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Edited by A. Dold and B. Eckmann

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Calvin H. Wilcox

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*CONTENTS*

# PREFACE

These lecture notes are the written version of a series of lectures given at Tulane University during the spring semester of 1974 and, in expanded form, at the University of Stuttgart during the summer semester of 1974. The goal of the lectures was to present a complete and self-contained exposition of the mathematical theory of scattering for a simple, but typical, wave propagation problem of classical physics. The problem selected for this purpose was that of the scattering of acoustic waves by a bounded rigid obstacle immersed in a homogeneous fluid. When formulated mathematically the problem becomes an initial-boundary value problem for the d'Alembert wave equation in an exterior domain. The lecture notes present a simple approach to this problem based on a selfadjoint extension of the Laplacian in Hilbert space. The principal results presented in the notes are the construction of eigenfunction expansions for the Laplacian and the calculation of the asymptotic form of solutions of the d'Alembert equation for large values of the time parameter.

The theory developed in the notes is an exposition and synthesis of results developed by several authors during a period of more than twelve years, unified and extended by a number of new results due to the author. A discussion of the related literature is given in Lecture

1. The new results which are presented here for the first time include the results on asymptotic wave functions for the d'Alembert equation (Lectures 2 and 8), the direct proof of the existence and structure of the wave operators based on the eigenfunction expansion (Lecture 7) and the proofs of the limiting absorption principle and eigenfunction expansion theorem for domains with non-smooth boundaries (Lectures 4 and 6). One interesting feature of the method developed here is that it makes no use of coercivity or elliptic estimates near the boundary for the Laplacian. In fact these concepts are not even mentioned in the lecture notes.

Before preparing these notes the author benefited from a number of discussions with Dr. William C. Lyford concerning eigenfunction expansions and scattering theory in domains with non-smooth boundaries. It is a pleasure to acknowledge his assistance.

The author would like to thank Professors E. D. Conway, J. A. Goldstein and S. I. Rosencrans of Tulane University and Professor Peter Werner of the University of Stuttgart for the opportunity to present these lectures at their universities. The lectures at Tulane were supported by a grant from the Ford Foundation. Moreover, the preparation of the written version of the lecture notes was supported in part by the U. S. Office of Naval Research. This support is hereby gratefully acknowledged.

Calvin H. Wilcox

## *LECTURE 1. INTRODUCTION*

These lectures deal with the physical problem of the scattering of acoustic waves by a bounded rigid obstacle $\Gamma$ immersed in an unlimited homogeneous fluid. It is assumed that a small-amplitude perturbation of the fluid exists at time $t = 0$ (due, for example, to forces acting during $t < 0$). The basic physical problem is to predict the evolution of the resulting acoustic wave during $t > 0$. This problem is solved below for arbitrary initial states with finite energy and a class of obstacles with irregular (non-smooth) surfaces. The class of allowable obstacles includes all of the simple, but non-smooth, surfaces that arise in applications, such as polyhedra, finite sections of cylinders, cones, spheres, disks, etc. One of the principal results of the analysis is that every wave with finite energy is asymptotically equal for $t \to \infty$ to a diverging spherical wave. Moreover, it is shown how the profile of this wave can be calculated from the initial state. These results are then used to calculate the asymptotic distribution of the energy for $t \to \infty$.

The following notation will be used in the mathematical formulation of acoustic wave propagation problem. $\mathbb{R}$ denotes the field of real numbers, $\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R}$ ($n$ factors), $t \in \mathbb{R}$ and $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$. $\Omega \subset \mathbb{R}^n$ denotes an exterior domain; that is, $\Omega$

is an open connected subset of $\mathbb{R}^n$ and $\Gamma = \mathbb{R}^n - \Omega$ is bounded.

The acoustic wave propagation problem, when formulated mathematically, leads to the following initial-boundary value problem for the d'Alembert wave equation [9]. A function

$$(1.1) \qquad u : \mathbb{R} \times \Omega \to \mathbb{R}$$

is sought such that

$$(1.2) \quad D_0^2 u - (D_1^2 u + D_2^2 u + \dots + D_n^2 u) = 0 \quad \text{for} \quad t \in \mathbb{R},\ x \in \Omega,$$

$$(1.3) \quad D_\nu u \equiv \nu_1 D_1 u + \nu_2 D_2 u + \dots + \nu_n D_n u = 0 \quad \text{for } t \in \mathbb{R},\ x \in \partial\Omega,$$

$$(1.4) \quad u(0,x) = f(x) \quad \text{and} \quad D_0 u(0,x) = g(x) \quad \text{for} \quad x \in \Omega.$$

Here $D_0 = \partial/\partial t$, $D_j = \partial/\partial x_j$ for $j = 1,2,\dots,n$, $\partial\Omega$ represents the boundary of $\Omega$ and $\nu = (\nu_1, \nu_2, \dots, \nu_n)$ represents a unit vector normal to $\partial\Omega$ at $x$. The functions $f(x)$ and $g(x)$ are prescribed real-valued functions on $\Omega$.

These equations and variables have the following physical interpretation for the acoustic wave propagation problem. $n$ is the space-dimension ($n = 1,2$ or $3$ in the applications). $\Omega$ represents the homogeneous fluid with sound speed $c = 1$ and $\Gamma = \mathbb{R}^n - \Omega$ represents the scattering obstacle. The function $u(t,x)$ is the acoustic potential. Thus $\vec{v} = \nabla u = (D_1 u, D_2 u, \dots, D_n u)$ represents the fluid velocity and $p = D_0 u$ represents the excess pressure of the acoustic disturbance. The d'Alembert wave equation

(1.2) is a consequence of the linearized equations of fluid dynamics; see, e.g. [9]. The Neumann boundary condition (1.3) describes the condition that $\Gamma$ is rigid. The prescribed functions $f(x)$ and $g(x)$ represent the initial state of the acoustic field.

If $K \subset \Omega$ then the quantity

$$(1.5) \qquad E(u,K,t) = \int_K \sum_{k=0}^{n} \{D_k u(t,x)\}^2 \, dx,$$

where $dx = dx_1 dx_2 \ldots dx_n$, is interpreted as the acoustic energy in the set $K$ at time $t$. In particular, solutions of (1.2), (1.3) satisfy the principle of conservation of energy:

$$(1.6) \qquad E(u,\Omega,t) = E(u,\Omega,0) = \text{const.}, \quad t \in \mathbb{R},$$

where the constant may be finite or infinite. These lectures are concerned primarily with solutions with finite energy or "solutions wFE" as they will be called for brevity. The primary goal of these lectures is to study the asymptotic behavior for $t \to \infty$ of these solutions and the associated theory of scattering. In particular, the following topics are treated.

THE TRANSIENCY OF THE ENERGY IN BOUNDED SETS. Physical intuition suggests that if $u$ is a solution wFE in an exterior domain $\Omega$ then

$$(1.7) \qquad \lim_{t\to\infty} E(u, K\cap\Omega, t) = 0 \quad \text{if } K \text{ is bounded;}$$

that is, the energy ultimately propagates out of every bounded set. A proof of this property is given in Lecture 5.

ASYMPTOTIC WAVE FUNCTIONS. Each initial state wFE satisfies

$$(1.8)\quad E(u,\Omega,0) = \int_\Omega \left( \sum_{k=1}^{n} \{D_k f(x)\}^2 + g(x)^2 \right) dx < \infty .$$

Such states are "quasi-localized" in the sense that to each $\epsilon > 0$ there corresponds a radius $R = R(\epsilon)$ such that

$$(1.9)\quad E(u,\Omega,0) - \epsilon < E(u, B(R) \cap \Omega, 0) < E(u,\Omega,0)$$

where

$$(1.10)\qquad B(R) = \{x : |x| < R\}.$$

It follows that at any time $t_0 > 0$ the energy is contained in $B(R + t_0) \cap \Omega$, apart from a wave of total energy less than $\epsilon$. Moreover, the energy propagates outward, by (1.7). This suggests that the velocity $\vec{v} = (D_1 u, D_2 u, \ldots, D_n u)$ and pressure $p = D_0 u$ behave asymptotically like a diverging spherical wave:

$$(1.11)\quad \begin{cases} D_k u(t,x) \sim u_k^\infty (t,x), \quad t \to \infty, \quad \text{where} \\ u_k^\infty (t,x) \equiv |x|^{\frac{1-n}{2}} F_k(|x| - t, x/|x|), \end{cases}$$

in the sense that, for $k = 0, 1, 2, \ldots, n$,

$$(1.12)\quad \lim_{t\to\infty} \int_\Omega \{D_k u(t,x) - u_k^\infty (t,x)\}^2 \, dx = 0.$$

Functions $u_k^\infty (t,x)$ of the form defined in (1.11) will be called "asymptotic wave functions". It is shown below that each solution wFE has unique asymptotic wave functions such that (1.12) holds. Moreover, the "wave profiles" $F_k(r,\eta)$, with $r \in \mathbb{R}$, $\eta \in S^{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$, are calculated from the initial state $f,g$.

ASYMPTOTIC ENERGY DISTRIBUTIONS. Consider a cone

$$C = \{x = r\eta : r > 0,\ \eta \in C_0 \subset S^{n-1}\}. \tag{1.13}$$

Properties (1.7), (1.11), (1.12) imply that the fraction of the total energy $E(u,\Omega,0)$ which is contained in $C$ at time $t$ tends to a limit as $t \to \infty$. More precisely,

$$\lim_{t\to\infty} E(u,\ C\cap\Omega,\ t) = \int_{\mathbb{R}} \int_{C_0} \sum_{k=0}^{n} \{F_k(r,\eta)\}^2 \, d\eta \, dr \tag{1.14}$$

where $d\eta$ denotes the element of area on $S^{n-1}$. This behavior is verified below and the ultimate energy distribution (1.14) is calculated as a function of the initial state $f,g$.

SCATTERING THEORY. The time-dependent theory of scattering deals with the asymptotic equality of two systems for $t \to \infty$. Consider two exterior domains, $\Omega_1$ and $\Omega_2$, and corresponding solutions wFE, $u_1(t,x)$ and $u_2(t,x)$. The solutions will be said to be "asymptotically equal in energy" for $t \to \infty$ if

$$\lim_{t\to\infty} E(u_1 - u_2,\ \Omega_1 \cap \Omega_2,\ t) = 0. \tag{1.15}$$

Note that if $u_1(t,x)$ and $u_2(t,x)$ are asymptotically equal to the same solution wFE $u_0(t,x)$ in $\mathbb{R}^n$ (no obstacle) then (1.15) follows by the triangle inequality. Hence, to study asymptotic equality it is enough to compare solutions $u(t,x)$ in exterior domains $\Omega$ with solutions $u_0(t,x)$ in $\mathbb{R}^n$. Moreover, $u_0(t,x)$ will also have asymptotic wave functions, say

$$(1.16)\qquad \begin{cases} D_k u_0(t,x) \sim u_{k0}^{\infty}(t,x), \quad t \to \infty, \quad \text{where} \\ u_{k0}^{\infty}(t,x) \equiv |x|^{\frac{1-n}{2}} F_{k0}(|x| - t,\ x/|x|). \end{cases}$$

Hence, the relation

$$(1.17)\qquad \lim_{t\to\infty} E(u - u_0, \Omega, t) = 0$$

will follow from (1.11) and (1.16) if the initial state $f_0, g_0$ for $u_0$ can be adjusted so that the profiles $F_{k0}$ and $F_k$ coincide. It is shown below that this is always possible. The initial states $f_0, g_0$ and $f,g$ are related by a "wave operator" in the sense of the time-dependent theory of scattering.

The remaining lectures are organized as follows.

LECTURE 2. SOLUTIONS OF THE D'ALEMBERT EQUATION IN $\mathbb{R}^n$. Here solutions in $\mathbb{R}^n$ are constructed by means of the Plancherel theory and used to derive the asymptotic behavior (1.11), (1.12) for the special case of solutions WFE in $\mathbb{R}^n$ (no boundary).

LECTURE 3. SOLUTIONS OF THE D'ALEMBERT EQUATION IN ARBITRARY DOMAINS. In this lecture $\Omega \subset \mathbb{R}^n$ represents an arbitrary domain ( = open connected subset). A selfadjoint operator $A$ on the Hilbert space $L_2(\Omega)$ is defined by the negative Laplacian, acting on a domain of functions which satisfy the Neumann condition in a suitably generalized sense. The spectral theorem for A is then used to discuss "solutions in $L_2(\Omega)$" and "solutions wFE" of the initial-boundary value problem.

LECTURE 4. STEADY-STATE SCATTERING THEORY IN EXTERIOR DOMAINS AND THE LIMITING ABSORPTION PRINCIPLE. Here the steady-state scattering problem is formulated for the d'Alembert equation in exterior domains and the uniqueness and existence of solutions is proved. The existence theorem is a corollary of a "limiting absorption theorem" which is the principal result of this lecture. It states that the resolvent $R_z = (A - z)^{-1}$ has limits, in a certain topology, when $z$ tends to points of the spectrum of A. The proof of the theorem, and of most of the results in the subsequent lectures, are based on a version of the Rellich selection theorem [1,31] for the domain $\Omega$. This is the only point in the theory where a restriction is imposed on the boundary of $\Omega$. The Rellich theorem is established here for a class of domains called "domains with the finite tiling property". The class contains all the domains with piece-wise smooth boundaries having edges and corners that occur in applications, as well as many domains with highly singular boundaries.

LECTURE 5. TIME-DEPENDENT SCATTERING THEORY IN EXTERIOR DOMAINS. The lecture begins with F. Rellich's classical theorem that the operator A has no point spectrum. Then the limiting absorption theorem is shown to imply