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FOREWORD

Substantial progress towards the demonstration of the scientific feasibility of controlled fusion and towards the design and construction of the first controlled thermonuclear power reactor is shown in the three volumes of these Proceedings of the Fifth IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research.

The Conference, held in Tokyo from 11 to 15 November 1974, was organized by the Agency with the assistance and cooperation of the Japanese Government and the Japan Atomic Energy Research Institute. Nearly 500 participants from 24 countries and three international organizations attended. A total of 187 papers was presented on research ranging from plasma physics to the design of fusion reactors. The papers are published here in the original language; English translations of the Russian papers will be published in a Special Supplement of the Nuclear Fusion Journal.

By regularly organizing conferences on controlled nuclear fusion and by holding seminars and specialists' meetings on selected topics the Agency promotes the close international collaboration among plasma physicists of all countries. These activities will, we hope, contribute to the rapid use of this new source of energy by mankind.

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by G.A. SAWYER as Rapporteur

Papers E 4-1 and E 4-2 (Belt pinch) were presented
by H. ZWICKER as Rapporteur

MHD-STABILITY OF THE SCYLLAC CONFIGURATION

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Abstract

MHD-STABILITY OF THE SCYLLAC CONFIGURATION.

The results of a stability analysis for a diffuse high-β, $\ell = 1$ helical configuration are presented. It is shown that there exists a gross m = 1 mode whose properties are quite similar to those predicted by the sharp-boundary model. In addition, two new classes of m = 1 modes are found, one localized on the inside of the plasma, the other one outside. For any monotonic pressure profile, these modes are unstable although their growth rates are very small. A further study suggests that small changes in the profile may stabilize these modes.

I. Introduction

The basic idea of the Scyllac configuration is to create a toroidal high β equilibrium possessing many of the desirable properties of linear θ pinches but eliminating the end loss. Toroidal equilibrium is achieved [1,2] by superposing a small ℓ=1 helical field and an even smaller ℓ=0 bumpy field on the basic θ pinch configuration.

With regard to stability the θ pinch field is essentially neutral. The ℓ=0 field required for equilibrium is so small that in many cases it can be neglected in the stability analysis. Consequently it is the ℓ=1 field which determines the stability of the system.

Most of the theory to date has been concerned with the m=1, k=0 mode of a straight, ℓ=1, helical system, as this appears to be the most important mode observed experimentally.[3,4] In addition, almost all of the calculations have treated the sharp boundary, constant pressure model. The results of these theories indicate that the m=1, k=0 instability is very subtle. Several different mathematical expansions have been devised and in each case instabilities, when they exist, occur as higher order effects in some appropriate expansion parameter.

The first of these theories,[5,6] referred to as the "old ordering", treats the case where the helical fields are assumed small but allows a finite helical period. In this case an instability arises basically due to the bad curvature associated with the ℓ=1 field. A second theory,[7] referred to as the "new ordering" assumes that the helical fields are finite but requires a long helical period. Here, instabilities can occur if the harmonic content of the ℓ=1 field is not properly adjusted. Finally a hybrid calculation[8] which includes the destabilizing effects of the ℓ=1 curvature and a particular harmonic content in the ℓ=1 field, indicates that the ℓ=1 curvature effect dominates in the range of current experimental parameters. This result is consistent with experimental measurements.[3,4]

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All of the sharp boundary models also predict a dipole wall stabilization term. In current experiments with highly compressed plasmas, this effect is negligible. It will become very important, however, in future experiments.[9]

Diffuse $\ell=1$ stability calculations[10,11] have been performed, but only using a very primitive ordering scheme which excludes all the known destabilizing terms. They do include the wall stabilization effect and the results are qualitatively similar to those of sharp boundary theory.

In the present calculation we examine the stability of a diffuse $\ell=1$ system using the old ordering expansion, since this theory appears to give rise to the most dangerous modes. Growth rates and eigenfunctions are calculated for a wide range of parameters. We find that there exists a gross mode which behaves qualitatively the same as the sharp boundary predictions, although the growth rates are somewhat lower. This mode can also be wall stabilized. In addition, for monotonic pressure profiles, two new classes of modes are found. One is localized on the inside of the plasma and the other on the outside. In general their growth rates are quite small. The exterior mode can be wall stabilized but the interior mode is present for any monotonic profile. A further study suggests that very slightly hollow profiles on the inside and flat profiles (i.e., zero θ pinch current) on the outside could eliminate the new classes of unstable modes.

II. Equilibrium

We consider the equilibrium of a diffuse high β helically symmetric system governed by the ideal MHD equations. Because of the helical symmetry, it is possible to describe the equilibrium quantities in terms of a single flux function, ψ,[12] which satisfies a partial differential equation of the form

$$\frac{1}{r}\frac{\partial}{\partial r}\frac{r}{1+h^2r^2}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} = \frac{2\,I}{(1+h^2r^2)^2} - \frac{1}{h^2}\left[\mu_0 p' + \frac{1}{2}\,\frac{(I^2)'}{1+h^2r^2}\right] \tag{1}$$

where $\phi = \theta + hz$ is the helical angle and $p(\psi)$, $I(\psi)$ are two arbitrary functions of ψ.

This equation is solved using the old Scyllac expansion: that is we write $\psi(r,\phi) = \psi_0(r) + \psi_1(r)\cos\phi$ ..... . It then follows that $\psi_1(r)$ must satisfy the following equation

$$\left(\frac{r\psi_1'}{1+h^2r^2}\right)' - \left[\frac{1}{r} + \frac{1}{B}\left(\frac{rB'}{1+h^2r^2}\right)'\right]\psi_1 = 0 \tag{2}$$

where $B(r) \equiv \psi_0'(r)/r$, the leading order θ pinch field, is an arbitrary function of r. The boundary conditions on $\psi_1(r)$ are

$$\begin{aligned}\psi_1(0) &= 0\\ \psi_1(b) &= \frac{2I_1'(hb)}{hb}\,\frac{B_\ell}{B_0}\end{aligned} \tag{3}$$

where I_1 is the modified Bessel function, h is the helical pitch number, B_0 is the amplitude of the applied θ pinch field far away from the plasma, b is the mean radius of the outer conducting shell and B_ℓ is the amplitude of the helical field[3] on axis without any plasma.

The magnetic fields are given in terms of ψ_1 as follows

$$
\begin{aligned}
B_r(r,\phi) &= B_r(r)\sin\phi\ , \qquad B_r(r) = \frac{h\psi_1}{r} \\
B_\theta(r,\phi) &= B_\theta(r)\cos\phi\ , \qquad B_\theta(r) = \frac{1}{1+h^2r^2}\left[\psi'_1 - \frac{B'}{B}\,\psi_1\right]
\end{aligned}
\tag{4}
$$

The results presented here were obtained by solving Eq. (2) numerically for the rigid rotor profile

$$
\begin{aligned}
B(r) &= B_0 \tanh\,(r^2/\gamma+\alpha) \\
\rho(r) &= \rho_0 \mathrm{sech}^2(r^2/\gamma+\alpha)/\mathrm{sech}^2\alpha
\end{aligned}
\tag{5}
$$

where $\rho(r)$ is the density, $\alpha = \tanh^{-1}\sqrt{1-\beta}$ (here β is measured on axis), ρ_0 is the density on axis, and $\gamma = a^2[1 + \sqrt{1-\beta}\,]$ with a the equivalent line density radius (i.e., line density $N = \pi\rho_0 a^2$).

III. Stability

The equilibrium just described was tested for stability in the ideal MHD model. By using a proper normalization, we are able to compute actual growth rates. This is important, because the simpler question of determining thresholds (i.e., marginal stability) is essentially impossible as a result of the complicated nature of the frequency spectrum.

Since the equilibrium is two dimensional, it is possible to Fourier analyze with respect to any independent (but not necessarily orthogonal) third coordinate. Thus all perturbed quantities can be assumed to vary as $\exp[i(m\theta+kz)]$, m and k arbitrary but k/m not commensurate with h. In this work we treat only m=1, because (1) m=1 appears to be the most dangerous mode experimentally (2) theory predicts that m≥2 modes, which are MHD unstable may well likely be stabilized by finite gyro radii effects.[13] In addition we treat the k=0 mode, since it can be shown theoretically[14] that this is the worst k. Under these conditions the m=1 eigenfunction has the form

$$
\begin{aligned}
\xi_r &= \xi\sin\theta + \eta_1\sin\theta\cos\phi + \eta_2\cos\theta\sin\phi \\
\xi_\theta &= (r\xi)'\cos\theta + \bar{\eta}_1\sin\theta\sin\phi + \bar{\eta}_2\cos\theta\cos\phi
\end{aligned}
\tag{6}
$$

Here ξ represents the basic m=1, k=0 displacement and η_1, η_2, $\bar{\eta}_1$, $\bar{\eta}_2$ are small sideband distortions, consistent with the old ordering, and which are required to produce instability. Note that there are two additional sidebands possible (with conjugate phase) for both ξ_r and ξ_θ. These terms are not included because they are always stabilizing.[14]

Equation (6) is substituted into the MHD energy principle with proper normalization and a minimization is performed with respect to $\bar{\eta}_1$ and $\bar{\eta}_2$. After a lengthy calculation, we arrive at the linearized equations of motion which can be written in the following convenient form

$$
\begin{aligned}
\underline{Z}'_1 &= \underline{Z}_2 \\
\left[(\underline{\underline{A}}-\omega^2\underline{\underline{D}})\cdot\underline{Z}_2\right]' &= (\underline{\underline{B}}-\omega^2\underline{\underline{E}})\cdot\underline{Z}_1 + \underline{\underline{C}}\cdot\underline{Z}_2
\end{aligned}
\tag{7}
$$

where $\underline{Z}_1$ and $\underline{Z}_2$ are the vectors

$$\underline{Z}_1 = \begin{pmatrix} \xi \\ \chi \\ \phi \end{pmatrix}, \qquad \underline{Z}_2 = \begin{pmatrix} \xi' \\ \chi' \\ \phi' \end{pmatrix}$$

with χ and ϕ related to ξ, η_1 and η_2 by

$$\chi = - hr(\eta_1+\eta_2)/2 + (B_r/B)r\xi'$$

$$\phi = hr(\eta_1-\eta_2)/2$$

The matrices $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, $\underline{\underline{D}}$, and $\underline{\underline{E}}$ are given by

$$\underline{\underline{A}} = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_2 & a_9 & 0 \\ a_3 & 0 & a_7 \end{pmatrix}, \quad \underline{\underline{B}} = \begin{pmatrix} 0 & -a_4' & -a_5' \\ 0 & a_8 & 0 \\ 0 & 0 & a_8 \end{pmatrix}$$

$$\underline{\underline{C}} = \begin{pmatrix} 0 & -a_4 & -a_5 \\ a_4 & 0 & 0 \\ a_5 & 0 & 0 \end{pmatrix}, \quad \underline{\underline{D}} = \begin{pmatrix} a_{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \underline{\underline{E}} = \begin{pmatrix} a_6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$a_1 = \frac{2r[B_r-B_\theta-h^2r^2B_\theta/2]^2}{4 + h^2r^2} + \frac{h^2r^3B_\theta^2}{2}, \qquad a_6 = - 2\omega^2r^2\rho$$

$$a_2 = \frac{2rB[B_r-B_\theta-h^2r^2B_\theta/2]}{4 + h^2r^2}, \qquad a_7 = 2B^2/h^2r$$

$$a_3 = rBB_\theta, \qquad a_8 = 2B^2/r$$

$$a_4 = B(B_r-B_\theta), \qquad a_9 = \frac{2rB^2}{4+h^2r^2}$$

$$a_5 = - B(B_r+B_\theta), \qquad a_{10} = 2\omega^2r^3\rho$$

The boundary conditions are $\underline{Z}_1(0) = 0$ and $\underline{Z}_1(b) = 0$ which corresponds to a perfectly conducting wall at a mean radius b. Note, that the plasma extends to the wall, and thus there is no vacuum region.