hamiltonian Function	23
*7c. The Schrödinger Equation and the Hamilton-Jacobi Equation	24
*7d. The Wave Equation for a System of Charged Particles in a Classical External Electromagnetic Field	26
8. The Physical Interpretation of the Wave Function and the Normalization Condition.	29
8a. Probability and Quadratic Integrability.	29
8b. Normalization and Mass Current Density.	31
8c. A System Consisting of Two Independent Parts.	33

CHAPTER II

WAVE PACKETS AND THE RELATION BETWEEN CLASSICAL MECHANICS AND WAVE MECHANICS.	35
9. Wave Packets and Group Velocity in a One-dimensional Homogeneous Medium	35
9a. The Fourier Integral Theorem.	35
9b. Derivation of Group-velocity Formula	37
10. Wave Packets in Three Dimensions	41
*11. Wave Surfaces and the Hamilton-Jacobi Equation of the Classical Dynamics.	43
*12. Wave Packets and the Motion of Particles in a Force Field; Fermat's Principle.	46
13. Direct Rigorous Proof of Newton's Second Law of Motion for Wave Packets	49

* An asterisk before the number of a section or subsection indicates that the section or subsection so marked may be omitted or skimmed to advantage on first reading.

	PAGE
14. The Statistical Interpretation of the Wave Theory of Matter	51
14a. Review of Assumptions	51
14b. Necessity of Introducing Assemblages	53
14c. Multiplicity of Energy and Momentum Values for a Definite State	56
15. The Wave Function and Measurements of Linear Momentum	58
15a. Operational Definition of Momentum for Free Particles	58
15b. Computation of Momentum Probabilities from a Wave Function	60
*15c. Momentum of Center of Gravity	63
*15d. The Measurement of the Momenta of Particles Moving in a Force Field	66
*15e. Individual Momenta of Particles in a System	68
15f. Summary: The Determination of the Wave Function of an Assemblage	69
16. The Heisenberg Uncertainty Principle	72

CHAPTER III

ONE-DIMENSIONAL ENERGY-LEVEL PROBLEMS	78
17. Boundary and Continuity Conditions; Eigenvalues and Eigenfunctions	78
18. The One-Dimensional Anharmonic Oscillator	81
19. The Qualitative Behavior of the Integral Curves: Existence of Class A Eigenfunctions	82
19a. Behavior of Integral Curves in Regions of Positive and Negative Kinetic Energy	82
19b. The Discrete Eigenfunctions	84
19c. The Continuous Spectrum of Class B Eigenfunctions	85
19d. The Paradox of the Nodes	86
20. The Planck Ideal Linear Oscillator	87
20a. The Sommerfeld Polynomial Method	87
20b. Determination of the Eigenvalues	89
20c. The Eigenfunctions and Their Properties	89
21. An Approximation Method Which Correlates the Eigenvalues of Wave Mechanics with the Energy Levels of the Bohr Theory	90
21a. The B. W. K. Approximations for $\psi(x)$	90
21b. Application to Eigenvalue Problem	93
*21c. Zwaan's Method and the Stokes Phenomenon	95
*21d. Analysis of the Stokes Phenomenon	97
*21e. Derivation of the Connection Formulas	100
*21f. Derivation of the Sommerfeld Phase-integral Quantum Condition	103
*21g. Higher Approximations	107
*21h. Modification of Method for Radial Motion in Two-particle Problem	107
21i. Asymptotic Agreement of Wave Theory and Classical Theory Regarding Position of Particle	108
21j. The Transmission of Progressive Matter Waves through a Potential Hill	109

CHAPTER IV

THE MATHEMATICAL THEORY OF COMPLETE SYSTEMS OF ORTHOGONAL FUNCTIONS	113
22. Scalar Products and Systems of Orthogonal Functions	113
22a. Expansion in a Series of Functions	113
22b. Comparison of Properties of Vectors and Functions	114

CONTENTS

xi

	PAGE
22c. Scalar Products of Vectors	115
22d. Scalar Products of Quadratically Integrable Functions of m Variables	116
22e. Spaces of Infinitely Many Dimensions	119
22f. Proof of Orthogonality of Eigenfunctions of the One-dimensional Anharmonic Oscillator Problem	120
23. Self-adjoint Operators and Equations. The Sturm-Liouville Problem	121
23a. Self-adjoint Differential Operators in One Dimension	121
23b. Orthogonality with Respect to a Density Function ρ	123
23c. The Sturm-Liouville Problem	124
23d. Singular-point Boundary Conditions	125
*23e. Existence of Discrete Eigenvalues for Sturm-Liouville Problems with Singular End Points	128
24. Reduction of Eigenvalue Problems Based on Self-adjoint Differential Equations to Variational Form	130
25. Completeness of System of Discrete Eigenfunctions of a Sturm-Liouville Problem	132
*25a. The Eigenvalues as Absolute Minima	132
*25b. The Expansion of Arbitrary Functions in Terms of Eigenfunctions	135

CHAPTER V

THE DISCRETE ENERGY SPECTRUM OF THE TWO-PARTICLE CENTRAL-FIELD PROBLEM	140
26. The Behavior of Solutions of an Ordinary Second-order Differential Equation near a Singular Point	140
27. The Legendre Polynomials	143
27a. General Properties of the Legendre Equation	143
27b. Explicit Determination of Eigenvalues and Eigenfunctions	144
28. The Energy Levels of the Two-particle Problem	146
28a. The Wave Equation	146
28b. Separation of the Variables	146
28c. The Azimuthal Factor of the Wave Function	147
28d. Determination of $\Theta(\theta)$ and Its Eigenvalues	148
28e. Completeness of System of Eigenfunctions	149
28f. Physical Interpretation of Quantum Numbers l and m	150
28g. Behavior of Radial Wave Functions at Boundary Points	152
28h. The Dumbbell Model of the Diatomic Molecule	155
29. The Hydrogenic Atom	157
29a. Application of the B. W. K. Method	157
29b. Application of Polynomial Method	158
29c. Generalized Laguerre Polynomials	160
29d. The Most General Eigenfunction	161

CHAPTER VI

THE CONTINUOUS SPECTRUM AND THE BASIC PROPERTIES OF SOLUTIONS OF THE MANY-PARTICLE PROBLEM	162
30. The Continuous Spectrum in One-dimensional Problems	162
30a. The Nature and Use of the Eigenfunctions of the Continuous Spectrum	162
30b. The Weyl Theory	163
*30c. Formal Treatment of Continuous Spectrum as the Limit of a Discrete Spectrum	165
*30d. The Spacing of Energy Levels in Problems β and α	168

	Page
*30e. The Eigendifferentials	169
*30f. Passage from the Completeness Theorem for Problem β to That for Problem α	171
30g. The Fourier Integral Formulas a Special Case	173
30h. Normal Packet Functions in One Dimension	174
30i. Normal Packet Functions for the Two-particle Problem	174
*30j. Normalization of the Class B Radial Eigenfunctions for the Hydrogenic Atom	176
31. Weak Quantization. Theory of Radioactive Emission of Alpha Particles	178
31a. Weak Quantization in General	178
31b. A Model for Alpha Particle Disintegration	179
31c. Resonant Energy Intervals	181
31d. Energy Distribution in Weakly Quantized States	186
31e. The Disintegration Process	187
*31f. Complex Eigenvalues	192
32. The Existence and Properties of Solutions of the Many-particle Schrödinger Eigenvalue-eigenfunction Problem	195
32a. Introduction	195
32b. New Boundary Conditions for Physically Admissible Wave Functions	197
*32c. Approximating Arbitrary Quadratically Integrable Functions by Means of Class D Functions	201
32d. Hermitian Character of the Hamiltonian Operator	202
32e. Reduction of the Eigenvalue-eigenfunction Problem for Discrete Spectra to Variational Form	206
32f. A Lower Bound for the Energy Integral	207
*32g. Behavior of Solutions of the Differential Equation at Singular Domains	208
*32h. The Auger Effect	213
32i. The Discrete Eigenfunctions of the Differential Equation as Minimizing Functions	214
32j. The Continuous Spectrum and the Completeness of the System of Eigenfunctions	215
32k. Degeneracy	217

CHAPTER VII

DYNAMICAL VARIABLES AND OPERATORS	219
33. The Mean Values of the Cartesian Coordinates and Conjugate Linear Momenta	219
33a. The Statistical Mean Values of the Coordinates	219
33b. The Linear Momentum Operator	220
34. The Angular-momentum Operators	224
34a. Definition of Operators	224
34b. Hermitian Character of Angular-momentum Operators	225
34c. The Expansion Theorem	226
34d. Angular Momentum of a System of Particles	227
34e. Mean Values	229
34f. The Vector Angular Momentum and Its Square; the Symmetric Top	230
35. The Energy Operators	234
35a. Calculation of Probabilities and Mean Values of Energy	234
*35b. Transformation of Hamiltonian Operator	237
36. Dynamical Variables in General	240

CONTENTS

xiii

	PAGE
36a. Remarks on the Value of the General Theory	240
36b. Possibility of Defining Physical Quantities by Operators	242
36c. The Transformation of Probability Amplitudes and Dynamical Variables	245
36d. Type 1 Operators as Dynamical Variables	248
36e. Calculation of Probabilities	256
36f. Type 2 Operators as Dynamical Variables; the Method of von Neumann	259
36g. The Method of Dirac and Jordan	265
*36h. Multiplication Operators in Many Dimensions	268
36i. Transformation of Probability Amplitudes from One Arbitrary Coordinate Scheme to Another	270
36j. Dynamical Variables with Complex Eigenvalues	275

CHAPTER VIII

COMMUTATION RULES AND RELATED MATTERS	278
37. Simultaneous Eigenfunctions and the Commutation of Dynamical Variables	278
37a. Operator Algebra	278
37b. Functions of a Single Operator	279
37c. Commutative Operators	281
37d. Functions of a Normal Set of Commuting Dynamical Variables	286
38. The Conservation Laws	288
38a. Conservation of Energy	288
38b. Variation of Energy When the Hamiltonian Depends on the Time	289
38c. Conservation of an Arbitrary Dynamical Variable	290
38d. Commutation Properties of the Hamiltonian and the Angular Momentum	291
39. Conjugate Dynamical Variables and Quantum-mechanical Equations of Motion	293
39a. Conjugate Dynamical Variables	293
39b. Functions of Non-commuting Linear Operators	300
39c. An Operator Form of Hamilton's Equations of Motion	301
40. Symmetry Properties of the Wave Equation	303
40a. Symmetry Properties in General	303
40b. The Reflection Operators	305
40c. The Rotation Operator	306
40d. The Permutation Operators	308
40e. Degeneracy and the Integrals of the Schrödinger Equation	310
40f. The Normal Degeneracy of the Energy Levels of Free Atomic Systems	313

CHAPTER IX

THE MEASUREMENT OF DYNAMICAL VARIABLES	318
41. General Theory of Measurement	318
41a. Fundamental Characteristics of Measurements	318
41b. Pure States and Mixtures	320
41c. Postulates Regarding Retrospective and Predictive Measurements	322
41d. The Reduction of the Wave Packet	326
41e. Classical Orbits and Wave Packets	331
42. More About Measurements	334

	PAGE
42a. Conjugate Variables and Measurements.	334
42b. Impossibility of Measurements Which Imply Distinction between Particles of Same Species.	335
42c. A Classification of Observations	341
42d. Measurements as Correlations.	342
42e. The Observing Mechanism Not Entirely Classical	343
 CHAPTER X 	
MATRIX THEORY	348
43. Matrix Algebra	348
44. Matrices and Operators	352
44a. The Derivation of Matrices from Operators.	352
44b. Canonical Matrix Transformations.	355
44c. Matrix Form of the Eigenvalue-eigenfunction Problem.	359
*44d. Matrices with Continuous Elements	363
45. The Matrix Theory of Heisenberg, Born, and Jordan	366
45a. Fundamental Postulates	366
45b. Correlation of the Heisenberg and Schrödinger Theories	368
45c. Solution of Matrix Equations of Motion for an Ideal Linear Oscillator.	370
45d. Reduction of the Fundamental Problem of Matrix Mechanics to a Principal-axis Transformation	372
46. The Bohr Correspondence Principle and Its Relationship to Matrix Theory	374
46a. The Bohr Postulates	374
46b. The Bohr Correspondence Principle and the Heisenberg Matrix Theory.	375
 CHAPTER XI 	
THEORY OF PERTURBATIONS WHICH DO NOT INVOLVE THE TIME	380
47. The Perturbation Theory for Nondegenerate Problems.	380
47a. First-order Perturbations	380
47b. Second-order Perturbations	384
47c. An Example: The Diatomic Molecule.	386
48. The Perturbation Theory for Degenerate Problems	388
48a. First-order Energy Perturbations.	388
48b. Second-order Energy Perturbations.	391
*48c. Van Vleck's Method for Second-order Perturbations	394
48d. Simplification of Perturbation Calculations by Means of Integrals of the Perturbed Hamiltonian	396
49. The Energy Levels of an Hydrogenic Atom in a Uniform Magnetic Field (Spin Neglected)	398
49a. Derivation of Hamiltonian Operator	398
49b. Legitimacy of the Perturbation Method.	399
49c. First-order Energy Correction; Relation to Magnetic Moment and Larmor Precession	400
49d. The Second-order Energy Correction.	402
50. The Energy Levels of an Hydrogenic Atom in a Uniform Electric Field.	403
51. The Variational Method	408
51a. Reduction of the Variational Problem to Algebraic Form.	408
51b. The Ritz Method	410
*51c. Higher Roots of the Secular Equation	415
51d. General Observations Regarding the Use of the Variational Method	416

CONTENTS

XV
PAGE

51e. Modifications of the Method; Construction of Eigenfunctions from Non-orthogonal System.	418
52. The Problem of the Hydrogen Molecule	419
52a. The Fixed-nuclei Problem.	419
52b. The Heitler and London Calculation	420
52c. The Method of James and Coolidge	425

CHAPTER XII

QUANTUM STATISTICAL MECHANICS AND THE EINSTEIN TRANSITION PROBABILITIES	427
53. Quantum Statistical Mechanics	427
53a. The General Theory of Perturbations Which May Involve the Time	427
53b. The Adiabatic Theorem.	431
53c. The Fundamental Problems of Quantum Statistical Mechanics	432
53d. The Conventional Characterization of a Chaotic Assemblage	434
53e. Transition Probabilities and Statistical Equilibrium for Chaotic Assemblages.	439
53f. The Gibbs Canonical Assemblage for Systems of the Most General Type.	446
54. The Absorption and Emission of Radiation: Perturbation of an Atomic System by a Classical Radiation Field	448
54a. The Einstein Derivation of the Planck Radiation Formula	448
54b. Elementary Approaches to the Quantum Theory of the Einstein Transition Probabilities.	450
54c. The Perturbing Hamiltonian for a Classical Radiation Field	452
54d. The Born Transition Probability.	454
54e. The Einstein Transition Probabilities.	458
54f. Spectroscopic Stability.	462
*54g. Magnetic Dipole and Electric Quadrupole Radiation	462
55. Some Elementary Selection Rules for Electric Dipole Radiation.	469
55a. The Harmonic Oscillator	469
55b. Selection Rules for the Two-particle Problem	470
55c. Fine Structure and Polarization of Spectrum Lines in Simple Zeeman Effect	471

CHAPTER XIII

INTRODUCTION TO THE PROBLEM OF ATOMIC STRUCTURE: ELECTRON SPIN.	474
56. The Atomic Problem as a Two-particle Problem.	474
56a. The Empirical Basis for the Idealized Bohr Atom Model	474
56b. Derivation of the Ritz Formula	478
57. The Bohr Assignment of Electronic Quantum Numbers.	481
57a. The Quantum Numbers of the Valence Electrons in the Spectra of the Alkalies and Alkaline Earths.	481
57b. Perturbation Theory and the Significance of an Assignment of Quan- tum Numbers to Inner Electrons.	484
58. The Electron-spin Hypothesis.	491
58a. The Empirical Fine Structure of Spectrum Lines.	491
58b. The Combination of Angular Momenta.	495
58c. The Landé Magnetic Core Theory	498
58d. Solution of the Fine-structure Problem by the Electron-spin Hypo- thesis.	500
59. The Fine Structure of the Spectra of Atomic Systems with a Single Valence Electron	503

	Page
60. The Approximate Relativistic Theory of the Hydrogen Atom	507
61. The Pauli Wave-mechanical Formulation of the Theory of Electron Spin	510
61a. Nature of the Configuration Space and Wave Functions	510
61b. Preliminary Discussion of Spin Operators and Spin Matrices	512
61c. Application of the Pauli Theory to the Alkali Doublets	519
CHAPTER XIV	
THE THEORY OF THE STRUCTURE OF MANY-ELECTRON ATOMS	523
62. General Formulation of the Problem	523
62a. The Configuration Space	523
62b. The Hamiltonian Operator	524
62c. The Perturbation Form of the General Atomic Problem	526
63. Problem B: The Spin-orbit Energy Neglected	528
63a. Integrals of the Motion	528
63b. Antisymmetric Functions and the Empirical Pauli Exclusion Rule	533
63c. Closed Shells	536
63d. Terms Originating in a Given Configuration	537
64. Selection Rules for Electric Dipole Radiation	540
64a. The Laporte Rule	540
64b. Selection Rules for the Central-field Problem	541
64c. Selection Rules for Problems B and C	543
65. The Helium Atom and Exchange Energy	547
65a. Two-electron Atoms	547
65b. The Exchange Phenomenon	553
66. Diagonal Sums and the Problem B Energy Levels	555
APPENDICES	557
A. The Calculus of Variations and the Principle of Least Action	557
B. Derivation of Equation (15.7)	564
C. Theorems Regarding the Linear Oscillator Problem	568
D. Mathematical Notes on the B.W.K. Method	572
E. The Reduction of Certain Boundary-value Problems Based on Self-adjoint Differential Equations to Variational Form	579
F. The Legendre Polynomials and Associated Legendre Functions	583
G. The Generalized Laguerre Orthogonal Functions	585
H. Two Theorems Relating to the Continuous Spectrum	588
I. Concerning the Expansion of H_f in Spherical Harmonics	592
J. The Jacobi Polynomials	594
K. Schlapp's Method	596
NAME INDEX	599
SUBJECT INDEX	603