JØRGEN BANG-JENSEN AND GREGORY GUTIN

Digraphs: Theory, Algorithms and Applications

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Digraphs

Theory, Algorithms and Applications

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We dedicate this book to our parents, especially to our fathers, Børge Bang-Jensen and the late Mikhail Gutin, who, through their very broad knowledge, stimulated our interest in science enormously.

Preface

Graph theory is a very popular area of discrete mathematics with not only numerous theoretical developments, but also countless applications to practical problems. As a research area, graph theory is still relatively young, but it is maturing rapidly with many deep results having been discovered over the last couple of decades.

The theory of graphs can be roughly partitioned into two branches: the areas of undirected graphs and directed graphs (digraphs). Even though both areas have numerous important applications, for various reasons, undirected graphs have been studied much more extensively than directed graphs. One of the reasons is that undirected graphs form in a sense a special class of directed graphs (symmetric digraphs) and hence problems that can be formulated for both directed and undirected graphs are often easier for the latter. Another reason is that, unlike for the case of undirected graphs, for which there are several important books covering both classical and recent results, no previous book covers more than a small fraction of the results obtained on digraphs within the last 25 years. Typically, digraphs are considered only in one chapter or by a few elementary results scattered throughout the book.

Despite all this, the theory of directed graphs has developed enormously within the last three decades. There is an extensive literature on digraphs (more than 3000 papers). Many of these papers contain, not only interesting theoretical results, but also important algorithms as well as applications. This clearly indicates a real necessity for a book, covering not only the basics on digraphs, but also deeper, theoretical as well as algorithmic, results and applications.

The present book is an attempt to fill this huge gap in the literature and may be considered as a handbook on the subject. It starts at a level that can be understood by readers with only a basic knowledge in university mathematics and goes all the way up to the latest research results in several areas (including connectivity, orientations of graphs, submodular flows, paths and cycles in digraphs, generalizations of tournaments and generalizations of digraphs). The book contains more than 700 exercises and a number of applications as well as sections on highly applicable subjects. Due to the fact that we wish to address different groups of readers (advanced undergraduate and graduate students, researchers in discrete mathematics and researchers in various areas including computer science, operations research, artificial intelligence, social sciences and engineering) not all topics will be equally interesting to all potential readers. However, we strongly believe that all readers will find a number of topics of special interest to them.

Even though this book should not be seen as an encyclopedia on directed graphs, we included as many interesting results as possible. The book contains a considerable number of proofs, illustrating various approaches and techniques used in digraph theory and algorithms.

One of the main features of this book is the strong emphasis on algorithms. This is something which is regrettably omitted in some books on graphs. Algorithms on (directed) graphs often play an important role in problems arising in several areas, including computer science and operations research. Secondly, many problems on (directed) graphs are inherently algorithmic. Hence, whenever possible we give constructive proofs of the results in the book. From these proofs one can very often extract an efficient algorithm for the problem studied. Even though we describe many algorithms, partly due to space limitations, we do not supply all the details necessary in order to implement these algorithms. The later (often highly non-trivial step) is a science in itself and we refer the reader to books on data structures.

Another important feature is the vast number of exercises which not only help the reader to train his or her understanding of the material, but also complements the results introduced in the text by covering even more material. Attempting these exercises (most of which appear in a book for the first time) will help the reader to master the subject and its main techniques.

Through its broad coverage and the exercises, stretching from easy to quite difficult, the book will be useful for courses on subjects such as (di)graph theory, combinatorial optimization and graph algorithms. Furthermore, it can be used for more focused courses on topics such as flows, cycles and connectivity. The book contains a large number of illustrations. This will help the reader to understand otherwise difficult concepts and proofs.

To facilitate the use of this book as a reference book and as a graduate textbook, we have added comprehensive symbol and subject indexes. It is our hope that the detailed subject index will help many readers to find what they are looking for without having to read through whole chapters. In particular, there are entries for open problems and conjectures. Every class of digraphs which is studied in the book has its own entry containing the majority of pages on which this class is treated. As sub-entries to the entry 'proof techniques' we have indexed different proof techniques and some representative pages where the technique is illustrated.

Due to our experience, we think that the book will be a useful teaching and reference resource for several decades to come.

Highlights

In this book we cover the majority of the important topics on digraphs ranging from quite elementary to very advanced ones. Below we give a brief outline of some of the main highlights of this book. Readers who are looking for more detailed information are advised to consult the list of contents or the subject index at the end of the book.

Chapter 1 contains most of the terminology and notation used in this book as well several basic results. These are not only used frequently in other chapters, but also serve as illustrations of digraph concepts. Furthermore, several applications of directed graphs are based on these elementary results. One such application is provided in the last section of the chapter. Basic concepts on algorithms and complexity can also be found in the chapter. Due to the comprehensive subject and notation indices, it is by no means necessary to read the whole chapter before moving on to other chapters.

Chapters 2 and 3 cover distances and flows in networks. Although the basic concepts of these two topics are elementary, both theoretical and algorithmic aspects of distances in digraphs as well as flows in networks are of great importance, due to their high applicability to other problems on digraphs and large number of practical applications, in particular, as a powerful modeling tool.

We start with the shortest path problem and a collection of classical algorithms for distances in weighted and unweighted digraphs. The main part of Chapter 2 is devoted to minimization and maximization of distance parameters in digraphs. We conclude the chapter by the following applications: the one-way street problem, the gossip problem and exponential neighbourhood local search, a new approach to find near optimal solutions to combinatorial optimization problems.

In Chapter 3 we cover basic, as well as some more advanced topics on flows in networks. These include several algorithms for the maximum flow problem, feasible flows and circulations, minimum cost flows in networks and applications of flows. We also illustrate the primal-dual algorithm approach for linear programming by applying it to the transportation problem. Although there are several comprehensive books on flows, we believe that our fairly short and yet quite detailed account of the topic will give the majority of readers sufficient knowledge of the area. The reader who masters the techniques described in this chapter will be well equipped for solving many problems arising in practice.

Chapter 4 is devoted to describing several important classes of directed graphs, such as line digraphs, the de Bruin and Kautz digraphs, series-parallel digraphs, generalizations of tournaments and planar digraphs. We concentrate on characterization, recognition and decomposition of these classes. Many properties of these classes are studied in more detail in the rest of the book.

In Chapter 5 we give a detailed account of results concerning the existence of hamiltonian paths and cycles in digraphs as well as some extensions to spanning collections of paths and cycles, in particular, the Gallai-Millgram theorem and Yeo's irreducible cycle factor theorem. We give a series of necessary conditions for hamiltonicity which 'converges' to hamiltonicity. Many results of this chapter deal with generalizations of tournaments. The reader will see that several of these much larger classes of digraphs share various nice properties with tournaments. In particular the hamiltonian path and cycle problems are polynomially solvable for most of these classes. The chapter illustrates various methods (such as the multi-insertion technique) for proving hamiltonicity.

In Chapter 6 we describe a number of interesting topics related to hamiltonicity. These include hamiltonian paths with prescribed end-vertices, pancyclicity, orientations of hamiltonian paths and cycles in tournaments and the problem of finding a strong spanning subdigraph of minimum size in a strong digraph. We cover one of the main ingredients in a recent proof by Havet and Thomassé of Rosenfeld's conjecture on orientations of hamiltonian paths in tournaments and outline a polynomial algorithm for finding a hamiltonian path with prescribed end-vertices in a tournament. We conclude the chapter with a brief introduction of a new approach to approximation algorithms, domination analysis. We illustrate this approach by applying results on hamiltonian cycles in digraphs to the travelling salesman problem.

Connectivity in (di)graphs is a very important topic. It contains numerous deep and beautiful results and has applications to other areas of graph theory and mathematics in general. It has various applications to other areas of research as well. We give a comprehensive account of connectivity topics in Chapters 7, 8 and 9 which deal with global connectivity issues, orientations of graphs and local connectivities, respectively.

Chapter 7 starts from basic topics such as ear-decompositions and the fundamental Menger's theorem and then moves on to advanced topics such as connectivity augmentation, properties of minimally k-(arc)-strong digraphs, highly connected orientations of digraphs and packing directed cuts in digraphs. We describe the splitting technique due to Mader and Lovász and illustrate its usefulness by giving an algorithm, due to Frank, for finding a minimum cardinality set of new arcs whose addition to a digraph D increases its arc-strong connectivity to a prescribed number. We illustrate a recent application due to Cheriyan and Thurimella of Mader's results on minimally k-(arc)-strong digraphs to the problem of finding a small certificate for k-(arc)-strong connectivity. Many of the proofs in the chapter illustrate the important proof technique based on the submodularity of degree functions in digraphs.

Chapter 8 covers important aspects of orientations of undirected and mixed graphs. These include underlying graphs of certain classes of digraphs. Nowhere zero integer flows, a special case of flows, related to (edge-)colourings of undirected graphs is discussed along with Tutte's 5-flow conjecture, which is one of the main open problems in graph theory. The famous theorem by Nash-Williams on orientations preserving a high degree of arc-strong connectivity is described and the weak version dealing with uniform arc-strong connectivities is proved using splitting techniques. Submodular flows form a powerful generalization of circulations in networks. We introduce submodular flows and illustrate how to use this tool to obtain (algorithmic) proofs of many important results in graph theory (including the Lucchesi-Younger theorem and the uniform version of Nash-Williams' orientation theorem). Finally we describe in detail an application, due to Frank, of submodular flows to the problem of orienting a mixed graph in order to maintain a prescribed degree of arc-strong connectivity.

Chapter 9 deals with problems concerning (arc-)disjoint paths and trees in digraphs. We prove that the 2-path problem is $\mathcal{NP}$-complete for arbitrary digraphs, but polynomially solvable for acyclic digraphs. Linkings in planar digraphs, eulerian digraphs as well as several generalizations of tournaments are discussed. Edmonds' theorem on arc-disjoint branchings is proved and several applications of this important result are described. The problem of finding a minimum cost out-branching in a weighted digraph generalizes the minimum spanning tree problem. We describe an extension, due to Frank, of Fulkerson's two-phase greedy algorithm for finding such a branching.

Chapter 10 describes results on (generally) non-hamiltonian cycles in digraphs. We cover cycle spaces, polynomial algorithms to find paths and cycles of 'logarithmic' length, disjoint cycles and feedback sets, including a scheme of a solution of Younger's conjecture by Reed, Robertson, Seymour and Thomas, applications of cycles in digraphs to Markov chains and the even cycle problem, including Thomassen's even cycle theorem. We also cover short cycles in multipartite tournaments, the girth of a digraph, chords of cycles and Ádám's conjecture. The chapter features various proof techniques including several algebraic, algorithmic, combinatorial and probabilistic methods.

Digraphs may be generalized in at least two different ways, by considering edge-coloured graphs or by considering directed hypergraphs. Alternating cycles in 2-edge-coloured graphs generalize the concept of cycles in bipartite digraphs. Certain results on cycles in bipartite digraphs, such as the characterization of hamiltonian bipartite tournaments, are special cases of results for edge-coloured complete graphs. There are useful implications in the other direction as well. In particular, using results on hamiltonian cycles in bipartite tournaments, we prove a characterization of those 2-edge-coloured complete graphs which have an alternating hamiltonian cycle. We describe an application of alternating hamiltonian cycles to a problem in genetics. Generalizations of the classical theorems by Camion, Landau and Rédei to hypertournaments are described.

Chapter 12 contains some topics that were not covered in other chapters. These include: an elementary proof of Seymour's second neighbourhood conjecture in the case of tournaments, various types of orderings of the vertices of digraphs of paired comparisons, kernels, a recent proof by Galvin of the Dinitz conjecture on list colourings using kernels in digraphs, and homomorphisms (an elegant generalization of colouring and also a useful vehicle for studying the borderline between $\mathcal{P}$ and $\mathcal{NP}$-complete problems). We describe basic concepts on matroids as well as questions related to the efficiency of matroid algorithms. We give a brief account on simulated annealing, a broadly applicable meta-heuristic which can be used to obtain near optimal solutions to optimization problems, in particular, on digraphs. We discuss briefly how to implement and tune simulated annealing algorithms so that they may produce good solutions.

Technical remarks

We have tried to rank exercises according to their expected difficulty. Marks range from (−) to (++) in order of increasing difficulty. The majority of exercises have no mark, indicating that they are of moderate difficulty. An exercise marked (−) requires not much more than the understanding of the main definitions and results. A (+) exercise requires a non-trivial idea, or involves substantial work. Finally, the few exercises which are ranked (++) require several deep ideas. Inevitably, this labelling is subjective and some readers may not agree with this ranking in certain cases. Some exercises have a header in bold face, which means that they cover an important or useful result not discussed in the text in detail.

We use the symbol □ to denote the end of a proof, or to indicate that either no proof will be given or is left as an exercise.

A few sections of the book require some basic knowledge of linear programming, in particular the duality theorem. A few others require basic knowledge of probability theory.

We would be grateful to receive comments on the book. They may be sent to us by email to jbj@imada.sdu.dk. We plan to have a web page containing information about misprints and other information about the book, see

http://www.imada.sdu.dk/Research/Digraphs/

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Odense, Denmark
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August 2000

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