PRINCIPLES OF PHYSICS SERIES

MECHANICS, HEAT AND SOUND

by

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FOREWORD

The first course in physics at the Massachusetts Institute of Technology extends through the first and second years. The subject matter covered in the first year consists of Mechanics, Heat, and Sound; in the second year Electricity and Magnetism, Optics, and Modern Physics are included. A two year course in Analytical Geometry and Calculus is given concurrently with the course in physics. This book has developed out of the author's experience in teaching the first year of the physics program.

The title of the book, Principles of Physics, has been chosen deliberately to indicate that its emphasis is on physical principles. Historical background and practical applications have been given a place of secondary importance.

The book opens with several chapters on statics in order that kinematics may be postponed until the student has acquired some familiarity with the concepts and notation of calculus. Beginning with Chapter 4, simple differentiation and integration are introduced to supplement and extend the algebraic development of the equations of linear motion with constant acceleration. From that point on, the calculus is used freely wherever its inclusion is warranted.

Three systems of units are used; the English gravitational because it is the one used in engineering work throughout this country, the cgs system because some familiarity with it is essential for any intelligent reading of the literature of physics, and the mks system because of its increasing use in electricity and magnetism as well as because it seems destined eventually to supplant the cgs system.

Many of the problems in the book are taken from examinations given in connection with the physics course at M.I.T. The author wishes to express his thanks to all of his colleagues who have shared in writing these examinations. In particular, he is indebted to Professor M. Stanley Livingston both for many stimulating and informative discussions arising from the latter's collaboration in presenting the experimental lectures in the course over a period of years, and for his encouragement in the task of developing a set of lecture notes into this book.

The multiflash photographs were taken with the advice and assistance of Professor Harold E. Edgerton, to whom the author is duly grateful. Collective acknowledgement is made to numerous contributors to the American Journal of Physics (formerly the American Physics Teacher) since its inception.

FRANCIS W. SEARS.

Cambridge, Mass.
March, 1944.

CONTENTS

CHAPTER 1

COMPOSITION AND RESOLUTION OF VECTORS

1.1 Force. Mechanics is the branch of physics and engineering which deals with the interrelations of force, matter, and motion. We shall begin with a study of forces. The term force, as used in mechanics, refers to what is known in everyday language as a *push* or a *pull*. We can exert a force on a body by muscular effort; a stretched spring exerts forces on the bodies to which its ends are attached; compressed air exerts a force on the walls of its container; a locomotive exerts a force on the train which it is drawing. In all of these instances the body exerting the force is in contact with the body on which the force is exerted, and forces of this sort are known as *contact* forces. There are also forces which act through empty space without contact, and are called *action-at-a-distance* forces. The force of gravitational attraction exerted on a body by the earth, and known as the *weight* of the body, is the most important of these for our present study. Electrical and magnetic forces are also action-at-a-distance forces, but we shall not be concerned with them for the present.

All forces fall into one or the other of these two classes, a fact that will be found useful later when deciding just what forces are acting on a given body. It is only necessary to observe what bodies are in contact with the one under consideration. The only forces on the body are then those exerted by the bodies in contact with it, together with the gravitational force or the weight of the body.

Those forces acting on a given body which are exerted by other bodies are referred to as *external* forces. Forces exerted on one part of a body by other parts of the same body are called *internal* forces.

1.2 Units and standards. The early Greek philosophers confined their activities largely to speculations about Nature, and to attempts to reconcile the observed behaviour of bodies with theological doctrines. What has been called the *scientific method* began to appear in the time of Galileo Galilei (1564–1642). Galileo's studies of the laws of freely falling bodies were made not in an attempt to explain *why* bodies fell toward the earth, but rather to determine *how far* they fell in a given time, and *how fast* they moved. Physics as it exists to-day has been called the science of measurement, and the importance of quantitative knowledge and reasoning has been expressed by Lord Kelvin (1824–1907) as follows: "I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of *Science*, whatever the matter may be."

The first step in the measurement of a physical quantity consists in choosing a *unit* of that quantity. As the result of international collaboration over a long period, practically all of the units used in physics are now the same throughout the world. The second step is an experiment that determines the ratio of the magnitude of the quantity to the magnitude of the unit. Thus, when we say that the length of a rod is 10 centimeters, we state that its length is ten times as great as the unit of length, the centimeter.

It is possible to simplify many of the equations of physics by the proper choice of units of physical quantities. Any set of units which is chosen so that these simplified equations can be used is called a *system* of units. We shall use three such systems in this book. They are, first, the *English gravitational* system; second, the meter-kilogram-second or *mks* system; and third, the centimeter-gram-second or *cgs* system. The units of these systems will be defined as the need for them arises.

Most of the fundamental units of physics are embodied in a physical object called a *standard*. One of the functions of the National Bureau of Standards in Washington, D. C. is to maintain in its vaults standards of various quantities with which commercial and technical measuring instruments can be compared for accuracy.

1.3 The pound. The unit of force which we shall use for the present is the English gravitational unit, the *pound*. Other units will be discussed in Chap. 5. This unit is embodied in a cylinder of platinum-iridium called the *standard pound*. The unit of force is defined as the weight of the standard pound. That is, it is a force equal to the force of gravitational attraction which the earth exerts on the standard pound. Since the earth's gravitational attraction for a given body varies slightly from one point to another on the earth's surface it is further stipulated that the unit force shall equal the weight of the standard pound *at sea level and 45° latitude.*[1]

In order that an unknown force can be compared with the force unit (and thereby measured) some measurable effect produced by a force must be used. One common effect of a force is to alter the dimensions or shape of a body on which the force is exerted; another is to alter the state of

[1] See Section 15.3 for a more precise definition.

motion of the body. Both of these effects are used in the measurement of forces. In this chapter we shall consider only the former; the latter will be discussed in Chap. 5.

The instrument used to measure forces is the spring balance, which consists of a coil spring enclosed in a case for protection and carrying at one end a pointer that moves over a scale. A force exerted on the balance increases the length of the spring. The balance can be calibrated as follows: The standard pound is first suspended from the balance and the position of the pointer marked 1 lb. Any number of duplicates of the standard can then be prepared by suspending each of them in turn from the balance and removing or adding material until the index stands at 1 lb. Then, when two, three, or more of these are suspended simultaneously from the balance, the force stretching it is 2 lbs, 3 lbs, etc., and the corresponding positions of the pointer can be labelled 2 lbs, 3 lbs, etc. This procedure makes no assumptions about the elastic properties of the spring, except that the force exerted on it is always the same when its index stands at the same point. The calibrated balance can then be used to measure any unknown force.

1.4 Graphical representation of forces. Vectors. Suppose we are to slide a box along the floor by pulling it with a string or pushing it with a stick, as in Fig. 1-1. That is, we are to slide it by exerting a force on it. The point of view which we now adopt is that the motion of the box is caused not by the *objects* which push or pull on it, but by the *forces* which these exert. For concreteness assume the magnitude of the push or pull to be 10 lbs. It is clear that simply to write "10 lbs" on the diagram would not completely describe the force, since it would not indicate the direction in which the force was acting. One might write "10 lbs, 30° above horizontal to the right," or "10 lbs, 45° below horizontal to the right," but all of the above information may be conveyed more briefly if we adopt the convention of representing a force by an arrow. The length of the arrow, to some chosen scale, indicates the size or magnitude of the force, and the direction in which the arrow points indicates the direction of the force. Thus Fig. 1-2 (in which a scale of ⅛ in. = 1 lb has been chosen) is the force diagram corresponding to Fig. 1-1. (There are other forces acting on the box, but these are not shown in the figure.)

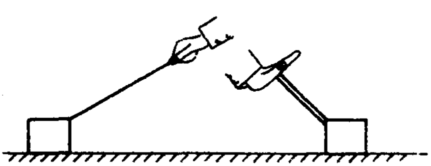

FIG. 1-1. The box is pulled by the string or pushed by the stick.

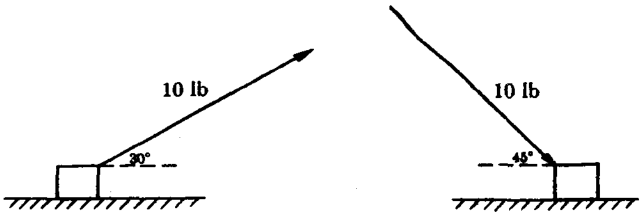

FIG. 1-2. The force diagram corresponding to Fig. 1-1.

Force is not the only physical quantity which requires the specification of direction as well as magnitude. For example, the velocity of a plane is not completely specified by stating that it is 300 miles per hour; we need to know the direction also. The concept of density, on the other hand, has no direction associated with it.

Quantities like force and velocity, which involve both magnitude and direction, are called *vector* quantities. Those like density, which involve magnitude only, are called *scalars*. Any vector quantity can be represented by an arrow, and this arrow is called a vector (or if a more specific statement is needed, a force vector or a velocity vector). We shall first consider force vectors only, but the ideas developed in dealing with them can be applied to any other vector quantity.

1.5 Components of a vector. When a box is pulled or pushed along the floor by an inclined force as in Fig. 1-1, it is clear that the effectiveness of the force in moving the box along the floor depends upon the direction in which the force acts. Everyone knows by experience that a given force is more effective for moving the box the more nearly the direction of the force approaches the horizontal. It is also clear that if the force is applied at an angle, as in Fig. 1-1, it is producing another effect in addition to moving the box ahead. That is, the pull of the string is in part tending to lift the box off the floor, and the push of the stick is in part forcing the box down against the floor. We are thus led to the idea of the *components* of a force, that is, the effective values of a force in directions other than that of the force itself.

The component of a force in any direction can be found by a simple graphical method. Suppose we wish to know how much force is available for sliding the box in Fig. 1-1 if the applied force is a pull of 10 lbs directed 30° above the horizontal. Let the given force be represented by the vector OA in Fig. 1-3, in the proper direction and to some convenient scale. Line OX is the direction of the desired component. From point A drop a perpendicular to OX, intersecting it at B. The vector OB, to the same scale as that used for the given vector, represents the component of OA in the direction OX. Measurements of the diagram show that if OA represents a force of 10 lbs, then OB is about 8.7 lbs. That is, the 10-lb force at an angle of 30° above the horizontal has an effective value of only about 8.7 lbs in producing forward motion.

The component OB may also be computed as follows. Since OAB is a right triangle, it follows that

$$\cos 30° = \frac{OB}{OA}$$

$$OB = OA \cos 30°$$

The lengths OB and OA, however, are proportional to the magnitudes of the forces they represent. Therefore the desired component OB, in pounds, equals the given force OA, in pounds, multiplied by the cosine of the angle between OA and OB. The magnitude of OB is therefore

$$\begin{aligned} OB \text{ (lbs)} &= OA \text{ (lbs)} \times \cos 30° \\ &= 10 \text{ lbs} \quad \times .866 \\ &= 8.66 \text{ lbs} \end{aligned}$$

This result agrees as well as could be expected with that obtained from measurements of the diagram. The superiority of the trigonometric method is evident, however, since it does not depend for accuracy on the careful construction and measurement of a scale diagram.

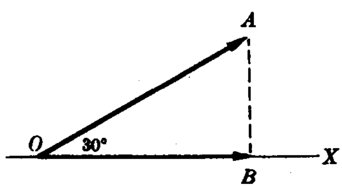

FIG. 1-3. Vector OB is the component of vector OA in the direction OX.

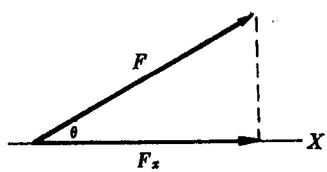

FIG. 1-4. $F_x = F \cos \theta$ is the X-component of F.

The line OX in Fig. 1-3 is called the X-axis, and the foregoing analysis may be generalized as follows. If a force F makes an angle θ with the X-axis (Fig. 1-4), its component F_x along the X-axis is

$$F_x = F \cos \theta \tag{1.1}$$

It should be obvious that if the force F is at right angles to the X-axis, its component along that axis is zero (since $\cos 90° = 0$), and if the force lies along the axis, its component is equal to the force itself (since $\cos 0° = 1$).

The lifting component of an inclined force can be found as in Fig. 1-5. Line OY, called the Y-axis, is constructed in a vertical direction through O and a perpendicular dropped to this axis from the head of the arrow F. Evidently

$$F_y = F \cos \phi \tag{1.2}$$

where ϕ is the angle between F and the Y-axis.

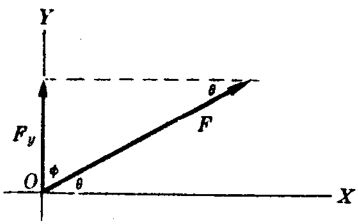

FIG. 1-5. $F_y = F \cos \phi = F \sin \theta$ is the Y-component of F.

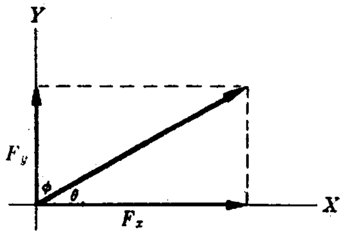

FIG. 1-6. The force F may be replaced by its rectangular components F_x and F_y.

It is also evident from Fig. 1-5 that

$$F_y = F \sin \theta \tag{1.3}$$

If $F = 10$ lbs. and $\theta = 30°$, then $\phi = 60°$ and $\cos \phi = \sin \theta = 0.50$. Hence $F_y = 5$ lbs.

Just as we may find the component of a given force in any direction, so may we find the component of any of its components, and so on. It will be seen from Fig. 1-6, however, that F_x has no component along the Y-axis and F_y has no component along the X-axis. No further resolution of the force into X- and Y-components is therefore possible. Physically this means that the two forces F_x and F_y, acting simultaneously, are equivalent in all respects to the original force F. Since the axes OX and OY are at right angles to one another, F_x and F_y are called the *rectangular components* of the force F. *Any force may be replaced by its rectangular components.* The fact that the force F has been replaced by its components F_x and F_y is indicated in Fig. 1-6 by crossing out lightly the vector F.

The process of finding the components of a vector is called the *resolution* of the vector, and one speaks of *resolving* a given vector into its rectangular components.

An experiment to show that a force may be replaced by its rectangular components is illustrated in Fig. 1-7. A small ring, to which are attached three cords, is placed on a pin set in a vertical board. Two of the cords pass over pulleys as shown. When weights of 8.66, 5, and 10 lbs are

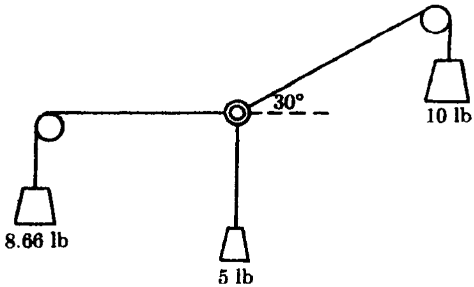

FIG. 1-7.

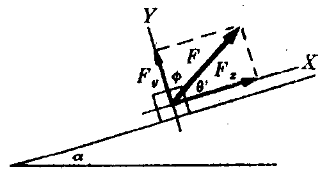

FIG. 1-8. F_x and F_y are the components of F, parallel and perpendicular to the surface of the plane.

suspended from the cords, with the cord carrying the 10-lb weight making an angle of 30° with the horizontal, it will be found that the pin can be removed and that the ring will remain at rest under the combined action of the pulls in the three cords. This shows that the 10-lb force, at an angle of 30° above the horizontal, is equivalent to a horizontal force of 8.66 lbs to the right and a vertical force of 5 lbs upward, since the ring can be held at rest by the application of two forces equal to these but oppositely directed.

It is frequently necessary to find the components of a force in other than horizontal and vertical directions. Thus in Fig. 1-8, where a block is being drawn up an inclined plane by the force F, it is desired to find the components of this force parallel and perpendicular to the surface of the plane. The X- and Y-axes are now drawn parallel and perpendicular to this surface, and the same procedure followed as before.

1.6 Composition of forces. When a number of forces are simultaneously applied at a point, it is found that the same effect can always be produced by a single force having the proper magnitude and direction. We wish to find this force, called the *resultant*, when the separate forces are known. The process is known as the *composition* of forces, and is evidently the converse problem to that of resolving a given force into components. Let us begin by considering some simple cases.

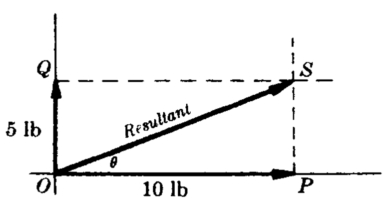

FIG. 1-9.

(1) Two forces at right angles. Suppose the two forces of 10 lbs and 5 lbs are applied simultaneously at the point O as in Fig. 1-9. To find the resultant force graphically, lay off the given forces OP and OQ to scale, and draw horizontal and vertical construction lines from P and Q,

intersecting at S. The arrow drawn from O to S represents the resultant of the given forces. Its length, to the same scale as that used for the original forces, gives the magnitude of the resultant, and the angle θ gives its direction.

Since the length PS or OQ represents 5 lbs, and the length OP represents 10 lbs, the magnitude of the resultant may be computed from the right triangle OPS. Thus

$$OS = \sqrt{\overline{OP}^2 + \overline{PS}^2} = \sqrt{10^2 + 5^2} = 11.2 \text{ lbs}$$

The angle θ may also be computed from either its sine, cosine, or tangent. Thus

$$\sin\theta = \frac{5}{11.2} = 0.447$$

$$\cos\theta = \frac{10}{11.2} = 0.893$$

$$\tan\theta = \frac{5}{10} = 0.500$$

Using any one of these values we find from tables of natural functions

$$\theta = 26.5°$$

We conclude, then, that a single force of 11.2 lbs, at an angle of 26.5° above the horizontal, will produce the same effect as the two forces of 10 lbs horizontally and 5 lbs vertically. Notice that the resultant is not the arithmetic sum of 5 lbs and 10 lbs. That is, the two forces are not equivalent to a single force of 15 lbs.

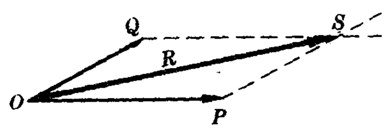

FIG. 1-10. Parallelogram method for finding the resultant of two vectors.

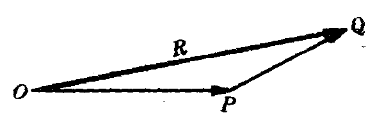

FIG. 1-11. Triangle method for finding the resultant of two vectors.

(2) Two forces not at right angles. (a) Parallelogram method. Let OP and OQ in Fig. 1-10 represent the forces whose resultant is desired. Draw construction lines from P parallel to OQ, and from Q parallel to OP, intersecting at S. The arrow OS represents the resultant R in magnitude and direction. Since $OPSQ$ is a parallelogram, this method is called the parallelogram method. The magnitude and direction of the resultant may

be found by measurement or may be computed from the triangle OPS with the help of the sine and cosine laws.

NOTE. The diagonal QP is *not* the resultant of the given forces.

(b) Triangle method. Draw one force vector with its tail at the head of the other as in Fig. 1-11 (the construction may be started with either vector), and complete the triangle. The closing side of the triangle, OQ, represents the resultant. A comparison of Figs. 1-11 and 1-10 will show that the same resultant is obtained by either method.

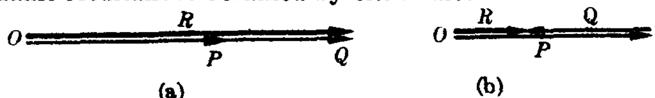

FIG. 1-12. Vector R is the resultant of vectors P and Q.

(3) Special case. Both forces in the same line. When both forces lie in the same straight line the triangle of Fig. 1-11 flattens out into a line also. To be able to see all of the force vectors, it is customary to displace them slightly as in Fig. 1-12. We then have Fig. 1-12 (a) or 1-12 (b), depending upon whether the two forces are in the same or opposite directions. Only in this case is the magnitude of the resultant equal to the sum (or difference) of the magnitudes of the components.

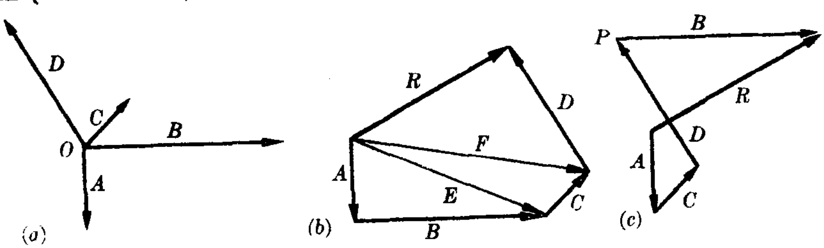

FIG. 1-13. Polygon method.

(4) More than two forces. Polygon method. When more than two forces are to be combined, one may first find the resultant of any two, then combine this resultant with a third, and so on. The process is illustrated in Fig. 1-13, which shows the four forces A, B, C, and D acting simultaneously at the point O. In Fig. 1-13 (b), forces A and B are first combined by the triangle method giving a resultant E; force E is then combined by the same process with C giving a resultant F; finally F and D are combined to obtain the resultant R. Evidently the vectors E and F need not have been drawn—one need only draw the given vectors in succession with the tail of each at the head of the one preceding it, and complete the polygon by a vector R from the tail of the first to the head

of the last vector. The order in which the vectors are drawn makes no difference as shown in Fig. 1-13 (c).

It has been assumed in the preceding discussion that all of the forces lie in the same plane. Such forces are called *co-planar*, and, except in a few instances, we shall consider only situations involving co-planar forces.

1.7 Composition of forces by rectangular resolution. While the polygon method is a satisfactory graphical one for finding the resultant of a number of forces, it is awkward for computation because one must work, in general, with oblique triangles. Therefore the usual method for finding the resultant of a number of forces is first to resolve all of the forces into their rectangular components along any convenient pair of axes; second, to find the algebraic sum of all of the X- and all of the Y-components; and third, combine these sums to obtain the final resultant. This process makes it possible to work with right triangles only, and is called the *method of rectangular resolution.* As an example, let us compute the resultant of the four forces in Fig. 1-14, which are the same as those in Fig. 1-13.

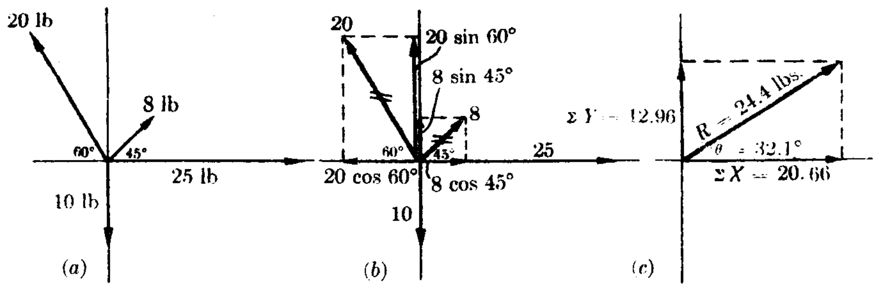

FIG. 1-14.

The forces are shown in Fig. 1-14 (b) resolved into X- and Y-components. The 25-lb and the 10-lb forces are already along the axes and need not be resolved. It is customary to consider X-components which are directed toward the right as positive and those toward the left, negative. Similarly Y-components in an upward direction are considered positive and those downward, negative. This convention of signs is not always adhered to, however. In general one chooses positive and negative directions so as to avoid minus signs if possible.

The X-component of the 8-lb force is $+8 \cos 45° = +5.66$ lbs, and its Y-component is $+8 \sin 45° = +5.66$ lbs. The X-component of the 20-lb force is $-20 \cos 60° = -10$ lbs, its Y-component is $+20 \sin 60° = +17.3$ lbs. The algebraic sum of the X-components is a force of $25 + 5.66 - 10 = + 20.66$ lbs toward the right. The algebraic sum of the Y-components is a force of 17.3 + 5.66 − 10 = + 12.96 lbs upward. The resultant is equal to the square root of the sums of the squares of the resultant X- and Y-components (Fig. 1-14 (c)). The angle which it makes with the X-axis can be found from its tangent. Thus

$$R = \sqrt{20.66^2 + 12.96^2} = 24.4 \text{ lbs}$$

$$\tan\theta = \frac{12.96}{20.66} = 0.627$$

$$\theta = 32.1°$$

While three separate diagrams are shown in Fig. 1-14 for clarity, in practice one would carry out the entire construction in a single diagram.

The mathematical symbol for the algebraic sum of the X- or Y-components is ΣX or ΣY. (Σ is the Greek letter sigma or S, meaning "the sum of".) Hence one can write in general

$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$$

$$\tan\theta = \frac{\Sigma Y}{\Sigma X}$$

1.8 Resultant of a set of non-concurrent forces. Fig. 1-15 represents a rod upon which are exerted the three forces F_1, F_2, and F_3. These forces are not all applied at the same point, and even if their lines of action are extended as shown by the dotted lines, these do not intersect at a common point. Nevertheless the three forces have a resultant in the sense that it is possible to find a single force which will produce the same effect as is produced by the simultaneous action of the given forces. This resultant may be found graphically as follows:

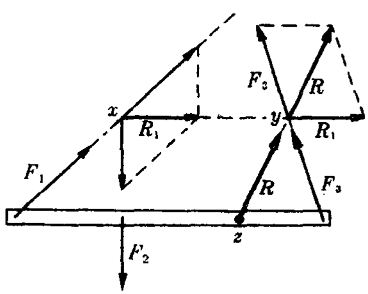

FIG. 1-15. Vector R is the resultant of F_1, F_2, and F_3.

Start with any two of the given forces, say F_1 and F_2, and extend their lines of action until they intersect (point x). Transfer F_1 and F_2 to point x, and find their resultant R_1 by any convenient method. The parallelogram method is used in the figure. Next, extend the lines of action of R_1 and F_3 until they intersect (point y) and combine R_1 and F_3 to find the resultant R. Finally, extend the line of action of R until it intersects the rod at point z. Then a single force having the magnitude and direction of R, and applied at point z of the rod, will produce the same effect as the given forces.