Experimental Methods in Magnetism

2. MEASUREMENT OF MAGNETIC QUANTITIES

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PREFACE

Only too often in the literature on magnetic phenomena little or no information is given on the measuring methods used. In particular it is not always clear to what extent the result is influenced by the methods of preparation and measurement of the sample. Literature on measuring apparatus is scattered in a great number of scientific periodicals and fluctuates considerably as regards what is considered as known and what is explained.

This book presents a treatment of the principles of a number of widely-used methods for measuring magnetic quantities. It is hoped that the book will be useful to those engaged in magnetic experiments, whether in scientific research or in routine measurements. The reader is presumed to be familiar with the basic concepts of magnetism and with elementary vector analysis. The latter, however, is not necessary if the proofs are taken for granted and only the results are used.

To increase the practical value of this book much attention has been given to those details of a method where errors are liable to be introduced and how this can be prevented.

The book is certainly not meant to give a complete survey of the literature. The examples are mainly chosen from the immediate neighbourhood of the author; this does not of course imply that there may not be better or earlier ones. However, several chapters are provided with a bibliography of books and survey articles consulted by the author; these contain fairly complete lists of the literature.

For technical reasons the book is split into two parts. The first part contains two chapters on the theory of the magnetic potential needed for the understanding (though not for the application) of several subjects discussed in the subsequent chapters of the book. It further contains a chapter on the generation of magnetic fields by ironless solenoids. This chapter is rather detailed as it is considered that the experimenter is often faced with the design and construction of coils of all sorts when building his apparatus. On the other hand a chapter on iron-core magnets is kept very elementary as these magnets are commonly bought and thus only a limited understanding of their differences is required.

Part 2 of the book deals with the measurement of magnetic quantities. For the discussion of the various methods use is often made of the results obtained in Part 1. Therefore the two parts should be considered as one whole. Two important fields are not treated, namely neutron diffraction and domain techniques. This is because the experimental techniques used in the former field are covered in detail in a book by BACON [1955] and two books on the latter subject have appeared recently (CRAIK and TEBBLE [1965] and CAREY and ISAAC [1966]). The chapter on resonance methods (Ch. 6) is kept very elementary because here too several books are available, mentioned at the end of that chapter.

As to the remaining subjects it is hoped that the book presents a useful supplement to the already existing literature. If so, this is in no small measure due to the generous help I received from my colleagues K. Compaan, P. Cornelius, U. Enz, W.P.J. Fontein, N.J. Freedman, P.R. Locher, A.L. Luiten, G.W. Van Oosterhout, R.P. Van Stapele, D.L.A. Tjaden, J.S. Van Wieringen, D. Wilkinson and from the Editor of this series E.P. Wohlfarth. Their assistance is gratefully acknowledged here.

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LIST OF MOST IMPORTANT SYMBOLS

The symbols may have other meanings, incidentally, than those mentioned below. The list applies to both Parts 1 and 2.

A	Area
$\mathbf{B}$, B	Magnetic induction (flux density)
C	Specific heat
C	Capacitance
C	Curie constant
E	Potential energy
$\mathbf{E}$	Electric fieldstrength
e	Electromotive force
$\mathbf{F}$, F	Force
f	Electric fieldstrength
f	Mössbauer fraction
f	Frequency
G	Shear modulus
G	Galvanometer constant
$\mathbf{H}$, H	Magnetic field
$\mathbf{I}$, I	Magnetization
i	Electric current
J_l	Bessel function of order l
J	Polar moment of inertia
$\mathbf{J}$, J	Angular momentum
K	Bulk modulus (modulus of compression)
K	Anisotropy constant

k	Torsion constant
k	Thermal conductivity
L	Inductance
l	Length
M	Mutual inductance
M	Molar mass
M	Mass
M	Torque
$\mathbf{m}$	Magnetic moment
N	Number of turns
N	Number of photons
$\|N\|$	Demagnetization tensor
N_f	Fluxmetric or ballistic demagnetization factor
N_m	Magnetometric demagnetization factor
n	Density of turns
P_l	Legendre polynomial
P_l^m	Associated Legendre polynomial
p	Pressure
p	Dipole density
p	Volume force density
Q	Factor of merit
Q_0	Magnetic charge
Q_l	Strength of magnetic 2^l-pole
q	Charge density
R	Resistance
R_H	Hall constant
r	Reduced resistivity
$\mathbf{r}$	Radius vector
S	Stress
s	Stress density
s	Angular momentum
T	Temperature
T	Volume
T	Torque

t	Time
V	Magnetic potential
V	Voltage
V	Volume
W	Power (energy per unit time)
Y	Young's modulus of elasticity
Y_l^m	Spherical harmonic
Z	Impedance
α	Dimensional ratio
α	Direction cosine
β	Dimensional ratio
β	Direction cosine
γ	Dimensional ratio
γ	Gyromagnetic ratio
Δ	Laplace operator
δ	Difference operator
δ	Packing density (volume fraction occupied by matter)
ε	Strain
ε	Absorption coefficient
ζ	Dimensional ratio
ζ	Heat transfer coefficient
η	Electrical resistivity
θ	Polar angle
θ	Debye temperature
ϑ	Reduced temperature
κ	Efficiency
λ	Packing density (volume fraction occupied by matter)
λ	Magnetostriction constant
μ	Absolute permeability
μ_0	Permeability of vacuum
μ_r	Relative permeability (vacuum$=1$)
ξ	Surfacial current density

ξ	Poisson's ratio of contraction
ξ	Lagrange's undetermined multiplier
ρ	Radius
ρ_V	Verdet's constant
σ	Area
σ	Absorbing cross-section
τ	Area
τ	Time constant, (relaxation time)
τ	Current density
Φ	Magnetic flux
φ	Azimuthal angle
χ	Magnetic susceptibility
ψ	Electric potential
ω	Angular frequency
∇	Gradient operator

CONTENTS

CHAPTER 1

MEASUREMENT OF MAGNETIC FIELDSTRENGTH

§ 1. INTRODUCTION

There are several ways of measuring magnetic fieldstrength. In this chapter a few will be treated, especially those that can be used more or less universally and can be easily adapted to special purposes.

The measurement of the fieldstrength in a solenoid is the most simple case. The fieldstrength is proportional to the current through the winding of the coil, provided that the dimensions and relative current distribution are independent of the strength of the current. The proportionality factor may be determined either by calculation (see Part 1, Ch. 3, § 1) or by comparison with a standard field. The field measurement then reduces to the measurement of the strength of a current.

In other magnets, however, there is not such a simple relation between current and field. We then use methods where a probe is brought into the field which gives a signal depending in a known way on the fieldstrength. The probe can be a moving coil and the signal the voltage induced in the winding of the coil (fluxmetric method, rotating and vibrating coils). We could also measure the effect of the Lorentz force exerted on a moving electron (Hall effect) or the rigidity of the coupling between the field and an electron spin moment or a nuclear magnetic moment (electron spin resonance and nuclear magnetic resonance), etc. In the first two cases we actually measure the induction B of the field which is related to the fieldstrength H by

$$B = \mu_0 H$$

in vacuum. Since the relative magnetic permeability of air and other gases at normal pressure and temperature does not deviate appreciably from unity ($<10^{-6}$) the difference between measurements in vacuum and in air will be ignored. Even when measuring in liquids correction will seldom be necessary. Liquid oxygen, for instance, being one of the strongest paramagnetic liquids, has a relative permeability of 1.0003.

§ 2. THE FLUXMETRIC METHOD

§ 2.1. *Principle*

The fluxmetric method consists of placing a coil in the magnetic field so that part of that field is enclosed as a magnetic flux through the coil area. Switching off the field or removing the coil from the field gives rise to an induction emf that can be derived from Maxwell's law

$$\text{curl}\, \boldsymbol{E} = -\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t}. \tag{1.1}$$

Application of Stokes' theorem gives

$$\oint \boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int \boldsymbol{B}\cdot\mathrm{d}\boldsymbol{s} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}, \tag{1.2}$$

where $\boldsymbol{E}$ is the electric fieldstrength to be integrated along a complete turn of the coil, $\boldsymbol{B}$ is the magnetic induction to be integrated over the area enclosed by that turn to give the magnetic flux Φ. This has to be done for each turn of the coil which results in

$$e_{\text{ind}} = -\sum_{N}\frac{\mathrm{d}\Phi}{\mathrm{d}t}, \tag{1.3}$$

where e_{ind} is the total electromotive force generated in the coil and the summation is made for all N turns of the coil. Integration of both sides of eq. (1.3) gives

$$\int_0^{t_0} e_{\mathrm{ind}}\,\mathrm{d}t = -\sum_N \Phi = AB\,, \tag{1.4}$$

where B is the induction along the axis of the coil and A is called the effective area.

The integration is extended from $t=0$, when the switching-off or the removal of the coil is started, until $t=t_0$ where the process has come to an end.

The time integral of e_{ind} can be measured by a suitable integrator of electric voltages, for instance a ballistic galvanometer or an electronic integrator for microvolts. These instruments will be treated in §§ 2.4 and 2.5 of this chapter.

§ 2.2. *Fluxmeter coils*

We see in eq. (1.4) that instead of measuring B at a certain point, we measure its mean value over a certain area. If B is not homogeneous this may cause errors that may be reduced by making the area of the coil small. This, however, reduces the sensitivity of the measurement.

We shall see in this and the next section how to make coils that approximate to measuring the fieldstrength B at a point, thereby conserving a good sensitivity.

An inhomogeneous field of which the induction of its z-component is required at a certain point, can be evaluated about this point in a three-dimensional Maclaurin's series:

$$B_z = B_z^0 + x\frac{\partial B_z}{\partial x} + y\frac{\partial B_z}{\partial y} + z\frac{\partial B_z}{\partial z} + \\ + \frac{x^2}{2!}\frac{\partial^2 B_z}{\partial x^2} + \frac{y^2}{2!}\frac{\partial^2 B_z}{\partial y^2} + \frac{z^2}{2!}\frac{\partial^2 B_z}{\partial z^2} + \cdots, \tag{1.5}$$

where B_z^0 is the value of B_z at the point under consideration and the derivatives are also taken at that point.

Now consider a cylindrical coil of radius ρ, length 2ζ, with its axis along the z-direction. The thickness of the layer of turns is assumed to be infinitesimally small (Fig. 1.1). The density of

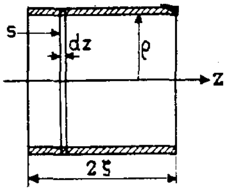

Fig. 1.1. Thin-walled fluxmeter coil.

turns is n per unit of axial length. The flux enclosed by one turn with area s is

$$\int_s B_z \, ds . \tag{1.6}$$

An elementary disk of height dz contains $n dz$ turns. So the flux enclosed by the winding of the disk is

$$n \, dz \int_s B_z \, ds \tag{1.7}$$

and the flux enclosed by the winding of the whole coil is

$$\Phi = n \int_T B_z \, d\tau , \tag{1.8}$$

where T is the volume enclosed by the cylindrical wall of the coil and its flat faces.

Now we substitute eq. (1.5) for B_z and calculate Φ by integrating eq. (1.8) term by term. If the point about which B_z is expanded coincides with the centre of the coil, all terms with odd powers of coordinates vanish by integration over the volume of the coil.