COSMICAL ELECTRO-DYNAMICS

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PREFACE

RECENT discoveries have revealed that electromagnetic phenomena are of greater importance in cosmic physics than used to be supposed. The time now seems to be ripe for an attempt to trace systematically the electromagnetic phenomena in the cosmos, and this is the reason for writing the present volume.

Cosmic physics is still in the stage where the most important task is to find out what are the dominating physical factors. Too many theories have been worked out with much mathematical skill on basic assumptions which were not physically tenable. Hence in this book the stress is always laid more on the physical than on the mathematical side. It is clearly understood that definite tests of any theory can be made only by means of rigorous mathematics, but the scope of this book is more to put the problems than to solve them.

The first four chapters are of fundamental character, the last three contain the applications. The reader is supposed to be familiar with the empirical results in this field. No attempt has been made to give an historical account of the development of the theories.

During a prolonged correspondence and many discussions Mr. Nicolai Herlofson has offered most valuable criticism from which I have profited. My thanks are also due to Mr. Stig Lundquist who has very kindly helped me with the preparation of the manuscript.

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I

GENERAL SURVEY

1.1. PHYSICS is mainly based on experience gained in the laboratory. When we try to apply to cosmic phenomena the laws in which this experience is condensed, we make an enormous extrapolation, the legitimacy of which can be checked only by comparing the theoretical results with observations. Classical mechanics was once extrapolated into the realm of astronomy so successfully that only the most refined observations of the last decades have revealed phenomena for which it does not hold. The application of atomic theory, especially spectroscopy, to cosmic phenomena has proved equally successful. In fact, classical mechanics and spectroscopy have been two invaluable tools in exploring the universe around us.

It seems very probable that electromagnetic phenomena will prove to be of great importance in cosmic physics. Electromagnetic phenomena are described by classical electrodynamics, which, however, for a deeper understanding must be combined with atomic physics. This combination is especially important for the phenomena occurring at the passage of current through gaseous conductors which are treated by the complicated theory of 'discharges' in gases. No definite reasons are known why it should not be possible to extrapolate the laboratory results in this field to cosmic physics. Certainly, from time to time, various phenomena have been thought to indicate that ordinary electrodynamic laws do not hold for cosmic problems. For example, the difficulty of accounting for the general magnetic fields of celestial bodies has led different authors, most recently Blackett (1947), to assume that the production of a magnetic field by the rotation of a massive body is governed by a new law of nature. If this is true, Maxwell's equations must be supplemented by a term which is of paramount importance in cosmic physics. The arguments in favour of a revision are still very weak. Thus it seems reasonable to maintain the generally accepted view that all common physical laws hold up to lengths of the order of the 'radius of the universe' and times of the order of the 'age of the universe', limits given by the theory of general relativity.

The discovery of sunspot magnetic fields (Hale, 1908) and later of the sun's general field (Hale, Seares, von Maanen, and Ellerman, 1918) has been of decisive importance to cosmic electrodynamics. More recently Babcock (1947) has shown that even stars possess strong magnetic fields. It may be said that if the sun and stars had no magnetic fields, electromagnetic phenomena would be of little importance to cosmic physics.

Celestial magnetic fields affect the motion of charged particles in space. Under certain conditions electromagnetic forces are much stronger than gravitation. In order to illustrate this, let us suppose that a particle moves at the earth's solar distance $R_♁$ with the earth's orbital velocity $v_♁$. If the particle is a neutral atom, it is acted upon only by the solar gravitation (the effect of the solar magnetic field upon an eventual atomic magnetic moment being negligible). If $M_\odot$ is the solar and m_A the atomic mass, and k is the constant of gravitation, this force is

$$f_A = kM_\odot m_A/R_♁^2.$$

If the atom becomes singly ionized, the ion as well as the electron (charge $= \pm e$) is subject to the force

$$\mathbf{f}_m = (e/c)[\mathbf{v}_♁ \mathbf{H}_♁]$$

from the solar magnetic field $\mathbf{H}_♁$. Under the assumption that this field is due to a dipole with the moment $a = 0{\cdot}42\,.\,10^{34}$ gauss cm.^{3}, we find $H_♁ = 1{\cdot}2\,.\,10^{-6}$ gauss. If m_A is the mass of a hydrogen atom it is easily found that

$$f_m/f_A \sim 10^5.$$

This illustrates the enormous importance of the solar magnetic field even at the earth's distance from the sun.

On the other hand, as f_m has opposite signs for electrons and for ions, in many cases the forces on electrons and ions may cancel each other. If we substitute for the particle an ionized cloud, containing the same number of electrons and ions, the resulting magnetic force on the cloud becomes zero to a first approximation. Second-order effects, e.g. due to the inhomogeneity of the magnetic field, may still be important. Further, the motion in the magnetic field produces a separation of the ions and electrons, but the resulting polarization causes an electric field which limits the separation. Under certain conditions the electric field may produce currents in adjacent conductors so that very complicated phenomena occur.

In the sun itself the magnetic field is of importance in several respects. In the outer layers, the chromosphere and the corona, the radius of curvature of the path of a charged particle with thermal velocity is smaller than the mean free path. Hence the magnetic field introduces an anisotropy, so that, for example, the electric conductivity is higher in the direction of the magnetic field than perpendicular to it (Cowling, 1932). In the photosphere, and in the sun's interior, the mean free path is small in comparison to the radius of curvature, which means that solar matter can be treated as an isotropic conductor. But even in this case an anisotropy is introduced by the fact that currents perpendicular to the magnetic field produce forces which accelerate the medium. A consequence of this is that magneto-hydrodynamic waves (see Chap. IV) move in the direction of the magnetic field.

The examples above demonstrate on the one hand the importance of electromagnetic forces in cosmic physics, and on the other the complexity of the electromagnetic phenomena. In our attempt to trace electromagnetic effects we shall start with a discussion of the magnetic and electric fields in cosmic physics. In Chapter II we shall treat the motion of a single particle in such fields. If several charged particles are present, forming an ionized gas, phenomena related to those studied in electric discharges are likely to occur. A survey of these phenomena is given in Chapter III. At densities so great that the ionized gas can be considered as an ordinary electrical conductor, the most important phenomenon in connexion with electromagnetic forces is probably that of magneto-hydrodynamic waves. These are treated in Chapter IV.

The results are applied to solar physics in Chapter V and to the theory of magnetic storms and aurora in Chapter VI. A discussion of the astrophysical aspect of cosmic radiation is given in Chapter VII.

It was originally intended to discuss an electromagnetic theory of the origin of the solar system (Alfvén, 1942, 1943, 1946) in an eighth chapter. This has been excluded, however, because it would require rather too much space.

It is a matter of judgement whether the physics of the ionosphere should be reckoned as cosmic electrodynamics or not. Certainly it has close connexions with, for example, the theory of the solar corona. On the other hand, it is still more closely related to the extensive field of the physics of the upper atmosphere. As even a superficial treatment of these problems would require too much space, the physics of the ionosphere has been excluded altogether.

1.2. Magnetic fields in cosmic physics

Every electric current, and what is equivalent to that, every magnet, gives a magnetic field which at great distances approximates to a dipole field. Hence in the absence of currents in the surroundings the fields of the earth and the sun are dipole fields at great distances. For the earth, and probably also for the sun, this approximation is rather close even at the surface, and hence everywhere above the surface.

A dipole with moment a situated at the origin and parallel to the z-axis

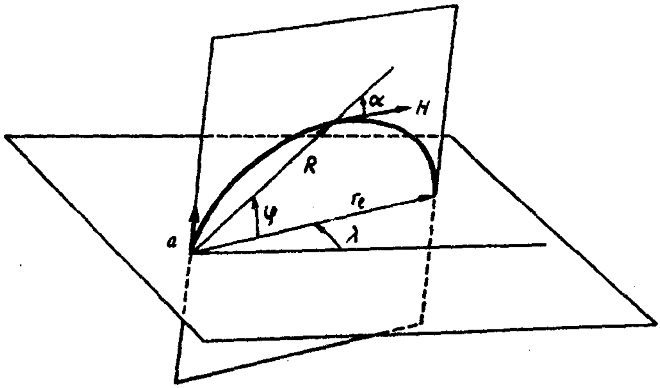

FIG. 1.1. Magnetic line of force from a dipole a.

gives a field, the components of which in a spherical coordinate system (R, φ, λ) are

$$H_R = H_p \sin\varphi, \tag{1}$$

$$H_\varphi = -\tfrac{1}{2}H_p \cos\varphi, \tag{2}$$

$$H_\lambda = 0,$$

$$H_p = 2a/R^3, \tag{3}$$

$$H = \sqrt{(H_R^2+H_\varphi^2+H_\lambda^2)} = a\phi/R^3 = \tfrac{1}{2}H_p\,\phi, \tag{4}$$

where
$$\phi = \sqrt{(1+3\sin^2\varphi)}. \tag{5}$$

H_R represents the 'vertical' and H_φ the 'horizontal' component of the field.† A magnetic line of force has the equation

$$R = r_e \cos^2\varphi, \tag{6}$$

$$\lambda = \text{const.},$$

where r_e is the distance from the origin of the point where it intersects the equatorial plane ($\varphi = 0$). The angle α between the line of force and the radius vector is given by

$$\tan\alpha = \tfrac{1}{2}\cot\varphi, \tag{7}$$

or
$$\sin\alpha = \frac{\cos\varphi}{\phi}, \tag{8}$$

$$\cos\alpha = \frac{2\sin\varphi}{\phi}. \tag{9}$$

The 'inclination' of the field is $\tfrac{1}{2}\pi-\alpha$.

† In terrestrial magnetism the 'vertical component' is counted positive if directed downwards.

The total strength of the field along a given line of force can also be written

$$H = \frac{a}{R^3}\phi = \frac{a}{r_e^3}(\cos\varphi)^{-6}\phi = \frac{a}{r_e^3}\eta, \tag{10}$$

where

$$\eta = \frac{\sqrt{(1+3\sin^2\varphi)}}{\cos^6\varphi}. \tag{11}$$

In a Cartesian system (x, y, z) we have

$$H_x = 3xz\frac{a}{R^5}, \tag{12}$$

$$H_y = 3yz\frac{a}{R^5}, \tag{13}$$

$$H_z = (3z^2-R^2)\frac{a}{R^5} \tag{14}$$

with $R^2 = x^2+y^2+z^2$.

If the *terrestrial field* is treated as the field from a dipole situated at the earth's centre, this dipole has the moment (see Chapman and Bartels, 1940, p. 645)

$$8\cdot1\,.\,10^{25}\ \text{gauss cm.}^3 \tag{15}$$

Its axis intersects the earth's surface in two antipodal points situated at latitude 78·5° S., longitude 111° E., and at latitude 78·5° N., longitude 69° W. The dipole moment (15) corresponds according to (3) to $H_p = 0{\cdot}63$ gauss.

A better approximation is obtainable if the condition that the dipole should be situated at the centre is dropped. The best agreement with the real field is obtained if the dipole is shifted 342 km. from the centre towards the point 6·5° N., 161·8° E. The axis of the eccentric dipole intersects the earth's surface at two points, 76·3° S., 121·2° E., and 80·1° N., 277·3° E.

The terrestrial field is subject to a slow (secular) variation. At present the magnetic moment seems to decrease by about 0·1 per cent. per year.

The *solar magnetic field* has been determined by measuring the Zeemann effect. The displacement of the sunspot zone (see § 5.31) and some other effects supply additional, although less direct, arguments for the existence of a general magnetic field. The properties of the field are discussed in § 5.22. The polar strength is likely to be about 25 gauss, corresponding to a dipole moment of

$$4\cdot2\,.\,10^{33}\ \text{gauss cm.}^3 \tag{16}$$

Because of the difficulty of exact measurements, this value may be in error by a factor 2, perhaps even more.

Sunspots are always associated with strong magnetic fields, as big as 4,000 gauss.

Stellar magnetic fields have been discovered by Babcock (1947) through Zeemann effect measurements. For 78 Virginis he finds a polar strength of 1,500 gauss corresponding to a moment of 4.10^{36} gauss cm.^{3}, and for the star BD 18°3789 (HD 1252 48) the field is no less than 5,500 gauss. The field of the latter object seems to be variable.

There are some arguments for the existence of a general *galactic magnetic field*. This problem is treated in § 7.5.

1.3. Induced electric field

In the presence of a magnetic field an electric field is defined only in relation to a certain coordinate system. If in a system 'at rest' the electric and magnetic fields are **E** and **H**, we can calculate by means of relativistic transformation formulae the fields **E**′, **H**′, in a system which moves in relation to the first with the velocity **v**. The components parallel to **v** remain unchanged, but the components perpendicular to **v** are transformed in the following way:

$$\mathbf{E}' = \frac{\mathbf{E}+c^{-1}[\mathbf{vB}]}{\sqrt{(1-c^{-2}v^2)}}, \tag{1}$$

$$\mathbf{H}' = \frac{\mathbf{H}-c^{-1}[\mathbf{vD}]}{\sqrt{(1-c^{-2}v^2)}} \tag{2}$$

($D = \epsilon E$, $B = \mu H$; reduced in a vacuum to $D = E$, $B = H$).

The astronomical velocities are much smaller than the velocity of light (c). Because of the good conductivity, electrostatic fields will usually be of little importance. Hence the electric fields are usually secondary to the magnetic fields, which, according to (1), means that the electric fields are much weaker than the magnetic fields. Consequently in cosmic physics we can usually to a good approximation write

$$\mathbf{E}' = \mathbf{E}+(1/c)[\mathbf{vH}], \tag{3}$$

$$\mathbf{H}' = \mathbf{H} \tag{4}$$

(where also the components parallel to v are included in the vectors).

Thus the magnetic fields are independent of the coordinate system, but *to speak of an electric field without defining exactly the coordinate system to which it refers is meaningless.*

These simple and fundamental principles seem to have attracted very little interest from astrophysicists and geophysicists. They are not very much to blame because the subject is omitted in most treatises on electromagnetism. Formulae (1) and (2) are found in books on the theory of relativity, e.g. Riemann–Weber (1927) and McCrea (1935).

The importance of the relativity of electric fields in cosmic physics is enormous. One of the consequences is that all celestial bodies with magnetic fields are on account of their rotation electrically polarized when seen from a system at rest. This phenomenon is well known from laboratory experiments and is usually called 'homopolar' or 'unipolar' induction. It was first studied by Faraday, and attracted much interest during the last century because it was thought that by investigating this subject it should be possible to ascertain whether the magnetic lines of force from a rotating magnet rotate with the magnet or not. At present the phenomenon seems to be half-forgotten, and most text-book authors do not mention it. A noteworthy exception is Cullwick (1939), who devotes a special appendix to it. It is also discussed by Becker (1933). Further, it should be mentioned that one of the best methods for absolute determination of the ohm employs a unipolar inductor (see Curtis, 1937). The device has also been developed electrotechnically as a direct current generator producing currents of thousands of amperes (see, for example, Arnold–la Cour, 1919).

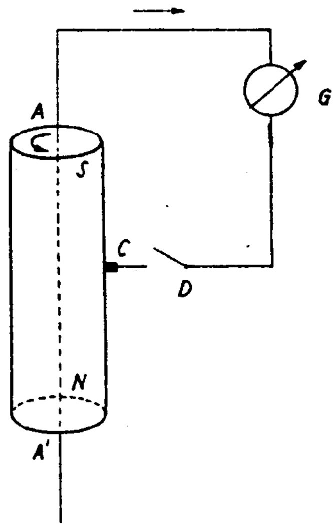

FIG. 1.2. Unipolar inductor. When a bar-magnet N–S is rotated around its axis AA' a current is obtained in a *fixed* circuit connecting the axis with a sliding contact at C.

A simple unipolar inductor is obtained by rotating a cylindrical bar-magnet N–S around its axis AA' (see Fig. 1.2). A fixed wire $AGDC$ connects the axis with a sliding contact C at the middle of the bar. If the switch D is closed the galvanometer G indicates a current as soon as the magnet rotates. The phenomenon can be treated *either* in a fixed system *or* in a rotating system. In the first case the magnetic lines of force are considered to be at rest *outside as well as inside* the magnet. The motion of the magnet produces a polarization inside it so that positive charge is accumulated near the axis and negative charge near the sliding contact. If the circuit is interrupted, this accumulation proceeds until the field from the charges neutralizes the polarization field, so that the resulting field E' becomes zero. This is necessary because the magnet is a conductor and when the current is zero the electric field seen from a system moving with a conductor must be zero. Then we have from (3)

$$\mathbf{E} = -(1/c)[\mathbf{vB}]. \tag{5}$$

The voltage difference between the sliding contact and the axis is

$$V = -(1/c) \int_A^C [\mathbf{vB}]\, \mathbf{ds}, \tag{6}$$

where **ds** is a line element.

If the switch is closed this voltage produces a current in the circuit.

This discussion is founded on the assumption that the magnetic field is 'at rest'. The problem can also be treated under the assumption that the lines of force take part in the rotation, and the result is the same. In this case no polarization is produced inside the magnet, but outside the magnet the wire $AGDC$ constantly cuts magnetic lines of force. Hence an e.m.f. is induced and it is easily shown that this has the value (6).

When we treat a problem in the rotating system we must observe that according to the general theory of relativity the electrodynamic equations for a rotating system do not have the usual form. In the presence of a magnetic field B the electric field deriving from a space charge ρ can be found from

$$4\pi\rho = \operatorname{div} E - (2/c)(\boldsymbol{\omega}\mathbf{B}),$$

where ω is the angular velocity.† Within a conductor we have $E = 0$, and the space charge is given by

$$\rho = -(\boldsymbol{\omega}\mathbf{B})/2\pi c.$$

The same result can also be obtained in the fixed system by taking the divergence of (5).

If the magnet is surrounded by an ideal insulator, we have outside the magnet $\rho = 0$. Hence, even in the rotating system, an electric field is produced. If, on the other hand, the surrounding medium has a conductivity which differs from zero, we have also $E = 0$ outside the magnet. In this case the magnetic lines of force may be considered as rotating with the magnet. As we have assumed $v \ll c$, the result does not, of course, hold for large distances from the axis.

It is not essential that the rotating body should be a permanent magnet. Any conductor will do if only a magnetic field is established in some way. In Fig. 1.3 a coil produces the magnetic field in which a copper disc rotates around the axis AA'. The e.m.f. is given by (6).

† I am indebted to Professor O. Klein for pointing this out to me.

It should be observed that if an instrument G' is placed on the disc, so that it takes part in the rotation and is connected between the axis and periphery, the voltage zero is read on this instrument.

After having discussed various types of earthly unipolar inductors we may be allowed to extrapolate the results to cosmic phenomena. It is obvious that the earth and the sun must be polarized in the same way as the bar-magnet or the copper disc. Let us consider the fields of these bodies as dipole fields with the magnetic axis coinciding with the rotational axis and neglect the non-uniform rotation of the sun. Because of the good electric conductivity the electrostatic potential must be the same at the poles as at the equator, *when measured in a system which takes part in the rotation*. Transforming to a system at rest (not partaking in the rotation) the bodies are electrically polarized according to (3). The electric field lies in the meridian plane and its horizontal component E_1 amounts to

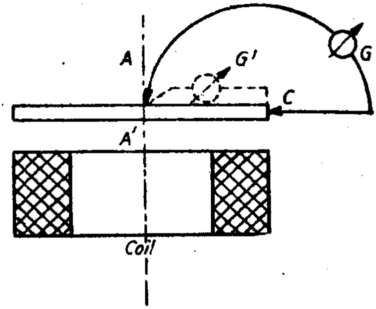

FIG. 1.3. Unipolar inductor, consisting of a rotating copper disc which is polarized by the field from a coil.

$$vH_R/c,$$

where H_R is the vertical component of the magnetic field. Putting

$$v = v_e \cos\varphi,$$

$$H_R = H_p \sin\varphi$$

(φ = latitude, v_e = equatorial velocity, H_p = polar field strength), we obtain

$$E_1 = \frac{v_e H_p}{2c} \sin 2\varphi. \qquad (7)$$

For the earth we have $v_e = 0{\cdot}5 . 10^5$ cm. sec.$^{-1}$, $H_p = 0{\cdot}6$ gauss. Hence we obtain for $\phi = 45°$, the field $E_1 = 0{\cdot}5 . 10^{-6}$ e.s.u. $= 150\ \mu$ volt cm.$^{-1}$ Integrating (7) from the equator to the pole, we find that the voltage difference between equator and pole is given by $V = \int E\,ds = 10^5$ volts if *measured from a system at rest.*

In the same way we find that *seen from a system at rest* there is a voltage difference between the equator and the poles of the sun of $1{\cdot}7 . 10^9$ volts. As in the case of the earth, the equator is negative in relation to both poles.

The surface charge of the rotating body produces an electric quadrupole field outside the body. In the case of a rotating sphere this field has been calculated by Davis (1947). If the body is surrounded by a conducting medium, the electric field is modified so that it becomes

$$E = vH/c,$$

with $v = r\omega$, which means that the surrounding medium tends to share the rotational state of the body.

The consequences of the unipolar action of celestial bodies will be discussed further in §§ 5.61 and 5.82.

Another example of unipolar induction is found in the solar atmosphere, where motions in the general magnetic field or sunspot fields may produce very large electromotive forces (see § 5.61).

When an ionized cloud moves in a magnetic field it becomes polarized according to (3). For example, the ionized clouds, which according to current ideas of the cause of magnetic storms are emitted from the sun (see § 6.1), are electrically neutral when seen from a coordinate system which moves with the cloud. When seen from the earth, which in this connexion may be considered as approximately at rest, they are electrically polarized (compare Becker, p. 336). As we shall see in Chapter VI, this field is probably of decisive importance in the theory of magnetic storms and aurorae.

For the production of electric fields according to (3) we use the term *polarization* or (with Cullwick) *motional induction*. Unipolar induction is one special case of this and the polarization of an ionized cloud another.

Electric fields may also be produced by a change in magnetic field. This type of induction is called by Cullwick *transformer induction*. The field may be calculated from Maxwell's equation

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

or from

$$\mathbf{E} = -\frac{d\mathbf{A}}{dt},$$

where $\mathbf{B} = \mu\mathbf{H}$ is the magnetic flux density and $\mathbf{A}$ the vector potential.

In cosmic physics large electromotive forces are produced either by motional induction or transformer induction. In special cases electrostatic fields must also be taken into consideration. Small voltage differences, due to diffusion, thermo-electric or electro-chemical effects, may in some cases be of importance.