

INTERNATIONAL SERIES  
IN  
PURE AND APPLIED PHYSICS

# INTRODUCTION TO THEORETICAL PHYSICS

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The late F. K. Richtmyer was Consulting Editor of the series from its inception in 1929 to his death in 1939. Lee A. DuBridge was Consulting Editor from 1939 to 1946; and G. P. Harnwell from 1947 to 1954.

## PREFACE

The general plan of a book is often clearer if one knows how it came to be written. This book started from two separate sources. First, it originated in a year's lecture course of the same title, covering about the first two-thirds of the ground presented here, the part on classical physics. This course grew out of the conviction that the teaching of theoretical physics in a number of separate courses, as in mechanics, electromagnetic theory, potential theory, thermodynamics, tends to keep a student from seeing the unity of physics, and from appreciating the importance of applying principles developed for one branch of science to the problems of another. The second source of this book was a projected volume on the structure of matter, dealing principally with applications of modern atomic theory to the structure of atoms, molecules, and solids, and to chemical problems. As work progressed on this, it became evident that the structure of matter could not be treated without a thorough understanding of the principles of wave mechanics, and that such an understanding demanded a careful grounding in classical physics, in mechanics, wave motion, the theory of vibrating systems, potential theory, statistical mechanics, where many principles needed in the quantum theory are best introduced. The ideal solution seemed to be to combine the two projects, including the classical and the more modern parts of theoretical physics in a coherent whole, thus, further increasing the unity of treatment of which we have spoken.

Two general principles have determined the order of presenting the material: mathematical difficulty, and order of historical development. Mechanics and problems of oscillations, involving ordinary differential equations and simple vector analysis, come first. Then follow vibrations and wave motion, introducing partial differential equations which can be solved by separation of variables, and Fourier series. Hydrodynamics, electromagnetic theory, and optics bring in more general partial differential equations, potential theory, and differential vector operations. Wave mechanics uses almost all the mathematical machinery which has been developed in the earlier part of the book. It is natural that the historical order is in general the

same as the order of increasing mathematical difficulty, for each branch of physics as it develops builds on the foundation of everything that has gone before. In cases where the two arrangements do not coincide, we have grouped together subjects of mathematical similarity, thus emphasizing the unity of which we have spoken.

In a book of such wide scope, it is inevitable that many important subjects are treated in a cursory manner. An effort has been made to present enough of the groundwork of each subject so that not only is further work facilitated, but also the position of these subjects in a more general scheme of physical thought is clearly shown. In spite of this, however, the student will of course make much use of other references, and we give a list of references, by no means exhaustive, but suggesting a few titles in each field which a student who has mastered the material of this book should be able to appreciate.

At the end of each chapter is a set of problems. The ability to work problems, in our opinion, is essential to a proper understanding of physics, and it is hoped that these problems will provide useful practice. At the same time, in many cases, the problems have been used to extend and amplify the discussion of the subject matter, where limitations of space made such discussion impossible in the text. The attempt has been made, though we are conscious of having fallen far short of succeeding in it, to carry each branch of the subject far enough so that definite calculations can be made with it. Thus a far surer mastery is attained than in a merely descriptive discussion.

Finally, we wish to remind the reader that the book is very definitely one on theoretical physics. Though at times descriptive material, and descriptions of experimental results, are included, it is in general assumed that the reader has a fair knowledge of experimental physics, of the grade generally covered in intermediate college courses. No doubt it is unfortunate, in view of the unity which we have stressed, to separate the theoretical side of the subject from the experimental in this way. This is particularly true when one remembers that the greatest difficulty which the student has in mastering theoretical physics comes in learning how to apply mathematics to a physical situation, how to formulate a problem mathematically, rather than in solving the problem when it is once formulated. We have tried wherever possible, in problems and text, to bridge the gap

between pure mathematics and experimental physics. But the only satisfactory answer to this difficulty is a broad training in which theoretical physics goes side by side with experimental physics and practical laboratory work. The same ability to overcome obstacles, the same ingenuity in devising one method of procedure when another fails, the same physical intuition leading one to perceive the answer to a problem through a mass of intervening detail, the same critical judgment leading one to distinguish right from wrong procedures, and to appraise results carefully on the ground of physical plausibility, are required in theoretical and in experimental physics. Leaks in vacuum systems or in electric circuits have their counterparts in the many disastrous things that can happen to equations. And it is often as hard to devise a mathematical system to deal with a difficult problem, without unjustifiable approximations and impossible complications, as it is to design apparatus for measuring a difficult quantity or detecting a new effect. These things cannot be taught. They come only from that combination of inherent insight and faithful practice which is necessary to the successful physicist. But half the battle is over if the student approaches theoretical physics, not as a set of mysterious formulas, or as a dull routine to be learned, but as a collection of methods, of tools, of apparatus, subject to the same sort of rules as other physical apparatus, and yielding physical results of great importance. The title of this book might have been aptly extended to "Introduction to the Methods of Theoretical Physics," for the aim has constantly been, not to teach a great collection of facts, but to teach mastery of the tools by which the facts have been discovered and by which future discoveries will be made.

In a subject about which so much has been written, it seems hardly practicable to acknowledge our indebtedness to any specific books. From many of those mentioned in the section on suggested references, and from many others, we have received ideas, though the material in general has been written without conscious following of earlier models. We wish to express thanks to several of our colleagues for suggestions, and particularly to Professors P. M. Morse and J. A. Stratton, who have read the manuscript with much care and have contributed greatly by their discussions.

CAMBRIDGE, MASS.,  
September, 1933.

J. C. S.  
N. H. F.

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