DEVELOPMENTS IN GEOTECHNICAL ENGINEERING 53

NONLINEAR ANALYSIS IN SOIL MECHANICS

Theory and Implementation

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PREFACE

A complete analysis of stress and strain in a structure, as the load is increased to failure, is known to be very complicated. This is particularly true in soil mechanics and soil–structure interaction problems where, unlike traditional structural engineering, the analysis almost always involves two- and three-dimensional continua. With the present state of development of finite-element computer softwares and high-speed digital computer hardwares, we can confidently say that an almost unlimited range of solutions can now be obtained. These are not limited to linear elastic small-deformation solid mechanics but can be extended to include problems of various kinds involving material and geometric nonlinearities. This book is concerned with the development of numerical tools for solutions of nonlinear analysis problems in soil mechanics.

The mathematical theories of elasticity and plasticity are employed for the constitutive modeling of the soil behavior. The theoretical foundation and basic concepts of material modeling are described in details in PART II including critical discussions of the Theory of Elasticity and Modeling in Chapter 3, the Theory of Perfect Plasticity and Modeling in Chapter 4, and the Theory of Hardening Plasticity and Modeling in Chapter 5.

The material nonlinearity is represented in PART III by an elastic-plastic cap-type model which can treat either strain-hardening or strain-softening materials. The plasticity formulation and calibration (Chapter 6) together with the numerical algorithm developed for its implementation and predictions (Chapter 7) provide a general format for incorporating various plasticity models described previously in PART II into the cap-type of plasticity relationship of PART III suitable for direct finite-element applications. In particular, the Drucker-Prager model with an elliptic hardening cap illustrated in details in Chapter 6 and coded in Chapter 7 as a SUBROUTINE CAPMDL represents an adequate constitutive model for the short-time behavior of many geological materials over a wide range of loadings.

The geometric nonlinearity for large-deformation analyses is described in details in PART IV. The theoretical foundation of large-deformation formulation and its simplification are described in details in Chapter 8. The nonlinear finite-element equations for large deformation and material inelasticity are solved by a combined incremental and iterative solution technique (Chapter 9). Two types of iterations are carried out, the first being iteration on the material parameters and the second equilibrium iterations. The method of analysis is based on a displacement formulation of the finite-element method. Large displacements are accounted for using a total Lagrangian formulation and an updated Lagrangian formulation.

Applications of the nonlinear theory are presented for a wide range of numerical examples throughout the book. In particular, several case studies using the computer program developed are presented in PART V (Chapter 10). First the material nonlinearities involving associated as well as non-associated flow rules with different procedures for the determination of the material parameters from the experimental data are examined for strip footings on stratum of clay. The algorithms for the behavior of the clay are then tested and checked for the simplification of small-strain small-rotation formulation. Finally, small-strain large-rotation analyses of elasto-plastic behavior are carried out for vertical slopes under seismic loading. This analytical simplification is reasonable for most large-displacement problems in geotechnical engineering applications and results in symmetric governing equations in the large-displacement finite-element analysis. The complete progressive failure behavior of slopes at all stress levels leading up to collapse is obtained. The collapse loads of the slopes and their associated failure mechanisms by the finite-element method are compared with those of the limit analysis method.

The book can be used for nonlinear analysis courses in geotechnical engineering of various lengths, involving mathematical modeling of materials (PART II and PART III), large-deformation finite-element description of structures (PART IV), and computer implementation and numerical predictions (PART V). In writing this book, we have endeavored to make the prerequisites as few as possible. Some background on the theory of linear elasticity and the finite-element method is assumed. The book is aimed to the graduate student in civil engineering who has sufficient background in soil mechanics and is learning about inelastic behavior of materials and large-displacement description of structures for the first time. The inclusion of computer subroutines for soil cap models is intended to encourage the reader to try out these models in a direct manner.

May, 1988

W.F. Chen
E. Mizuno

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Part I

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Chapter 1

INTRODUCTION

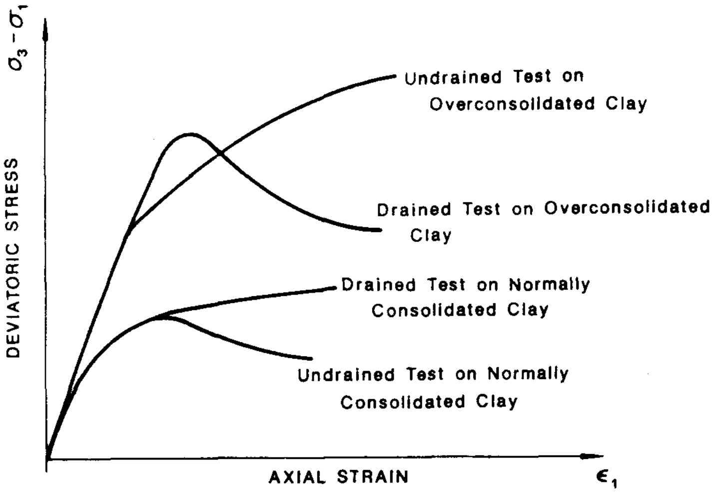
1.1 INTRODUCTION

For a long time, soil mechanics has been based on Hooke's law of linear elasticity for stress and deformation analysis for a soil mass under a footing, or behind a retaining wall, when no failure of the soil is involved. This is known as the *elasticity problem* in soil mechanics. On the other hand, the theory of *perfect plasticity* is used to deal with the conditions of ultimate failure of a soil mass. Problems of earth pressure, retaining walls, bearing capacity of foundations, and stability of slopes are all considered in the realm of perfect plasticity. These are called *stability problems*. Long-term settlement problems and consolidation problems, however, are treated in soil mechanics as essentially *viscoelastic problems*.

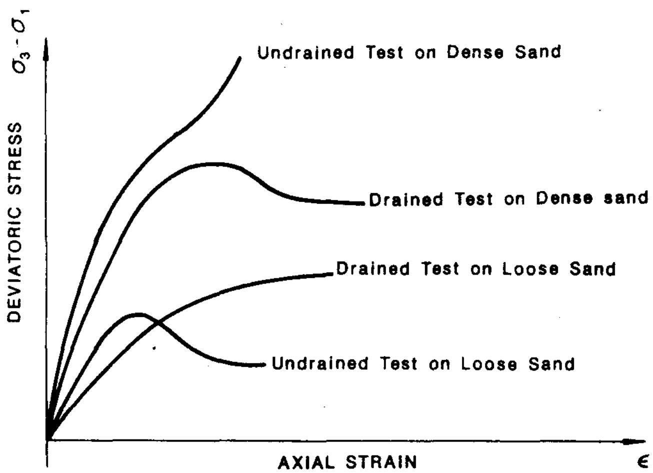
Partly for simplicity in practice and partly because of the historical development of mechanics of solids, the elasticity problems and the stability problems in soil mechanics are treated separately and in unrelated ways. The essential connection between the elasticity problems and the stability problems is known as the *progressive failure problems*. The progressive failure problems deal with the elastic-plastic transition from the initial linear elastic state to the ultimate state of the soil by plastic flow. The essential set of equations for the solutions of progressive failure problems is the *constitutive equations* of soils, which give a unique relationship of stress and strain for different geotechnical materials. These relationships and applications to soils and rocks are discussed in the following Chapters.

1.2 CHARACTERISTICS OF SOIL BEHAVIOR

Some typical stress-strain curves for soils in the triaxial tests are shown in Fig. 1.1. As can be seen from Fig. 1.1a, the relation of the deviatoric stress σ_3-σ_1 v.s. axial strain ϵ_1 for a *normally consolidated* clay in a *drained test* is characterized by a nonlinear response curve which rises at a slower rate after reaching a certain stress level. Here, the further straining is always associated with an increase in stress. This phenomenon is known as *strain-hardening*. The stress-strain curve for *overconsolidated* clay in an *undrained test* exhibits the same behavior as that of normally consolidated clay in a drained test. The overconsolidated clay in a drained test and normally consolidated clay in an undrained test behave differently from those mentioned above. The stress-strain curves both have a clearly defined peak occurring at a low strain level. An element of these clays, strained beyond the strain



a) Behavior of Clay



b) Behavior of Sand

Fig. 1.1. Typical soil stress-strain curves.

corresponding to the peak stress point, becomes weaker than it was at this peak point. This phenomenon is known as *strain-softening*.

Similar conclusions can be made from Fig. 1.1b for the behavior of sand. Dense sand in an undrained test and loose sand in a drained test show the strain-hardening. Dense sand in a drained test and loose sand in an undrained test show a peak stress followed by strain-softening to a residual stress. The behavior of dense sand in a drained test is similar to that of overconsolidated clay in a drained test. The similar behavior also occurs in the other corresponding cases for clay and sand.

1.3 IDEALIZATIONS AND MATERIAL MODELING

The typical stress-strain behavior of soils presented in the previous Section is not linearly elastic for the entire range of loading of practical interest. In fact, actual behavior of soils is much complicated and they show a great variety of behavior when subjected to different conditions. Drastic idealizations are therefore essential in order to develop simple mathematical models for practical applications. No mathemetical model can completely describe the complex behavior of real soils under all conditions. Each soil model aims at a certain class of phenomena, captures their essential features, and disregards what is considered to be of minor importance in that class of applications. Thus, this constitutive model meets its limits of applicability where a disregarded influence becomes important. As mentioned previously, Hooke's law has been used successfully in soil mechanics to describe the general behavior of soil media under short-term working load condition, but it fails to predict the behavior and strength of a soil–structure interaction problem near ultimate strength condition, because plastic deformation at this load level attains a dominating influence, while elastic deformation becomes of minor importance.

Under a short-term loading, soil behavior may be idealized as time-independent where the effects of time can be neglected. This time-independent idealization of soils can be further idealized as *elastic* behavior and *plastic* behavior. As the first step in constitutive modeling of soils, it is therefore logical to utilize and refine the classical theories of elasticity and plasticity as developed for such an idealized material. However, there are in many cases considerable differences between the properties of soils and those of the idealized bodies. These differences may have a significant influence on the solution of some boundary value problems in soil mechanics. In such cases, the classical theories must be modifed and extended so that the special properties of soils in certain practical applications are taken into consideration.

A material for which there exists a one-to-one coordination between stress and strain is known as *elastic material.* Thus, a body that consists of this idealized material returns to its original shape whenever all stresses are reduced to zero. The linear theory of elasticity used most commonly is the *Hooke* type or *Cauchy* type of constitutive models for soils, in spite of its obvious shortcomings. These linear elastic models can be significantly improved by assuming bilinear or higher polynomial type of nonlinear fit for the stress-strain relationship of soil in the form of *secant formulations*. This is known as the *Hooke type* or *Cauchy type of elastic formulation*. These types of elastic models must be combined with criteria defining "failure" of the soil.

In a more restricted sense, an elastic material must satisfy the energy equation of thermodynamics. The elastic material characterized by this additional requirement is known as *hyperelastic material*. On the other hand, the minimal requirement for a material to qualify as elastic in any sense is that there exists a one-to-one coordination between stress increment and strain increment. Thus, a body that consists of this material returns to its original state of deformation whenever all stress increments are reduced to zero. This reversibility in the infinitesimal sense justifies the use of the term *hypoelastic* for elastic materials satisfying only this minimal requirement. The *incremental* constitutive formulations based on hypoelastic models have been increasingly used in recent years by geotechnical engineers for soils in which the state of stress is generally a function of the current state of stresses and strains as well as of the stress path followed to reach that state (Chen and Saleeb, 1982).

The elastic modeling in the form of secant or incremental formulations can be quite accurate for soils sustaining proportional loading. However, this *reversibility* associated with these formulations is not the case for a *plastic material*. Thus, these formulations fail to identify plastic deformations when unloading occurs. This can, to some extent, be rectified by introducing loading criteria as in the *deformation theory of plasticity*. Although the deformation theory and the existence of loading function are incompatible even in the most limited sense, it is still a very attractive alternative for solution of large classes of soil and soil–structure interaction problems, because of its simplicity.

The *flow theory of plasticity* represents a necessary and correct extension of elastic stress-strain relations into the plastic range at which permanent plastic strain is possible in addition to elastic strain. This plastic strain remains when the stresses are removed. Thus, the strain in a plastic material may be considered as the sum of the reversible elastic strain and the permanent irreversible plastic strain. Since an elastic stress-strain law as mentioned above is usually assumed to provide the relation between the incremental changes of stress and elastic strain, the stress-strain law for a plastic material reduces, essentially, to a relation involving the current state of stresses and strains and the incremental changes of stress and plastic strain. This relation is generally assumed to be homogeneous and linear in the incremental changes of the components of stress and plastic strain. This assumption precludes viscosity effects and thus constitutes the time-independent idealization.

The first step towards such a mathematical model is to establish the yield limit of an elastic material. This is known as the *yield function* which is a certain function of the stress components. A plastic material is called *perfectly plastic* or *work-hardening* or *softening* according to whether the yield function as represented by a certain hypersurface in six-dimensional stress space is fixed or it admits changes (expansion or contraction) as plastic strain develops. For moderate strains, mild steel behaves approximately like a perfectly plastic material. It is therefore not surprising that in early years (1950–1965) this perfect plasticity model was used almost exclusively and extensively in the analysis and design of steel structures because of its simplicity. The general theorems of limit analysis, developed on the basis of perfect