Advances in Fuzzy Mathematics and Engineering

# Fuzzy Sets and Fuzzy Information-Granulation Theory

Key Selected Papers by Lotfi A. Zadeh

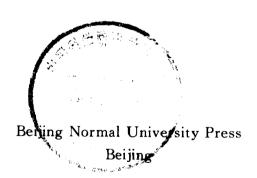
Beijing Normal University Press

51.98

## Fuzzy Sets and Fuzzy Information-Granulation Theory

Key selected papers by Lotfi A. Zadeh

Edited by Da Ruan Chongfu Huang



### 图书在版编目(CIP)数据

模糊集与模糊信息粒理论:英文版/(美)扎德(Zadeh, L. A.)著.—北京:北京师范大学出版社,2000.5 (模糊教学与工程新进展国际系列丛书) ISBN 7-303-05324-7

Ⅰ.模… Ⅰ.扎… Ⅱ.模糊集-文集-英文 №.0159-53

中国版本图书馆 CIP 数据核字(2000)第 02100 号 北京市版权局著作权合同登记图字 01-2000-0187 号

北京师范大学出版社出版发行 (北京新街口外大街 19 号 邮政编码:100875) 出版人:常汝吉

北京师范大学印刷厂印刷 全国新华书店经销 开本:890mm×1 240mm 1/32 印张:16:375 字数:472千字 2000年5月第1版 2000年5月第1次印刷 三价: 20.00 元

## The International Committee of the Series

Etienne E. Kerre (Belgium)

Da Ruan (Belgium)

Madan M. Gupta (Canada)

Liu Yingming (China)

Wang Peizhuang (China)

Huang Chongfu (China)

Didier Dubois (France)

Hans J. Zimmermann (Germany)

Yee Leung (HK, China)

Hiroshi Inoue (Japan)

Masahara Mizumoto (Japan)

Zeungnam Bien (Korea)

A. J. van der Wal (Netherlands)

Janusz Kacprzyk (Poland)

Bart Kosko (USA)

George Klir (USA)

Paul P. Wang (USA)

(Note: the order is according to the countries' names)

### **Preface**

Advances in Fuzzy Mathematics and Engineering is a new international series dedicated to the support and development of the theory of fuzzy mathematics and related areas and their industrial applications in general and in engineering in particular. The series is supported and published by Beijing Normal University Press, Beijing, China.

This book, Fuzzy Sets and Fuzzy Information-Granulation Theory, is the third volume of Collected Papers by Lotfi A. Zadeh. The first volume, entitled Fuzzy Sets and Applications, was published in 1987 by John Wiley. Its editors, Ronald R. Yager, Sergei Ovchinnikov, Richard M. Tong, and Hung T. Ngugen undertook the project on the occasion of the 20th anniversary of the publication of the first paper on fuzzy set by Lotfi A. Zadeh. The second volume, entitled Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems, was published in 1996 by World Scientific. Its editors, George J. Klir and Bo Yuan selected, from among all papers by Lotfi Zadeh not included in the first volume, those papers on fuzzy sets, fuzzy logic, and fuzzy systems whose easy accessibility would likely be of benefit to those working in these areas. The previous two volumes in English have proved to be great utility to anyone interested in fuzzy set theory and its applications.

1

Considering the largest number of the readers related to Fuzzy Mathematics and Engineering in China, we felt that a Chinese version of the key selected papers by Lotfi Zadeh Fuzzy Sets and Fuzzy Information-Granulation Theory would fit well into the book series on Advances in Fuzzy Mathematics and Engineering by Beijing Normal University Press. The book collects Zadeh's original perception which may be viewed as an evolution of ideas rooted in his 1965 paper on fuzzy sets; 1971 paper on fuzzy systems; 1973 — 1976 papers on linguistic variables, fuzzy if-then rules and fuzzy graphs; 1979 paper on fuzzy sets and information granularity; 1986 paper on generalized constrains; 1996 paper on computing with words and 1997 papers on theory of fuzzy information granulation.

The purpose of this book is twofold. Firstly, it is intended as a quick reference for those working in Fuzzy Mathematics and Engineering in China as well as in the world. Secondly, it is expected to play a major role in-Research and Development of Fuzzy Mathematics and Engineering, as a useful source of supplementary readings in this new book series. We hope this volume will benefit many readers around the world.

Da Ruan, Chongfu Huang Editors

## Note to the Reader

This book is the third volume of collected papers by Lotfi A. Zadeh. The first volume entitled Fuzzy Sets and Applications, was published in 1987 by John Wiley (Editors: Ronald R. Yager, Sergei Ovchinnikov, Richard M. Tong, and Hung T. Ngugen). The second volume entitled Fuzzy Sets. Fuzzy Logic, and Fuzzy Systems, was published in 1996 by World Scientific (Editors: George J. Klir and Bo Yuan).

The current edited-book is a set of key selected papers by Lotfi Zadeh. Both English and Chinese versions of these papers are available.

## Acknowledgements

This book consists of the following reprinted (both retyped and translated) papers. The relevant copyright owners whose permissions to reproduce the papers in this book are gratefully acknowledged.

#### Academic Press:

- L. A. Zadeh, "Fuzzy sets." Information and Control, 8(3), pp. 338-353, 1965.
- L. A. Zadeh, "A fuzzy-algorithmic approach to the definition of complex or imprecise concepts," Internat. J. Man-Machine Stud. 8.pp.  $249 \sim 291$ . 1976.

#### Elsevier Science:

- L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning, I, II, III." Information Sciences, 8(3), pp. 199 ~ 251, (4)301~357;9, pp. 43~80, 1975.
- L. A. Zadeh, "Fuzzy sets and information granularity." In: Gupta, M. M., R. K. Ragade, and R. R. Yager, eds., Advances in Fuzzy Set Theory and Applications. North-Holland, New York, pp. 3~18,1979.
- L. A. Zadeh, "Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic." Fuzzy Sets and Systems 90 (2), pp. 111~127,1997.

### Institute of Electrical and Electronics Engineers:

L. A. Zadeh, "Outline of a new approach to the analysis of complex

system and decision processes." *IEEE Trans. On Systems*, Man, and Cybernectics, 1(1), pp. 28~44,1973.

L. A. Zadeh, "Fuzzy logic = computing with words." *IEEE Trans. On Fuzzy Systems* 4.pp. 103~111.1996.

### Oxford University Press Inc. :

L. A. Zadeh, "Towards a theory of fuzzy systems." In Kalman, R. E. and N. Declaris, eds. Aspects of Networks and Systems Theory. Holt, Rinehart &. Winston, New pp. 469~490, 1971.

#### Springer Verlag:

L. A. Zadeh, "Outline of a computational approach to meaning and knowledge representation based on the concept of a generalized assignment statement." In: Thoma, M. And W. Wyner, eds., Proc. Of the International Seminar on AI and Man-Machine Systems. Springer-Verlag, Heidelberg, pp. 198~211,1986.

## Lotfi A. Zadeh's biography

Lotfi A. Zadeh joined the Department of Electrical Engineering at the University of California, Berkeley, in 1959, and served as its chairman from 1963 to 1968. Earlier, he was a member of the electrical engineering faculty at Columbia University. In 1956, he was a visiting member of the Institute for Advanced Study in Princeton, New Jersey. In addition, he held a number of other visiting appointments, among them a visiting professorship in Electrical Engineering at MIT in 1962 and 1968; a visiting scientist appointment at IBM Research Laboratory, San Jose, CA, in 1968, 1973, and 1977; and visiting scholar appointments at the AI Center, SRI International, in 1981, and at the Center for the Study of Language and Information, Stanford University, in 1987—1988. Currently he is a Professor in the Graduate School, and is serving as the Director of BISC (Berkeley Initiative in Soft Computing).

Until 1965. Dr. Zadeh's work had been centered on system theory and decision analysis. Since then his research interests have shifted to the theory of fuzzy sets and its applications to artificial intelligence linguistics logic decision analysis, control theory, expert systems and neural networks. Currently, his research is focused on fuzzy logic, soft computing and computing with words.

An alumnus of the University of Teheran, MIT, and Columbia

University, Dr. Zadeh is a fellow of the IEEE, AAAS, ACM and AAAI, and a member of the National Academy of Engineering. He was the recipient of the IEEE Education Medal in 1973 and a recipient of the IEEE Centennial Medal in 1984. In 1989, Dr. Zadeh was awarded the Honda Prize by the Honda Foundation, and in 1991 received the Berkeley Citation, University of California.

In 1992.Dr. Zadeh was awarded the IEEE Richard W. Hamming Medal "For seminal contributions to information science and systems including the conceptualization of fuzzy sets." He became a Foreign Member of the Russian Academy of Natural Sciences (Computer Sciences and Cybernetics Section) in 1992 and received the Certificate of Commendation for AI Special Contributions Award from the International Foundation for Artificial Intelligence. Also in 1992, he was awarded the Kampe de Feriet Medal and became an Honorary Member of the Austrian Society of Cybernetic Studies.

In 1993, Dr. Zadeh received the Rufus Oldenburger Medal from the American Society of Mechanical Engineers "For seminal contributions in system theory decision analysis, and theory of fuzzy sets and its applications to AI, linguistics, logic, expert systems and neural networks." He was also awarded the Grigore Moisil Prize for Fundamental Researches, and the Premier Best Paper Award by the Second International Conference on Fuzzy Theory and Technology. In 1995, Dr. Zadeh was awarded the IEEE Medal of Honor "For pioneering development of fuzzy logic and its many diverse applications." In 1996, Dr. Zadeh was awarded the Okawa Prize "For Outstanding contribution to information science through the development of fuzzy logic and its applications."

In 1997, Dr. Zadeh was awarded the B. Bolzano Medal by the Academy of Sciences of the Czech Republic "For outstanding achievements in fuzzy mathematics." He also received the J. P. Wohl Outstanding Career

8

Achievement Award of the IEEE Systems, Man and Cybernetics Society. He served as a Lee Kuan Yew Distinguished Visitor, lecturing at the National University of Singapore and the Nanyang Technological University in Singapore, and as the Gulbenkian Foundation Visiting Professor at the New University of Lisbon in Portugal.

Dr. Zadeh holds honorary doctorates from Paul-Sabatier University. Toulouse, France; State University of New York, Binghamton, NY: University of Dortmund, Dortmund, Germany; University of Oviedo, Oviedo, Spain; University of Granada, Granada, Spain; Lakehead University, Canada; University of Louisville, KY; Baku State University, Azerbaijan; and the Silesian Technical University, Gliwice, Poland.

Dr. Zadeh has authored close to two hundred papers and serves on the editorial boards of over fifty journals. He is a member of the Technology Advisory Board, U. S. Postal Service; Advisory Committee, Department of Electrical and Computer Engineering, UC Santa Barbara; Advisory Board, Fuzzy Initiative, North Rhine-Westfalia, Germany; Fuzzy Logic Research Center, Texas A & M University, College Station, Texas; Advisory Committee, Center for Education and Research in Fuzzy Systems and Artificial Intelligence, Iasi, Romania; Senior Advisory Board, International Institute for General Systems Studies; the Board of Governors, International Neural Networks Society.

#### Contents

## Part 1: Fuzzy Sets

Fuzzy sets 3

## Part 2: Fuzzy Systems

Towards a theory of fuzzy systems 29

Outline of a new approach to the analysis of complex systems and decision processes 62

## Part 3: Linguistic variable and approximate reasoning

The concept of a linguistic variable and its application to approximate reasoning, I, I, I 121

## Part 4: Fuzzy-algorithmic approach and information granularity

A fuzzy-algorithmic approach to the definition of complex or imprecise concepts 307

Fuzzy sets and information granularity 384

Outline of a computational approach to meaning and knowledge representation based on the concept of a generalized assignment statement 412

## Part 5:Soft computing with words

Fuzzy logic=Computing with words 435

Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic 461

What is soft computing? 504

## Part 1: Fuzzy Sets

## **Fuzzy Sets**

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

#### 1. Introduction

More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.

Clearly, the "class of all real numbers which are much greater than 1," or "the class of beautiful women," or "the class of tall men," do not constitute classes or sets in the usual