The History of Mathematics

An Introduction

Fourth Edition

David M. Burton

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Preface

Since many excellent treatises on the history of mathematics are available, there may seem little reason for writing still another. But most current works are severely technical, written by mathematicians for other mathematicians or for historians of science. Despite the admirable scholarship and often clear presentation of these works, they are not especially well adapted to the undergraduate classroom. (Perhaps the most notable exception is Howard Eves's popular account, *An Introduction to the History of Mathematics*.) There seems to be room at this time for a textbook of tolerable length and balance addressed to the undergraduate student, which at the same time is accessible to the general reader interested in the history of mathematics.

In the following pages, I have tried to give a reasonably full account of how mathematics has developed over the past 5000 years. Because mathematics is one of the oldest intellectual instruments, it has a long story, interwoven with striking personalities and outstanding achievements. This narrative is basically chronological, beginning with the origin of mathematics in the great civilizations of antiquity and progressing through the later decades of the twentieth century. The presentation necessarily becomes less complete for modern times, when the pace of discovery has been rapid and the subject matter more technical.

Considerable prominence has been assigned to the lives of the people responsible for progress in the mathematical enterprise. In emphasizing the biographical element, I can say only that there is no sphere in which individuals count for more than the intellectual life, and that most of the mathematicians cited here really did tower over their contemporaries. So that they will stand out as living figures and representatives of their day, it is necessary to pause from time to time to consider the social and cultural framework that animated their labors. I have especially tried to define why mathematical activity waxed and waned in different periods and in different countries.

Writers on the history of mathematics tend to be trapped between the desire to interject some genuine mathematics into a work and the desire to make the reading as painless and pleasant as possible. Believing that any mathematics textbook should concern itself primarily with teaching mathematical content, I have favored stressing the mathematics. Thus, assorted problems of varying degrees of difficulty have been interspersed throughout. Usually these problems typify a particular historical period, requiring the procedures of that time. They are an integral part of the text, and you will, in working them, learn some interesting mathematics as well as history. The level of maturity needed for this work is approximately the mathematical background of a college junior or senior. Readers with more extensive training in the subject must forgive certain explanations that seem unnecessary.

The title indicates that this book is in no way an encyclopedic enterprise. Neither does it pretend to present all the important mathematical ideas that arose during the vast sweep of time it covers. The inevitable limitations of space necessitate illuminating some outstanding landmarks instead of casting light of equal brilliance over the whole landscape. In keeping with this outlook, a certain amount of judgment and self-denial has to be exercised, both in choosing mathematicians and in treating their contributions. Nor was material selected exclusively on objective factors; some personal tastes and prejudices held sway. It stands to reason that not everyone will be satisfied with the choices. Some readers will raise an eyebrow at the omission of some household names of mathematics that have been either passed over in complete silence or shown no great hospitality; others will regard the scant treatment of their favorite topic as an unpardonable omission. Nevertheless, the path that I have pieced together should provide an adequate explanation of how mathematics came to occupy its position as a primary cultural force in Western civilization. The book is published in the modest hope that it may stimulate the reader to pursue the more elaborate works on the subject.

Anyone who ranges over such a well-cultivated field as the history of mathematics becomes so much in debt to the scholarship of others as to be virtually pauperized. The chapter bibliographies represent a partial listing of works, recent and not so recent, that in one way or another have helped my command of the facts. To the writers and to many others of whom no record was kept, I am enormously grateful.

New to This Edition

Readers familiar with the previous editions of *The History of Mathematics* will find that this edition maintains the same overall organization and content. Nevertheless, the preparation of a fourth edition has provided the occasion for a variety of small improvements as well as several more significant ones. The largest change is a more extensive treatment of the mathematics developed in parts of the world other than the West. This is presented in a new section, "Mathematics in the Near and Far East." The section discussing American mathematics in the latter half of the nineteenth century has been expanded and retitled "The Emergence of American Mathematics." Other smaller additions (including accounts of the Königsberg Bridge Problem, Kepler's sphere-packing conjecture, and modern computer development) thread their way through the text.

Another notable difference is the increased coverage given to several figures touched upon too lightly in past editions. Specifically, material has been added concerning John Napier, Christiaan Huygens, Charles Babbage, Benjamin Peirce, and Augustus De Morgan.

Beyond these textual modifications, there are a number of relatively minor changes; further exercises have been introduced, bibliographies brought up to date, and certain numerical information kept current. Needless to say, an attempt has been made to correct all errors, typographical and historical, which crept into the earlier versions.

Acknowledgments

I am indebted to a number of people in deciding which alterations would be desirable in this edition. Extensive and constructive comments were provided by:

Mark Coffey, University of Colorado
Martin Flashman, Humboldt State University

Many other friends, colleagues, and readers—too numerous to mention individually—were also kind enough to offer their suggestions. Although not all recommendations were adopted, they were all seriously considered in the revision process. Lastly, I am deeply grateful to my wife, Martha Beck Burton, for her constant encouragement and assistance in the project from beginning to end.

D.M.B.

Contents

CHAPTER 1

Early Number Systems and Symbols

To think the thinkable—that is the mathematician's aim.

C. J. KEYSER

1.1 Primitive Counting

The root of the term *mathematics* is in the Greek word *mathemata*, which was used quite generally in early writings to indicate any subject of instruction or study. As learning advanced, it was found convenient to restrict the scope of this term to particular fields of knowledge. The Pythagoreans are said to have used it to describe arithmetic and geometry; previously, each of these subjects had been called by its separate name, with no designation common to both. The Pythagoreans' use of the name would perhaps be a basis for the notion that mathematics began in Classical Greece during the years from 600 to 300 B.C. But its history can be followed much further back. Three or four thousand years ago, in ancient Egypt and Babylonia, there already existed a significant body of knowledge that we should describe as mathematics. If we take the broad view that mathematics involves the study of issues of a quantitative or spatial nature—number, size, order, and form—it is an activity that has been present from the earliest days of human experience. In every time and culture, there have been people with a compelling desire to comprehend and master the form of the natural world around them. To use Alexander Pope's words, "This mighty maze is not without a plan."

It is commonly accepted that mathematics originated with the practical problems of counting and recording numbers. The birth of the idea of number is so hidden behind the veil of countless ages that it is tantalizing to speculate on the remaining evidences of early humans' sense of number. Our remote ancestors of some 20,000 years ago—who were quite as clever as we are—must have felt the need to enumerate their livestock, tally objects for barter, or mark the passage of days. But the evolution of counting, with its spoken number words and written number symbols, was gradual and does not allow any determination of precise dates for its stages.

Anthropologists tell us that there has hardly been a culture, however primitive, that has not had some awareness of number, though it might have been as rudimentary as the distinction between one and two. Certain Australian aboriginal tribes, for instance, counted to two only, with any number larger than two called simply "much" or "many." South American Indians along the tributaries of the Amazon were equally destitute of number words. Although they ventured further than the aborigines in being able to count to six, they had no independent number names for groups of three, four, five, or six. In their counting vocabulary, three was called "two-one," four was "two-two," and so on. A similar system has been reported for the Bushmen of South Africa, who counted to ten ($10 = 2+2+2+2+2$) with just two words; beyond ten, the descriptive phrases became too long. It is notable that such tribal groups would not willingly trade, say, two cows for four pigs, yet had no hesitation in exchanging one cow for two pigs and a second cow for another two pigs.

The earliest and most immediate technique for visibly expressing the idea of number is tallying. The idea in tallying is to match the collection to be counted with some easily employed set of objects—in the case of our early forebears, these were fingers, shells, or stones. Sheep, for instance, could be counted by driving them one by one through a narrow passage while dropping a pebble for each. As the flock was gathered in for the night, the pebbles were moved from one pile to another until all the sheep had been accounted for. On the occasion of a victory, a treaty, or the founding of a village, frequently a cairn, or pillar of stones, was erected with one stone for each person present.

The term *tally* comes from the French verb *tailler*, "to cut," like the English word *tailor*; the root is seen in the Latin *taliare*, meaning "to cut." It is also interesting to note that the English word *write* can be traced to the Anglo-Saxon *writan*, "to scratch," or "to notch."

Neither the spoken numbers nor finger tallying have any permanence, although finger counting shares the visual quality of written numerals. To preserve the record of any count, it was necessary to have other representations. We should recognize as human intellectual progress the idea of making a correspondence between the events or objects recorded and a series of marks on some suitably permanent material, with one mark representing each individual item. The change from counting by assembling collections of physical objects to counting by making collections of marks on one object is a long step, not only toward abstract number concept, but also toward written communication.

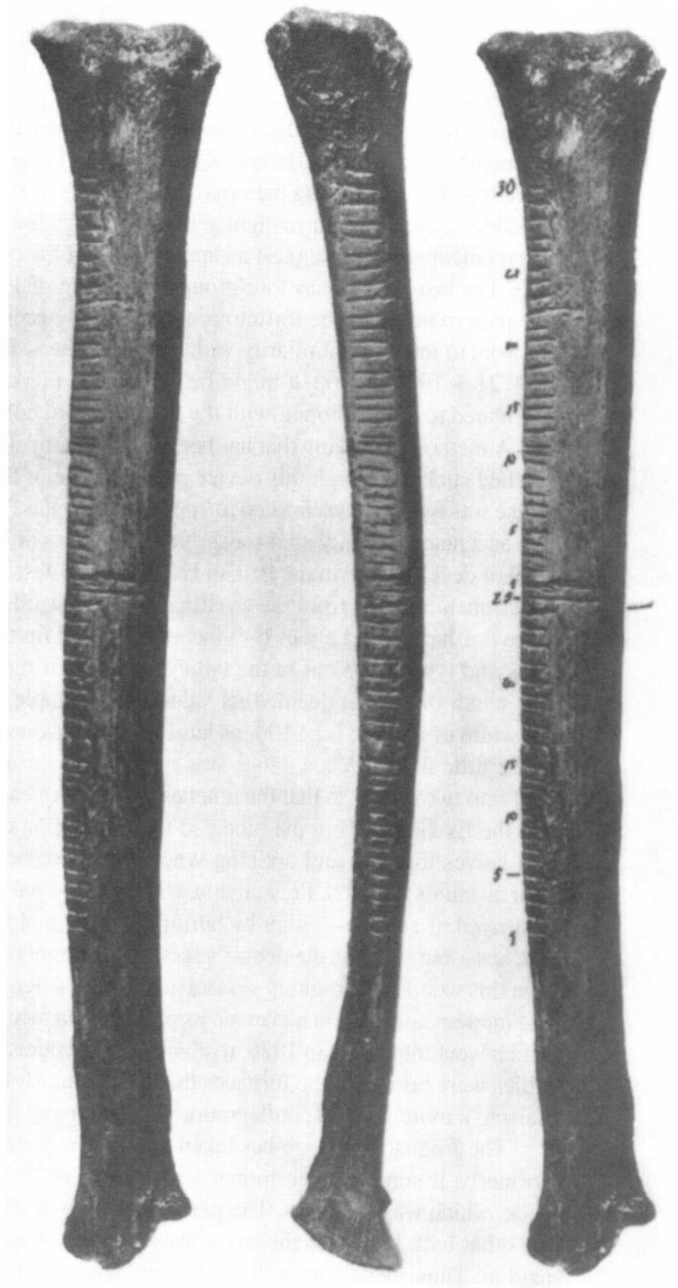
Counts were maintained by making scratches on stones, by cutting notches in wooden sticks or pieces of bone, or by tying knots in strings of different colors or lengths. When the numbers of tally marks became too unwieldy to visualize, primitive people arranged them in easily recognizable groups such as groups of five, for the fingers of a hand. It is likely that grouping by pairs came first, soon abandoned in favor of groups of 5, 10, or 20. The organization of counting by groups was a noteworthy improvement on counting by ones. The practice of counting by fives, say, shows a tentative sort of progress toward reaching an abstract concept of "five" as contrasted with the descriptive ideas "five fingers" or "five days." To be sure, it was a timid step in the long journey toward detaching the number sequence from the objects being counted.

Bone artifacts bearing incised markings seem to indicate that the people of the Old Stone Age had devised a system of tallying by groups as early as 30,000 B.C. The most impressive example is a shinbone from a young wolf, found in Czechoslovakia in 1937; about 7 inches long, the bone is engraved with 55 deeply cut notches, more or less equal in length, arranged in groups of 5. (Similar recording notations are still used, with the strokes bundled in fives, like 𝍸. Voting results in small towns are still counted in the manner devised by our remote ancestors.) For many years such notched bones were interpreted as hunting tallies and the incisions were thought to represent kills. A more recent theory, however, is that the first recordings of ancient people were concerned with reckoning time. The markings on bones discovered in French cave sites in the late 1880s are grouped in sequences of recurring numbers that agree with the numbers of days included in successive phases of the moon. One might argue that these incised bones represent lunar calendars.

Another arresting example of an incised bone was unearthed at Ishango along the shores of Lake Edward, one of the headwater sources of the Nile. The best archeological and geological evidence dates the site to 17,500 B.C., or some 12,000 years before the first settled agrarian communities appeared in the Nile valley. This fossil fragment was probably the handle of a tool used for engraving, or tattooing, or even writing in some way. It contains groups of notches arranged in three definite columns; the odd, unbalanced composition does not seem to be decorative. In one of the columns, the groups are composed of 11, 21, 19, and 9 notches. The underlying pattern may be $10 + 1$, $20 + 1$, $20 - 1$, and $10 - 1$. The notches in another column occur in eight groups, in the following order: 3, 6, 4, 8, 10, 5, 5, 7. This arrangement seems to suggest an appreciation of the concept of duplication, or multiplying by 2. The last column has four groups consisting of 11, 13, 17, and 19 individual notches. The pattern here may be fortuitous and does not necessarily indicate—as some authorities are wont to infer—a familiarity with prime numbers. Because $11 + 13 + 17 + 19 = 60$ and $11 + 21 + 19 + 9 = 60$, it might be argued that markings on the prehistoric Ishango bone are related to a lunar count, with the first and third columns indicating two lunar months.

A method of tallying that has been used in many different times and places involves the notched stick. Although this device provided one of the earliest forms of keeping records, its use was by no means limited to "primitive peoples," or for that matter, to the remote past. The acceptance of tally sticks as promissory notes or bills of exchange reached its highest level of development in the British Exchequer tallies, which formed an essential part of the government records from the twelfth century onward. In this instance, the tallies were flat pieces of hazelwood about 6–9 inches long and up to an inch thick. Notches of varying sizes and types were cut in the tallies, each notch representing a fixed amount of money. The width of the cut decided its value. For example, the notch of £1000 was as large as the width of a hand; for £100, as large as the thickness of a thumb; and for £20, the width of the little finger. When a loan was made, the appropriate notches were cut and the stick split into two pieces so that the notches appeared in each section. The debtor kept one piece and the Exchequer kept the other, so the transaction could easily be verified by fitting the two halves together and noticing whether the notches coincided (whence the expression "our accounts tallied"). Presumably, when the two halves had been matched, the Exchequer destroyed its section—either by burning it or by making it smooth again by cutting off the notches—but retained the debtor's section for future record. Obstinate adherence to custom kept this wooden accounting system in official use long after the rise of banking institutions and modern numeration had made its practice quaintly obsolete. It took an act of Parliament, which went into effect in 1826, to abolish the practice. In 1834, when the long-accumulated tallies were burned in the furnaces that heated the House of Lords, the fire got out of hand, starting a more general conflagration that destroyed the old Houses of Parliament.

The English language has taken note of the peculiar quality of the double tally stick. Formerly, if someone lent money to the Bank of England, the amount was cut on a tally stick, which was then split. The piece retained by the bank was known as the foil, whereas the other half, known as the stock, was given the lender as a receipt for the sum of money paid in. Thus, he became a "stockholder" and owned "bank stock" having the same worth as paper money issued by the government. When the holder would return, the stock was carefully checked and compared against the foil in the bank's possession; if they agreed, the owner's piece would be redeemed in currency. Hence, a written certificate that was presented for remittance and checked against its security later came to be called a "check."



Three views of a Paleolithic wolfbone used for tallying. *(The Illustrated London News Picture Library.)*

Using wooden tallies for records of obligations was common in most European countries and continued there until fairly recently. Early in this century, for instance, in some remote valleys of Switzerland, "milk sticks" provided evidence of transactions among farmers who owned cows in a common herd. Each day the chief herdsman would carve a six- or seven-sided rod of ashwood, coloring it with red chalk so that incised lines would stand out vividly. Below the personal symbol of each farmer, the herdsman marked off the amounts of milk, butter, and cheese yielded by a farmer's cows. Every Sunday after church, all parties would meet and settle the accounts. Tally sticks—in particular, double tallies—were recognized as legally valid documents until well into the 1800s. France's first modern code of law, the Code Civil, promulgated by Napoleon in 1804, contained the provision:

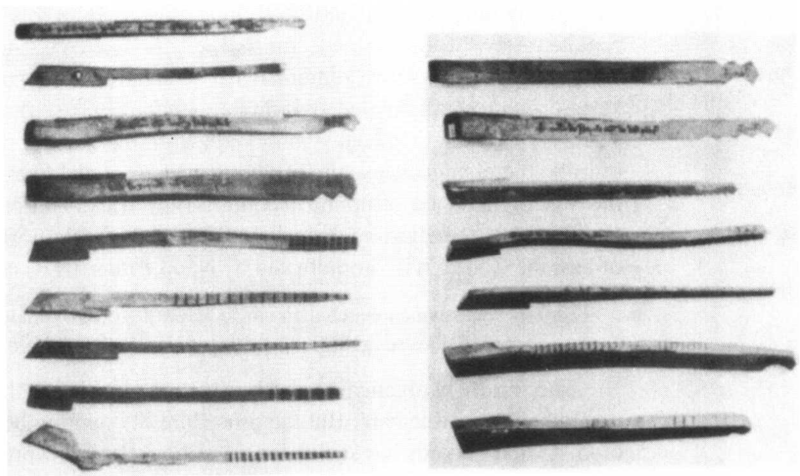
> The tally sticks which match their stocks have the force of contracts between persons who are accustomed to declare in this manner the deliveries they have made or received.

The variety in practical methods of tallying is so great that giving any detailed account would be impossible here. But the procedure of counting both days and objects by means of knots tied in cords has such a long tradition that it is worth mentioning. The device was frequently used in ancient Greece, and we find reference to it in the work of Herodutus (fifth century B.C.). Commenting in his *History*, he informs us that the Persian king Darius handed the Ionians a knotted cord to serve as a calendar:

> The King took a leather thong and tying sixty knots in it called together the Ionian tyrants and spoke thus to them: "Untie every day one of the knots; if I do not return before the last day to which the knots will hold out, then leave your station and return to your several homes."

In the New World, the number string is best illustrated by the knotted cords, called *quipus*, of the Incas of Peru. When the Spanish conquerors arrived in the sixteenth century, they observed that each city in Peru had an "official of the knots," who maintained complex accounts by means of knots and loops in strands of various colors. Performing duties not unlike those of the city treasurer of today, the quipu keepers recorded all official transactions concerning the land and subjects of the city and submitted the strings to the central government in Cuzco. The quipus were important in the Inca Empire, because apart from these knots no system of writing was ever developed there. The quipu was made of a thick main cord or crossbar to which were attached finer cords of different lengths and colors; ordinarily the cords hung down like the strands of a mop. Each of the pendent strings represented a certain item to be tallied; one might be used to show the number of sheep, for instance, another for goats, and a third for lambs. The knots themselves indicated numbers, the values of which varied according to the type of knot used and its specific position on the strand. A decimal system was used, with the knot representing units placed nearest the bottom, the tens appearing immediately above, then the hundreds, and so on; absence of a knot denoted zero. Bunches of cords were tied off by a single main thread, a summation cord, whose knots gave the total count for each bunch. The range of possibilities for numerical representation in the quipus allowed the Incas to keep incredibly detailed administrative records, despite their ignorance of the written word. More recent (1872) evidence of knots as a counting device occurs in India; some of the Santal headsmen, being illiterate, made knots in strings of four different colors to maintain an up-to-date census.

Over the long sweep of history, it seems clear that progress in devising efficient ways of retaining and conveying numerical information did not take place until primitive people



Thirteenth-century British Exchequer tallies. *(By courtesy of the Society of Antiquaries of London.)*

abandoned the nomadic life. Incised markings on bone or stone may have been adequate for keeping records when human beings were hunters and gatherers, but the food producer required entirely new forms of numerical representation. Besides, as a means for storing information, groups of markings on a bone would have been intelligible only to the person making them, or perhaps to close friends or relatives; thus, the record was probably not intended to be used by people separated by great distances.

Deliberate cultivation of crops, particularly cereal grains, and the domestication of animals began, so far as can be judged from present evidence, in the Near East some 10,000 years ago. Later experiments in agriculture occurred in China and in the New World. A widely held theory is that a climatic change at the end of the last ice age provided the essential stimulus for the introduction of food production and a settled village existence. As the polar ice cap began to retreat, the rain belt moved northward, causing the desiccation of much of the Near East. The increasing scarcity of wild food plants and the game on which people had lived forced them, as a condition of survival, to change to an agricultural life. It became necessary to count one's harvest and herd, to measure land, and to devise a calendar that would indicate the proper time to plant crops. Even at this stage, the need for means of counting was modest; and tallying techniques, although slow and cumbersome, were still adequate for ordinary dealings. But with a more secure food supply came the possibility of a considerable increase in population, which meant that larger collections of objects had to be enumerated. Repetition of some fundamental mark to record a tally led to inconvenient numeral representations, tedious to compose and difficult to interpret. The desire of village, temple, and palace officials to maintain meticulous records (if only for the purposes of systematic taxation) gave further impetus to finding new and more refined means of "fixing" a count in a permanent or semipermanent form.

Thus, it was in the more elaborate life of those societies that rose to power some 6000 years ago in the broad river valleys of the Nile, the Tigris-Euphrates, the Indus, and the Yangtze that special symbols for numbers first appeared. From these, some of our most elementary branches of mathematics arose, because a symbolism that would allow expressing large numbers in written numerals was an essential prerequisite for computation and measurement. Through a welter of practical experience with number symbols, people gradually recognized certain abstract principles; for instance, it was discovered that in the fundamental operation of addition, the sum did not depend on the order of the summands. Such discoveries were hardly the work of a single individual, or even a single culture, but more a slow process of awareness moving toward an increasingly abstract way of thinking.

We shall begin by considering the numeration systems of the important Near Eastern civilizations—the Egyptian and the Babylonian—from which sprang the main line of our own mathematical development. Number words are found among the word forms of the earliest extant writings of these people. Indeed, their use of symbols for numbers, detached from an association with the objects to be counted, was a big turning point in the history of civilization. It is more than likely to have been a first step in the evolution of humans' supreme intellectual achievement, the art of writing. Because the recording of quantities came more easily than the visual symbolization of speech, there is unmistakable evidence that the written languages of these ancient cultures grew out of their previously written number systems.

1.2 Number Recording of the Egyptians and Greeks

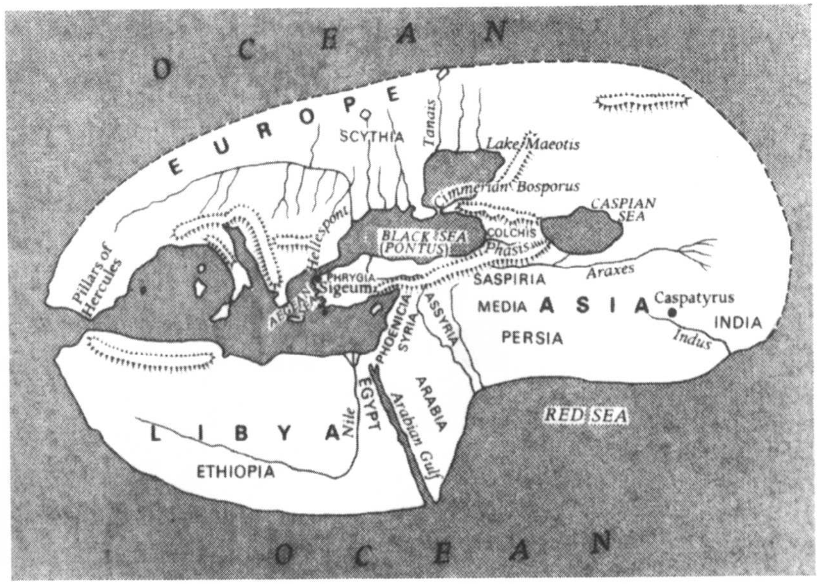
The writing of history, as we understand it, is a Greek invention; and foremost among the early Greek historians was Herodotus. Herodotus (circa 485–430 B.C.) was born at Halicarnassus, a largely Greek settlement on the southwest coast of Asia Minor. In early life, he was involved in political troubles in his home city and forced to flee in exile to the island of Samos, and thence to Athens. From there Herodotus set out on travels whose leisurely character and broad extent indicate that they occupied many years. It is assumed that he made three principal journeys, perhaps as a merchant, collecting material and recording his impressions. In the Black Sea, he sailed all the way up the west coast to the Greek communities at the mouth of the Dnieper River, in what is now Russia, and then along the south coast to the foot of the Caucasus. In Asia Minor, he traversed modern Syria and Iraq, and traveled down the Euphrates, possibly as far as Babylon. In Egypt, he ascended the Nile River from its delta to somewhere near Aswan, exploring the pyramids along the way. Around 443 B.C., Herodotus became a citizen of Thurium in southern Italy, a new colony planted under Athenian auspices. In Thurium, he seems to have passed the last years of his life involved almost entirely in finishing the *History of Herodotus*, a book larger than any Greek prose work before it. The reputation of Herodotus as a historian stood high even in his own day. In the absence of numerous copies of books, it is natural that a history, like other literary compositions, should have been read aloud at public and private gatherings. In Athens, some 20 years before his death, Herodotus recited completed portions of his *History* to admiring audiences and, we are told, was voted an unprecedentedly large sum of public money in recognition of the merit of his work.

Although the story of the Persian Wars provides the connecting link in the *History of Herodotus*, the work is no mere chronicle of carefully recorded events. Almost anything that concerned people interested Herodotus, and his *History* is a vast store of information on all manner of details of daily like. He contrived to set before his compatriots a general picture of the known world, of its various peoples, of their lands and cities, and of what they did and above all why they did it. (A modern historian would probably describe the *History* as a guidebook containing useful sociological and anthropolotical data, instead of a work of history.) The object of his *History*, as Herodotus conceived it, required him to tell all he had heard but not necessarily to accept it all as fact. He flatly stated, "My job is to report what people say, not to believe it all, and this principle is meant to apply to my whole work." We find him, accordingly, giving the traditional account of an occurrence and then offering his own interpretation or a contradictory one from a different source, leaving the reader to choose between versions. One point must be clear: Herodotus interpreted the state of the world at his time as a result of change in the past, and felt that the change could be described. It is this attempt that earned for him, and not any of the earlier writers of prose, the honorable title "Father of History."

Herodotus took the trouble to describe Egypt at great length, for he seems to have been more enthusiastic about the Egyptians than about almost any other people that he met. Like most visitors to Egypt, he was distinctly aware of the exceptional nature of the climate and the topography along the Nile: "For anyone who sees Egypt, without having heard a word about it before, must perceive that Egypt is an acquired country, the gift of the river." This famous passage—often paraphrased to read "Egypt is the gift of the Nile"—aptly sums up the great geographical fact about the country. In that sun-soaked, rainless climate, the river in overflowing its banks each year regularly deposited the rich silt washed down from the East African highlands. To the extreme limits of the river's waters there were fertile fields for crops and the pasturage of animals; and beyond that the barren desert frontiers stretched in all directions. This was the setting in which that literate, complex society known as Egyptian civilization developed.

The emergence of one of the world's earliest cultures was essentially a political act. Between 3500 and 3100 B.C., the self-sufficient agricultural communities that clung to the strip of land bordering the Nile had gradually coalesced into larger units until there were only the two kingdoms of Upper Egypt and Lower Egypt. Then, about 3100 B.C., these regions were united by military conquest from the south by a ruler named Menes, an elusive figure who stepped forth into history to head the long line of pharaohs. Protected from external invasion by the same deserts that isolated her, Egypt was able to develop the most stable and longest-lasting of the ancient civilizations. Whereas Greece and Rome counted their supremacies by the century, Egypt counted hers by the millennium; a well-ordered succession of 32 dynasties stretched from the unification of the Upper and Lower Kingdoms by Menes to Cleopatra's encounter with the asp in 31 B.C. Long after the apogee of Ancient Egypt, Napoleon was able to exhort his weary veterans with the glory of its past. Standing in the shadow of the Great Pyramid of Gizeh, he cried, "Soldiers, forty centuries are looking down upon you!"

As soon as the unification of Egypt under a single leader became an accomplished fact, a powerful and extensive administrative system began to evolve. The census had to be taken, taxes imposed, an army maintained, and so forth, all of which required reckoning

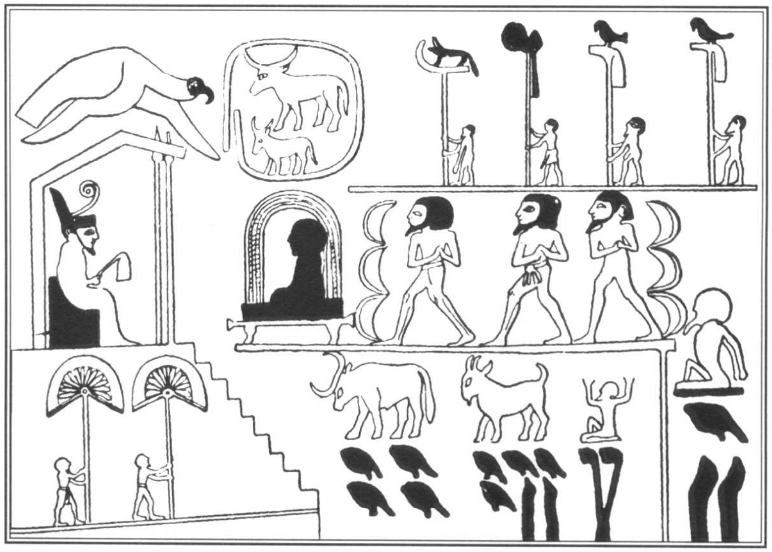


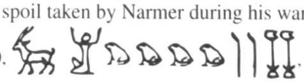
The habitable world according to Herodotus. *(From* Stories from Herodotus *by B. Wilson and D. Miller. Reproduced by permission of Oxford University Press.)*

with relatively large numbers. (One of the years of the Second Dynasty was named Year of the Occurrence of the Numbering of all Large and Small Cattle of the North and South.) As early as 3500 B.C., the Egyptians had a fully developed number system that would allow counting to continue indefinitely with only the introduction from time to time of a new symbol. This is borne out by the macehead of King Narmer, one of the most remarkable relics of the ancient world, now in a museum at Oxford University. Near the beginning of the dynastic age, Narmer (who, some authorities suppose, may have been the legendary Menes, the first ruler of the united Egyptian nation) was obliged to punish the rebellious Libyans in the western Delta. He left in the temple at Hierakonpolis a magnificent slate palette—the famous Narmer Palette—and a ceremonial macehead, both of which bear scenes testifying to his victory. The macehead preserves forever the official record of the king's accomplishment, for the inscription boasts of the taking of 120,000 prisoners and a register of captive animals, 400,000 oxen and 1,422,000 goats.

Another example of the recording of very large numbers at an early stage occurs in the *Book of the Dead*, a collection of religious and magical texts whose principle aim was to secure for the deceased a satisfactory afterlife. In one section, which is believed to date from the First Dynasty, we read (the Egyptian god Nu is speaking): "I work for you, o ye spirits, we are in number four millions, six hundred and one thousand, and two hundred, ..."

The spectacular emergence of the Egyptian government and administration under the pharaohs of the first two dynasties could not have taken place without a method of writing;



The scene above is taken from the great stone macehead of Narmer, which J. E. Quibell discovered at Hierakonpolis in 1898. There is a summary of the spoil taken by Narmer during his wars, namely "cows, 400,000,  goats, 1,422,000, , and captives, 120,000, ."

Scene reproduced from the stone macehead of Narmer, giving a summary of the spoil taken by him during his wars. *(From* The Dwellers on the Nile *by E. W. Budge, 1977, Dover Publications, N.Y.)*

and we find such a method both in the elaborate "sacred signs," or hieroglyphics, and in the rapid cursive hand of the accounting scribe. The hieroglyphic system of writing is a picture script, in which each character represents a concrete object, the significance of which may still be recognizable in many cases. In one of the tombs near the Pyramid of Gizeh there have been found hieroglyphic number symbols in which the number one is represented by a single vertical stroke, or a picture of a staff, and a kind of horseshoe, or heelbone sign ∩ is used as a collective symbol to replace ten separate strokes. In other words, the Egyptian system was a decimal one (from the Latin *decem*, "ten") which used counting by powers of 10. That 10 is so often found among ancient peoples as a base for their number systems is undoubtedly attributable to humans' ten fingers and to our habit of counting on them. For the same reason, a symbol much like our numeral 1 was almost everywhere used to express the number one.

Special pictographs were used for each new power of 10 up to 10,000,000: 100 by a curved rope, 1000 by a lotus flower, 10,000 by an upright bent finger, 100,000 by a tadpole,