LINEAR NETWORK DESIGN AND SYNTHESIS

By WAYNE H. CHEN

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Preface

This book and the author's "The Analysis of Linear Systems" were written in an attempt to present a unified analysis-synthesis sequence with uniform terminology and consistent viewpoint. Each book is self-contained and may be used independently of the other. A unique style of presentation is utilized in both books for clarity and easy reference. For example, as can be seen from the table of contents and the subtopics at all levels of the text, this book is organized so that the reader will see the relative importance of, and the proper relations between, topics.

The purpose of this book is to present the important methods for synthesizing an electrical network from a prescribed network characteristic or network function, or from a prescribed set of network parameters, with particular emphasis on underlying principles and certain design applications. To be able to synthesize a network, we must know whether the prescribed network characteristic, function, or parameters are realizable; if they are not, we must find a realizable "approximate" characteristic as a compromise. The conditions for physical realizability and approximation problems are discussed rather thoroughly here. Although some active networks are studied, this book is primarily devoted to the design and synthesis of passive, linear, lumped-constant networks. No attempt is made to discuss networks handling signals of a statistical nature.

Many network-synthesis methods are very complicated. To treat difficult synthesis problems which require solution rather than discussion, procedures consisting in clearly defined steps are formalized in this text and assigned section numbers or equation numbers for easy reference. They show the reader the methods for solution with minimum uncertainty and avoid wordy descriptions. However, these procedures are the results of treatment in the text; they are not merely stated as rules to be followed, as is the practice in handbooks. If a procedure is involved and contains earlier procedures in its individual steps, the reader is made aware of its genesis and detailed description through unmistakable references to the locations of the earlier procedures.

This text is divided into 10 parts. An introductory statement is included at the beginning of each part to indicate coverage and emphasis.

Part A includes the Introduction and a review of some network fundamentals.

Parts B and C constitute a study of two-terminal networks. Conditions for physical realizability are studied before the various synthesis methods are introduced. Emphasis is placed upon the principles underlying two-terminal network-synthesis methods, so that the reader will have a thorough understanding of these methods and may apply them later in four-terminal synthesis problems.

Parts D through H constitute a study of four-terminal networks. The conditions for physical realizability are discussed and are followed by a study of the methods of insertion-loss synthesis for modern filters and the methods of transfer-function synthesis with ladder, lattice, and parallel-ladder networks. Although the principles underlying these methods are emphasized, complete design procedures for modern filters of the Butterworth and Chebyshev types are studied and illustrated.

Parts I and J introduce the concept of potential analogy, enabling the reader to solve a network problem as an "analogous" potential problem. Some well-known approximation problems for modern filters are also studied.

Although primarily intended for use by graduate and advanced undergraduate students in electrical engineering and by practicing engineers, this book will also be found useful by applied mathematicians, control engineers, and system designers. Since the text is a comprehensive one, a section (Art. 1.3) is included in the Introduction to aid in the selection of reading material for use of this book as a textbook, a book for self-study, or a reference.

The material and method of presentation in this book have been classroom-tested at the University of Florida.

The preparation of this book has left the author indebted to many people. It is his pleasure to acknowledge the encouragement of Drs. J. H. Mulligan, F. E. Terman, W. W. Harman, and E. S. Kuh during the early and formative stages of the work; and to express his deep appreciation to Drs. W. W. Harman and J. G. Truxal for their valuable suggestions.

During the writing of the book, the author enjoyed the encouragement and assistance of his colleagues and graduate students at the University of Florida.

It is with much pleasure that he extends his thanks to Drs. M. E. Forsman and M. J. Larsen and Dean Joseph Weil for their continued encouragement and interest and to Drs. M. J. Larsen and W. E. Lear for many fruitful discussions about the manuscript.

The author is particularly indebted to Prof. R. C. Harden, Missouri School of Mines and Metallurgy, and Prof. J. L. Lowry, Auburn University, for their assistance while at the University of Florida. Professor Harden assisted in the preparation of the illustrations and proofread portions of the manuscript, and Professor Lowry read the complete manuscript and checked all derivations and examples for correctness; both have contributed in a variety of ways to the improvement of this work. The author is also appreciative of the excellent work of Mrs. Alice Harden, who, in a joint effort with the author's wife, typed the complete manuscript. Finally, to the many others who assisted in the preparation of this book, the author expresses his sincere appreciation.

The author's special thanks go to his wife, Dorothy, who has been in many ways an important partner in this venture.

Wayne H. Chen

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PART G: FOUR-TERMINAL NETWORKS: TRANSFER-FUNCTION SYNTHESIS WITH LADDER NETWORKS

part A Introduction and network fundamentals

Part A provides an introduction to the text and a review of some network fundamentals. The review is primarily intended to provide uniform notation and terminology throughout the book.

Since the text contains more material than is required for a one-semester course, a section (Art. 1.3) *is included in the Introduction to aid in the selection of reading material for use of this book as* (1) *a textbook,* (2) *a book for self-study, or* (3) *a reference.*

1 Introduction

The purpose of this book is to present the principles and methods of design and synthesis of passive networks. It is assumed that the reader is familiar with some of the elementary methods of network analysis.

But what are the problems of network analysis and synthesis?

1.1. Remarks about the problems of network analysis and synthesis

Problems of network analysis. We shall describe the problems of network analysis in this manner:

Network analysis:

$$[\textit{Network}] \xrightarrow[\text{find}]{\text{to}} \begin{bmatrix} \textit{Network} \\ \textit{function} \\ \text{or network} \\ \text{parameters} \end{bmatrix} \xrightarrow[\text{find}]{\text{to}} \begin{bmatrix} \textit{Network} \\ \textit{characteristics} \\ \text{or network} \\ \text{behavior} \end{bmatrix} \qquad (1.1)$$

Although "network function," "network parameters," and "network characteristics" are commonly used terms, we shall give their exact definitions and interpretations as a review in Chap. 2.

Problems of network synthesis. The problems of network synthesis are considered the converse of network-analysis problems; i.e.,

Network synthesis:

$$\begin{bmatrix} \textit{Network} \\ \textit{characteristics} \\ \text{or } \textit{specifications} \\ \text{in terms of} \\ \text{behavior} \end{bmatrix} \xrightarrow[\text{find}]{\text{to}} \begin{bmatrix} \textit{Network} \\ \textit{function} \\ \text{or network} \\ \text{parameters} \end{bmatrix} \xrightarrow[\text{find}]{\text{to}} [\textit{Network}] \qquad (1.2)$$

Questions of realizability and approximation. For the synthesis problems described in (1.2), we shall find it necessary to know the answers to the following questions:

Question 1. *Can we always realize a given network function or a given set of network parameters with a passive, linear, bilateral network?*

The answer to this question is "no." The given network function or given parameters must satisfy a set of *conditions for physical realizability.* These conditions will be studied in this text.

Question 2. *Is a given network characteristic (or a given specification) always realizable in the sense that we may obtain from it a realizable network function and a physical network?*

Again the answer is "no." This means that we must (1) compromise and find a realizable network characteristic which is a close approximation to the given non-realizable characteristic and (2) from this "approximated" realizable characteristic, obtain the realizable network function and the physical network. We shall also study *approximation problems* in this text.

Remarks about two-terminal and four-terminal networks. With reference to Fig. 1.1*c* and *d*, we note that (1) a two-terminal network may be a component of a four-terminal network and (2) a two-terminal network may be equivalent to a four-terminal network terminated in a load. This suggests that many a four-terminal network-synthesis problem is solved with the aid of two-terminal network-synthesis techniques. For example, (1) to synthesize the lattice arrangement in Fig. 1.1*c*, we actually

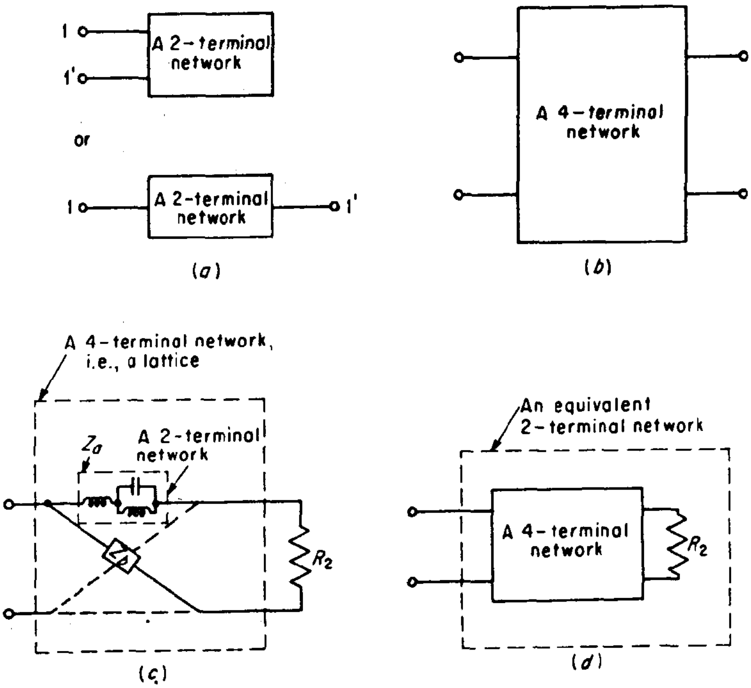

FIG. 1.1

synthesize Z_a and Z_b with two-terminal synthesis techniques, as will be studied in Chap. 17, and (2) realizing a modern filter terminated in a load as an "equivalent two-terminal network" as depicted in Fig. 1.1*d* is actually a part of Darlington synthesis, which will be studied in Chap. 12.

Figure 1.1*a* and *b* are standard representations for two-terminal and four-terminal networks, and are included to clarify the representations in Fig. 1.1*c* and *d*.

We shall consider two-terminal network synthesis as a category of problems in its own right, as well as the foundation for four-terminal network synthesis. We shall therefore study two-terminal network synthesis in detail in the earlier chapters of this book and then, with this background, investigate the problems of four-terminal network synthesis.

1.2. Remarks about the coverage of this book

A. Principal Subjects Covered in the Text

In view of the remarks in Art. 1.1, this book will cover the following principal subjects:

Coverage of this book:

a. Some network fundamentals which are essential to the study of network synthesis
b. Two-terminal network synthesis
c. Insertion-loss synthesis as a four-terminal synthesis approach, often referred to as "modern filter synthesis"
d. Transfer-function synthesis as another four-terminal synthesis approach
e. Approximation problems

Topics *b* through *e* are often considered to be the areas of study in *modern network synthesis;* in this sense, we may consider this book as one on modern network synthesis.

To cover these principal subjects adequately, this book is divided into 10 parts (Parts A through J). With reference to the table of contents, the descriptive title of each part indicates its own coverage and the general area to which it belongs.

B. Remarks about the Selection and Presentation of the Subject Matter

During the planning and writing of this book, attempts have been made to achieve the following:

1. To treat only those network problems which have direct or indirect practical application. For example, we study (1) two-terminal network synthesis (Chaps. 3 through 9) as a problem in its own right, as well as the foundation for four-terminal problems, (2) four-terminal network synthesis (Chaps. 10 through 18), including both insertion-loss and transfer-function synthesis methods, important for their applications in communication and control systems, and (3) potential-analogy and approximation problems (Chaps. 19 through 24), which find application in both network analysis and network synthesis.

The methods of synthesis with networks whose only asset is novel form are not studied.

2. To treat the "complete problem" in addition to the realization techniques. Too often a student knows how to synthesize a network function with a network, but not where the network function comes from.

In addition to the realization techniques, this book treats the "complete synthesis problem" as briefly described in (1.2). In the example of the modern filter, (1) the approximation problem in Chap. 24 "approximates" the network specification with a realizable network characteristic, and (2) this network characteristic is then realized with a network arrangement by the method of Darlington synthesis (Chap. 12). Design procedure and illustrations are provided in Chap. 13.

3. To develop methods for solving synthesis problems under several possible operating conditions rather than under a special condition. A student who knows how to synthesize an *RC* parallel-ladder network with an open-circuit transfer voltage-ratio function $G^*(s)$ may not know how to synthesize an *RC* parallel-ladder network with a transfer admittance $Y_T(s)$.

In order to solve synthesis problems under all possible operating conditions, we (1) define *eight* transfer functions of the response/excitation type in Eqs. (10.1), (2) study conditions for physical realizability in Chap. 10 for *all* of them, and (3) develop synthesis techniques with *LC*, *RC*, *RL*, and *RLC* ladder networks in Chaps. 14 to 16, and with lattice networks in Chap. 17, for *all* of them. Only three transfer functions $Y_T(s)$, $G(s)$, and $G^*(s)$ may be realized with parallel-ladder networks, and their synthesis techniques are studied in Chap. 18.

4. To stress the underlying principles of the various methods. Although synthesis and design procedures are important from the practical viewpoint, the principles behind these procedures are even more important and should not be obscured by the