Calculus of One Variable

SECOND EDITION

Stanley I. Grossman

CALCULUS OF ONE VARIABLE

SECOND EDITION

STANLEY I. GROSSMAN
University of Montana

Academic Press, Inc.

(Harcourt Brace Jovanovich, Publishers)
Orlando San Diego San Francisco New York
London Toronto Montreal Sydney Tokyo São Paulo

TO KERSTIN, ERIK, AND AARON

Academic Press, Inc.
Orlando, Florida 32887

United Kingdom edition published by
Academic Press, Inc. (London) Ltd.
24/28 Oval Road, London NW1 7DX

ISBN: 0-12-304390-5
Library of Congress Catalog Card Number: 85-71541
Printed in the United States of America

Preface

The study of calculus has been of central importance to scientists throughout most of recorded history. Our present understanding of the subject owes much to Archimedes of Syracuse (287–212 B.C.), who developed what were, for his time, incredibly ingenious techniques for calculating the areas enclosed by a great variety of curves. Archimedes' work led directly to the modern concept of the integral. Much later, toward the end of the seventeenth century, Sir Isaac Newton and Gottfried Leibniz independently showed how to calculate instantaneous rates of change and slopes to curves. This development inspired an enormous variety of mathematical techniques and theorems that could be used to solve problems in a number of diverse and often unrelated fields.

Many of these techniques and theorems are considered to be part of "the calculus." No textbook could discuss all the myriad results in even one-variable calculus discovered over the past several hundred years. Fortunately, a weeding out process has occurred, and there is now fairly widespread agreement as to what topics properly should be included in a two-semester (or three-quarter) introduction to the subject. This book includes all the standard topics. In the process of writing it, I have worked to achieve certain goals that make the text unique.

EXAMPLES

As a student, I learned calculus from seeing examples and doing exercises. *Calculus of One Variable, Second Edition,* contains 673 examples—many more than commonly found in standard one-variable calculus texts. Each example includes all the algebraic steps needed to complete the solution. As a student, I was infuriated by statements like "it now easily follows that . . . " when it was not at all easy for me. Students have a right to see the "whole hand," so to speak, so that they always know how to get from "a" to "b." In many instances, explanations are highlighted in color to make a step easier to follow.

EXERCISES

The text includes approximately 4800 exercises—including both drill and applied-type problems. More difficult problems are marked with an asterisk (*) and a few especially difficult ones are marked with a double asterisk (**). The exercises provide the most important learning tool in any undergraduate mathematics textbook. I stress to my students that no matter how well they think they understand my lectures or the textbook, they do not really know the material until they have worked problems. A vast difference exists between understanding someone else's solution and solving a new problem by yourself. Learning mathematics without doing problems is about as easy as learning to ski without going to the slopes.

CHAPTER REVIEW EXERCISES

At the end of each chapter, I have provided a collection of review exercises. Any student who can do these exercises can feel confident that he or she understands the material in the chapter.

APPLICATIONS

Calculus *is* applied mathematics. Consequently, this book includes many applied examples and exercises. Many calculus books draw examples exclusively from the physical sciences, even though calculus today is also used in the biological sciences, the social sciences, economics, and business. Thus, the examples and problems in this book, while including a great number from the physical sciences, also cover a wide range of other fields. Moreover, I have included "real-world" data wherever possible to make the examples more meaningful. For example, students are asked to find the escape velocity from Mars, the effective interest rate of a large installment purchase, and the optimal branching angle between two blood vessels. Finally, as most of the world uses the metric system and even the United States is reluctantly following suit, the majority of the applied examples and problems in the book make use of metric units.

OPTIONAL, LONGER APPLICATIONS

By necessity, many applications in a calculus text are short and, as a result, sometimes seem contrived. To solve this problem, I have included a variety of optional sections that discuss important applications in greater detail. There are extensive physical applications in Section 5.9 (Work, Power, and Energy), 7.3 (Periodic Motion), 9.6 (Moments of Inertia and Kinetic Energy), and 9.7 (Fluid Pressure). Two sections, Sections 4.6 and 6.7, contain applications in economics. Finally, new to this edition, is a section (6.8) that describes models of epidemics. In this section, students can learn about a topic that is much discussed in current research in mathematical biology.

REVIEW CHAPTER

Chapter 1 includes general discussions of several topics that are commonly taught in an intermediate algebra–college algebra course, including absolute value and inequalities, circles and lines, and an introduction to functions. This chapter also includes some detailed discussions of graphing techniques, including a unique section on shifting the graphs of functions—an extremely useful procedure.

INTUITION VERSUS RIGOR

Intuition rather than rigor is stressed in the early parts of the book. For example, in the introduction to the limit in Chapter 2, the student's intuition is appealed to in the initial discussion of limits, and the concept is introduced by means of examples, together with detailed tables. I believe that the "ϵ-δ" or "neighborhood" approach to limits can be appreciated only after some feeling for the limit has been developed. However, I also feel strongly that standard definitions must be included, since mathematics depends on rigorous, unambiguous proofs. To resolve this apparent contradiction, I have put much of the one-variable theory in separate optional sections (Sections 2.8, 5.10, and Appendix 5). These sections can be included at any stage, as the instructor sees fit, or omitted without loss of continuity if time is a problem. By the time students reach infinite series, they should have developed enough mathematical sophistication to understand and appreciate some of the subtleties of mathematical proof. Thus, in the discussion of convergence of sequences and series in Chapter 14, I have included ϵ's and δ's whenever necessary.

EARLY TRIGONOMETRY

The derivatives of all six trigonometric functions are computed in Section 3.5. Equally important, all six are then used in applications throughout the rest of the book—not just $\sin x$ and $\cos x$. See, for example, the application to blood flow in Example 4.6.10 (on page 246) that makes use of the derivatives of $\csc x$ and $\cot x$.

TRIGONOMETRY REVIEW

High school trigonometry is, of course, a prerequisite for calculus. Many students, however, come to calculus without having seen trigonometry for several years. Much has been forgotten. To fill this gap and to free the instructor from the necessity of taking class time to review trigonometry, I have included a four-section appendix (Appendix 1), which contains all the precalculus trigonometry a student needs.

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

I have introduced an element of choice in the way in which exponential and logarithm functions are first presented. In Section 6.2 a detailed introduction (without calculus) to exponential and logarithmic functions is provided. These functions are also introduced, in Section 6.4, via the definition of the natural logarithm function as an integral. This gives the instructor the option of choosing the introduction more suitable for his or her students, or both approaches may be covered to illustrate interesting relationships among seemingly disparate concepts in mathematics.

USE OF THE HAND-HELD CALCULATOR

Today, most students who study calculus own or have access to a hand-held calculator. As well as being useful in solving computational problems, a calculator can be employed as a learning device. For example, a student will develop a feeling for limits more quickly if he or she can literally *see* the limit being approached. Chapter 2 contains numerous tables that make use of a calculator to illustrate particular limits. At several places in the text a calculator has been used for illustrative purposes. Section 4.8 includes a discussion of Newton's method for finding roots of equations. This method is quite easy to employ with the aid of a calculator. In Section 6.7 Newton's method is used to estimate effective interest rates.

Examples, problems, and sections employing a hand calculator are marked with the symbol . Note, however, that the availability of the calculator is *not* a prerequisite for use of this text. The vast majority of problems do not require a calculator and the illustrative tables can be appreciated without independent verification. I recommend that the student who does wish to use a calculator find a calculator with function keys for the three basic trigonometric functions (sin, cos, tan), common (log) and natural (ln) logarithm keys, and an exponential key (usually denoted y^x), to allow easy calculation of powers and roots; in addition, a memory unit (so that numbers can be stored for easy retrieval) is a useful feature.

BIOGRAPHICAL SKETCHES

Mathematics becomes more interesting if one knows something about the historical development of the subject. I try to convince my students that, contrary to what they may believe, many great mathematicians lived interesting and often controversial lives. Thus, to make the subject more interesting and, perhaps, more fun, I have included a number of full-page biographical sketches of mathematicians who helped develop the calculus. In these sketches students will learn about the wonderful inventiveness of Archimedes, the dispute between Newton and Leibniz, the unproven theorem of Fermat, the reactionary behavior of Cauchy, and the love life of Lagrange. It is my hope that these notes will bring the subject to life.

ANSWERS AND OTHER AIDS

The answers to most odd-numbered exercises appear at the back of the book. In addition, a student's manual containing detailed solutions to all odd-numbered problems is available. Richard Lane at the University of Montana prepared the students' manual. Also, Leon Gerber at St. John's University in New York City prepared an instructor's manual containing detailed solutions to all even-numbered problems.

COMPUTER SUPPLEMENT

As many instructors will want to have their students use a computer in conjunction with their calculus courses, a supplement entitled *Computing for Calculus* has been prepared. This supplement, written by Mark Christensen at Georgia Institute of Technology, includes an introduction to BASIC and programs for implementing the numerical techniques (Newton's method, numerical integration) discussed in the text. It also contains a section on computer graphics.

NUMBERING AND NOTATION IN THE TEXT

Numbering in the book is fairly standard. Within each section, examples, problems, theorems, and equations are numbered consecutively, starting with 1. Reference to an example, problem, theorem, or equation outside the section in which it appears is by chapter, section, and number. Thus, Example 4 in Section 2.3 is called, simply, Example 4 in that section but outside the section is referred to as Example 2.3.4. As already mentioned, the more difficult problems are marked (*) or occasionally (**), and problems where the use of a calculator is advisable are marked . Sections that are more difficult and can be omitted without loss of continuity and sections that contain specialized applications are labeled "optional." Finally, the ends of examples and proofs of theorems are marked with a ■.

CHANGES IN THE SECOND EDITION

There have been many large and small changes in this edition. A number of reviewers suggested many ways to improve the clarity of the text. As a result of their suggestions, I made literally hundreds of changes in the examples, exercises, theorems, and explanations. The major changes include the following:

- The differentiation and subsequent use of all six trigonometric functions in Chapter 3. Some of the material previously found in Chapter 7 ("The Trigonometric Functions") now appears in Chapters 3, 4, 5, and 6.
- A rearrangement of Chapter 5 ("The Integral") so that antiderivatives are now discussed before definite integrals.
- A rearrangement of Chapter 6 so that inverse functions in general are discussed before exponential and logarithmic functions.
- A discussion of epidemic models in Section 6.8.
- The statement and proof of a uniqueness theorem for Taylor polynomials in Section 13.2. This section contains results that justify procedures for simplifying the computation of certain Taylor polynomials. This important material is absent from most elementary calculus texts.

- A paring of extraneous text and examples. While examples are an extremely important part of any mathematics text, redundant examples make the text harder to teach from. With the help of reviewers, I have cut approximately 10 percent of the examples. I also deleted explanatory material that was considered redundant. The result is a text that, I trust, remains a book for the student but covers the material in a more streamlined fashion.
- Detailed biographical sketches of mathematicians who were important in the development of the calculus.

ACCURACY

The success of a calculus book depends, to a large extent, on its accuracy. A few badly placed typographical errors can turn a good teaching tool into a source of confusion.

Every book has errors. This one has some too (although I'd love to know, as I write this, exactly where they are). Academic Press, however, has gone to considerable lengths to ensure that the book is as error free as possible.

Approximately twenty mathematics professors read portions of my original manuscript. They found a number of errors, which were corrected before the book was typeset. The checking of galley proofs, however, is the most important step in the process of finding and correcting errors. The galleys were checked in the following ways:

1. I read each set of galleys twice.
2. A proofreader compared the galleys to the original manuscript, looking for any discrepancies between the two.
3. A team of two faculty members at a community college read through the galleys, checking both mathematical accuracy and adherence to the manuscript.
4. A team consisting of a faculty member at a university in Florida and a graduate student read the galleys of the textual material.
5. A second team consisting of a faculty member and a graduate student read only the problem sets in the galleys.
6. A faculty member in New York, who prepared the instructor's manual, checked the problem sets for accuracy.

Some of these actions were repeated when the galleys were corrected and turned into page proofs. The result is a book that, while probably not error free, is as clean as could reasonably be expected in its first printing.

ACKNOWLEDGMENTS

Since the first edition of *Calculus* appeared in 1977, I have received a large number of helpful comments and suggestions from readers. The following people contributed constructive comments on the first edition: Boyd Benson, Rio Hondo College; Mike Burke, College of San Mateo; Ernest Fandreyer, Fitchburg State College; Bo Green, Abilene Christian University; Floyd F. Helton, College of the Pacific; David Hughes, Abilene Christian University; B. William Irlbeck, Xavier University of Louisiana; Raymond Knodel, Bemidji State University; Jane E. Lewenthal, College of the Pacific; David C. Mayne, Anne Arundel Community College; Charles Miers, University of Victoria; William F. Moss, Georgia Institute of Technology; Rosalie M. O'Mahony, College of San Mateo; Vincent G. Ryan, St. Joseph's College; Ruben D. Schwieger, Sterling College; and John F. Weiler, University of Wisconsin-Stevens Point.

In addition, reviewers of the first edition of this text included Don Gallagher, Central Oregon State University; David Hansen, Monterey Peninsula College; David Horowitz, Golden West College; John Jewett, Oklahoma State University; Beverly Marshman, University of Waterloo; Alexander Nagel, University of Wisconsin-Madison; Raymond C. Roan, Washington State University; William Rundell, Texas A&M University; Wayne Roberts, Macalester University; Thomas Schwartzbauer, Ohio State University; Houston H. Stokes, University of Illinois-Chicago Circle; and Professor Fredric I. Davis of the United States Naval Academy, who provided the computer sketches in Figures 11.3.4 and 11.3.5.

The following reviewers read all or part of the second edition manuscript and made many very useful suggestions: Duane W. Bailey, Amherst College; Paul F. Baum, Brown University; Don Bellairs, Grossmont College; Neil Berger, University of Illinois-Chicago Circle; Carl Cowen, Purdue University; Daniel Drucker, Wayne State University; Nat Grossman, University of California, Los Angeles; Howard Hamilton, California State University at Sacramento; Robert Lohman, Kent State University; James Maxwell, Oklahoma State University; Frank D. Pederson, Southern Illinois University at Carbondale; Thomas Schaffter, University of Nevada; Lindsey A. Skinner, University of Wisconsin-Milwaukee; Martin Sternstein, Ithaca College; William Ray Wilson, Central Piedmont Community College. The mathematical accuracy was checked by Howard Sherwood, Gwen Sherwood, Michael Taylor, and Kathy Burnett of the University of Central Florida, and by George Coyne and James D. Lange of Valencia Community College.

I am grateful to Charles Bryan at the University of Montana, who suggested the need for the uniqueness theorem for Taylor polynomials that now appears in Section 13.2.

I wish to thank the Addison-Wesley Publishing Company, Inc., for permission to use some material from my book (with William R. Derrick) *Elementary Differential Equations with Applications*, Second Edition, in Chapters 6 and 7. I am especially grateful to Leon Gerber at St. John's University in New York City, and Richard Lane of the University of Montana. Professor Gerber prepared the *Instructor's Manual*. In addition, he made an astonishing number of insightful suggestions that, I am confident, have made this a better book both for students and their instructors. Mr. Lane provided a large number of interesting problems for the first edition of this text. These problems, many of which appear in this edition, will challenge students to explore some additional implications of the mathematics they are studying.

Finally, I want to thank the editorial staff of Academic Press for providing much needed help and encouragement during the preparation of this second edition.

Stanley I. Grossman

To the Instructor

As an aid to the instructor, the following table indicates the interrelationships of the various parts of the text.

Chapter	Chapter Dependence	Comments
1	—	Section 1.8 provides extremely useful techniques for elementary curve sketching.
2	Chapter 1	Derivatives are first introduced as a rate of change in Section 2.1 with the analytic definition postponed until Section 2.5. Mathematically rigorous (i.e., $\epsilon-\delta$ arguments) definitions of limits are given in Section 2.8. This section is optional.
3	Chapters 1 and 2 (except Section 2.8)	After this chapter students should be able to differentiate any algebraic function as well as the six trigonometric functions.
4	Chapters 1, 2, and 3 (except Section 2.8)	This chapter provides applications of the derivative. Theorems relating to curve sketching depend on the mean value theorem proved in Section 4.2. The first and second derivative tests are treated separately in Sections 4.3 and 4.4, but the two sections together provide a rather complete procedure for sketching a curve. Section 4.8 on Newton's method makes extensive use of the hand calculator.
5	Chapters 1–4	The introduction to the integral. Antiderivatives are discussed in Section 5.2. The Σ notation is discussed in Section 5.3. The two fundamental theorems of calculus are given in Section 5.6. A mean value theorem for integrals and a rigorous proof of the second fundamental theorem of calculus are given in the optional Section 5.10. Section 5.9 is also optional.

Chapter	Chapter Dependence	Comments
6	Section 6.2 is algebra and depends only on Sections 1.7 and 1.8. The rest of the chapter depends on Chapters 1–3, 5, and parts of Chapter 4	Exponential and logarithmic functions are introduced algebraically in Section 6.2 and their derivatives and integrals are computed in Section 6.3. In Section 6.4 ln x is defined as $\int_1^x (1/t)dt$, making use of the second fundamental theorem of calculus. Then, from this definition, the basic properties of logarithmic and exponential functions are derived. If Sections 6.2 and 6.3 are covered, then Section 6.4 can be omitted, and conversely. Or, all three sections can be covered to show students how two seemingly unrelated ideas are really two sides of the same coin. Section 6.6 introduces the simplest differential equations and should be covered in courses with science majors. Section 6.7 is optional but should be covered in courses with economics and business majors. Section 6.8 contains an optional discussion of epidemic models.
7	This chapter presupposes the material in Appendix 1, the review of elementary trigonometry. Sections 7.1–7.3 require the material in Chapters 1–3, 5, and parts of Chapter 4, while Sections 7.4 and 7.5 also depend on Chapter 6	Section 7.3 is optional but should be covered in courses with science majors. Section 7.5 on the inverse hyperbolic functions should be covered if time permits, but is not used in other parts of the text.
8	Chapters 1–3, 5, 6, and 7	Section 8.6 describes the integration of rational functions with linear and quadratic denominators. Section 8.7 extends these results and can be omitted if time is a problem. Section 8.9 on using the integral tables should be covered. This is undoubtedly the most used technique for integration "in the field."
9	Chapters 1–3, 5–8, and parts of Chapter 4	Each section presents a particular geometric or physical concept and is optional.
10	Chapter 1 and Appendix 1	This chapter provides extensive descriptions of the conic sections and the techniques of translation and rotation of axes. No calculus is needed in the chapter.
11	Chapters 1–3, 5, 6, 7, and parts of Chapter 8	Section 11.5 is optional.
12	Chapters 1–3, 5, 6, 7, and parts of Chapter 8	Section 12.2 can be omitted without loss of continuity.
13	Chapters 1–3, 5, 6, 7, and parts of Chapters 8 and 12	Section 13.1 introduces Taylor's theorem as an approximation technique before a discussion of infinite series. Section 13.2 contains an important uniqueness theorem. Section 13.3 makes use of the calculator.

Chapter	Chapter Dependence	Comments
14	Chapters 1–3, 5, 6, 7, and parts of Chapters 8, 12, and 13	All sections in this chapter should be covered.

Contents

7 MORE ON TRIGONOMETRIC FUNCTIONS AND THE HYPERBOLIC FUNCTIONS 426

8 TECHNIQUES OF INTEGRATION 462

9 FURTHER APPLICATIONS OF THE DEFINITE INTEGRAL 527

10 TOPICS IN ANALYTIC GEOMETRY 577