THEORETICAL MECHANICS

JEANS

THEORETICAL MECHANICS

CHAPTER I

REST AND MOTION

INTRODUCTION

1. Uniformity of nature. If we place a stone in water, it will sink to the bottom; if we place a cork in water, it will rise to the top. These two statements will be admitted to be true not only of stones and corks which have been seen to sink or rise in water but of all stones and corks. Given a piece of stone which has never been placed in water, we feel confident that if we place it in water it will sink. What justification have we for supposing that this new and untried piece of stone will sink in water? We know that millions of pieces of stone have at different times been placed in water; we know that not a single one of these has ever been known to do anything but sink. From this we infer that nature treats all pieces of stone alike when they are placed in water, and so feel confident that a new and untried piece of stone will be treated by the forces of nature in the same way as the innumerable pieces of stone of which the behavior has been tested, and hence that it will sink in water. This principle is known as that of the *uniformity of nature;* what the forces of nature have been found to do once, they will, under similar condi tions, do again.

2. Laws of nature. The principle just stated amounts to saying that the action of the forces of nature is governed by certain laws; these we speak of as *laws of nature.* For instance, if it has been found that every stone which has ever been placed in water has sunk to the bottom, then, as has already been said, the principle of uniformity of nature leads us to suppose that every stone which at any future time is placed in water will sink to the bottom; and we can then announce, as a law of nature, that any stone, placed in water, will sink to the bottom.

That part of science which deals with the laws of nature is called *natural science.* Natural science is divided into two parts, experimental and theoretical. *Experimental science* tries to discover laws of nature by observing the action of the forces of nature time after time. *Theoretical science* takes as its material the laws of nature discovered by experimental science, and aims at reducing them, if possible, to simpler forms, and then discovering how to predict from these laws what the action of the forces of nature will be in cases which have not actually been subjected to the test of experiment. For example, experimental science discovers that a stone sinks, that a cork floats, and a number of similar laws. From these *theoretical physics* arrives at the simple laws of nature which govern all phenomena of sinking or floating, and, going further, shows how these laws enable us to predict, before the experiment has been actually tried, whether a given body will sink or float. For instance, experimental science cannot discover whether a 50,000-ton ship will float or sink, because no 50,000-ton ship exists with which to experiment. The naval architect, relying on the uniformity of nature, on the laws of nature determined by experimental science, and on the method of handling these laws taught by theoretical science, may build a 50,000-ton ship with every confidence that it will behave in the way predicted by theoretical science.

3. The science of mechanics. The branch of science known as *mechanics* deals with the motion of bodies in space, and with the forces of nature which cause or tend to cause this motion. The laws of nature which govern the action of these forces and the motion of bodies have long been known, and were reduced to their simplest form by Newton. Thus we may say that *experimental mechanics* is a completed branch of science.

The present book deals with *theoretical mechanics.* We start from the laws supplied by experimental mechanics, and have to discuss how these laws can be used to predict the motion of bodies, — for instance, the falling of bodies to the ground, the firing of projectiles, the motion of the earth and the planets round the sun. An important class of problems which we shall have to discuss will be those in which no motion takes place, the forces of nature which tend to cause motion being so evenly balanced that no motion occurs. Such problems are known as *statical.*

Motion of a Point

4. State of rest. Before we can reason about the motion of a body we have to determine what is meant by a body being at rest. In ordinary language we say that a train is at rest when the cars are not moving over the rails. We know, however, that the train, in common with the rest of the earth, is not actually at rest, but moving round the sun with a great velocity. Again, a fly crawling on the wall of a railway car might in one sense be said to be at rest, if it remained standing on the same spot of the wall. The fly, however, would not actually be at rest; it would share in the motion of the train over the country, the country would share in the motion of the earth round the sun, and the sun would share in the motion of the whole solar system through space.

These instances will show the necessity of attaching a clear and exact meaning to the conceptions of rest and motion. Obviously our statements would have been exact enough if we had said that in the first case the train was at rest *relatively to the earth,* and that in the second case the fly was at rest *relatively to the car.*

5. Frame of reference. Thus we find it necessary, before discussing rest and motion, to introduce the conception of a *frame of reference.* The earth supplied a frame of reference for the motion of the train, and when a train is not moving over the rails we may say that it is *at rest, the earth being taken as frame of* *reference.* So also we could say that the fly was *at rest, the car being taken as frame of reference.* Obviously any framework, real or imaginary, or any material body, may be taken as a frame of reference, provided that it is rigid, i.e. that it is not itself changing its shape or size.

We may accordingly say that a point is at rest relatively to any frame of reference when the distance of the point from each point of the frame of reference remains unaltered.

6. Motion relative to frame of reference. Having specified a frame of reference, we can discuss not only rest but also motion relative to the frame of reference. When the train has moved a mile over the tracks we say that it has moved a mile relatively to its frame of reference, the earth. When the fly has crawled from floor to ceiling of the car we say that it has moved, say, eight feet relatively to its frame of reference, the car.

In fixing the distance traveled by the fly relatively to the train in an interval between two instants t_1, t_2, we notice that the actual point from which the fly started is, say, a mile behind the present position of the train; but the point from which we measure is the point which occupies the same position in the car at time t_2 as this point did at time t_1. So, in general, to fix the distance moved relatively to a given frame of reference in the interval between times t_1 and t_2, we first find the point A which stands in the same position relative to the frame of reference at time t_2 as did the point from which the moving point started at time t_1. The distance from this point A to the point B, which is occupied by the moving point at instant t_2, is the distance moved relatively to the moving frame of reference.

By the motion of a particle B relative to a particle A, is meant the motion of B relative to a frame of reference moving with A.

7. Composition of motions. Suppose that in a given time the moving point moves a certain distance relatively to its frame of reference, while this frame of reference itself moves some other distance relatively to a second frame of reference, — as will, for instance, occur if a fly climbs up the side of a car while the car moves relatively to the earth.

Let us suppose that there is a frame of reference moving in the plane of the paper on which fig. 1 is drawn, and that the paper itself supplies a second frame of reference. Suppose that the moving point starts at A, and that during the motion that point of the first frame of reference which originally coincided with the moving point has moved from A to B, while the point itself has moved to C. Then the line AB represents the motion of frame 1 relative to frame 2, while BC represents the motion of the moving point relative to frame 1. The whole motion of the point relative to frame 2 is represented by AC. The motion AC is said to be *compounded* of the two motions AB, BC, or is said to be the *resultant* of the two motions. Thus:

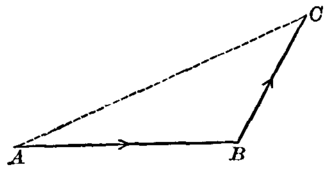

FIG. 1

If a point moves a distance BC relatively to frame 1, while frame 1 moves a distance AB relatively to frame 2, the resultant motion of the point relative to frame 2 will be the distance AC, obtained by taking the two distances AB, BC and placing them in position in such a way that the point B at which the one ends is also the point at which the other begins.

There is a second way of compounding two motions. Let x, y represent the two motions. The rule already obtained directs us to construct a triangle ABC, to have x, y for the sides AB, BC, and then AC will be the motion required. Having constructed such a triangle ABC, let us complete the parallelogram $ABCD$ by drawing AD, CD parallel to the side of the triangle. Then AD, being equal to BC, will also represent the motion y, so that we may say that the two edges of the parallelogram which meet in A represent the two motions to be compounded, while the diagonal AC through A has already been seen to represent the resultant motion. Thus we have the following rule for compounding two motions x, y:

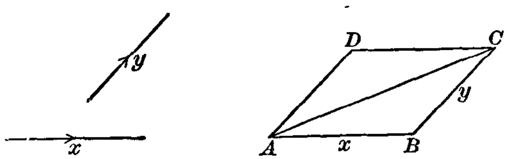

FIG. 2

Construct a parallelogram ABCD such that the two sides AB, AD which meet in A represent the two motions x, y to be compounded, as regards both magnitude and direction; then the diagonal AC which passes through A will represent the resultant obtained by compounding these two motions.

Velocity

8. Uniform and variable velocity. Velocity means simply rate of motion. It may be either uniform or variable. If a point moves in such a way that a feet are described in each second of its motion, no matter which second we select, we say that the velocity of the point is a *uniform* velocity of a feet per second. If, however, the point moves a feet in one second, b feet in another, c feet in a third, and so on, we cannot say that any one of the quantities a, b, or c measures the velocity. The velocity is now said to be *variable*: it is different at different stages of the motion. To define the velocity at any instant, we take an infinitesimal interval of time dt and measure the distance ds described in this time. We then define the ratio $\frac{ds}{dt}$ to be the velocity at the instant at which the interval dt is taken. If the velocity is uniform, $\frac{ds}{dt}$ is the space described in unit time, and so the present definition of velocity becomes the same as that already given.

Average velocity. If a point moves with variable velocity, and describes a distance of a feet in t seconds, we speak of $\frac{a}{t}$ as the "average velocity" of the moving point during the time t. This average velocity is the velocity which would have to be possessed by an imaginary point moving with uniform velocity, if it were to cover the same distance in time t as the actual point moving with variable velocity.

Units. In measuring a velocity we need to speak in terms of a unit of length and of a unit of time; for instance, in saying that a point has a velocity of a feet per second we have selected the foot as unit of length and the second as unit of time. We can find the amount of this same velocity in other units by a simple proportion.

Thus suppose it is required to express a velocity of a feet per second in terms of miles and hours.

The point moves a feet in one second, and therefore $a \times 60 \times 60$ feet in one hour, and therefore

$$\frac{a \times 60 \times 60}{3 \times 1760} = \frac{15\,a}{22} \text{ miles}$$

in one hour. Thus the velocity is one of $\frac{15\,a}{22}$ miles per hour.

EXAMPLES

1. A railway train travels a distance of 918 miles in 18 hours. What is its average velocity in feet per second?

2. Compare the velocities of a train and an automobile which move uniformly, the former covering 100 feet a second and the latter 1500 yards a minute.

3. A man runs 100 yards in 9½ seconds. What is his average speed in miles per hour?

4. The two hands of a town clock are 10 and 7 feet long. Find the velocities of their extremities.

5. Taking the diameter of the earth as 7927 miles, what is the velocity in foot-second units of a man standing at the equator (in consequence of the daily revolution of the earth about its axis)?

6. Two trains 230 and 440 feet long respectively pass each other on parallel tracks, the former moving with twice the speed of the latter. A passenger in the shorter train observes that it takes the longer train three seconds to pass him. Find the velocities of both trains.

9. Composition of velocities. All motion, as we have seen, must be measured relatively to a frame of reference. Thus velocity, or rate of motion, must also be measured relatively to a frame of reference. A point may have a certain velocity relative to a frame of reference, while the frame of reference itself has another velocity relative to a second frame. It may be necessary to find the velocity of the moving point with reference to the second frame, in other words, to *compound* the two velocities.

To do this we consider the motions which take place during an infinitesimal interval of time dt. Let the moving point have a velocity v_1 in a direction AB relative to the first frame, while the frame has a velocity v_2 in a direction AC relative to the second frame. Then in time dt the moving point describes a distance v_1dt, say the distance AD, along AB relative to the first frame, while the frame itself describes a distance v_2dt, say AE, along AC relative to the second frame. Let AF be the diagonal of the parallelogram of which AD, AE are two edges; then AF will be the resultant motion of the point in time dt relative to the second frame. Since the moving point describes a distance AF in time dt, the resultant velocity will be $\frac{AF}{dt}$.

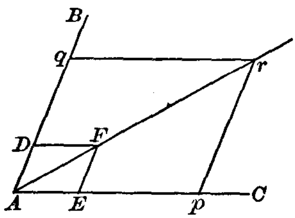

FIG. 3

Let us now agree that velocities are to be represented by straight lines, the direction of the line being parallel to that of the velocity and its length being proportional to the amount of the velocity, the lengths being drawn according to any scale we please; for example, we might agree that every inch of length is to represent a velocity of one foot per second, in which case a velocity of three feet a second will be represented by a line three inches long drawn parallel to the direction of motion.

In fig. 3 let Ap, Aq represent the velocities v_2, v_1 drawn on any scale we please. Since the scale is the same for both, we have

$$Ap : Aq = v_2 : v_1.$$

Now $AE = v_2dt$, $AD = v_1dt$, so that

$$AE : AD = v_2 : v_1,$$

and hence

$$Ap : Aq = AE : AD.$$

If we complete the parallelogram $Aprq$, the diagonal Ar will pass through F, and we shall have

$$Ar : Ap = AF : AE.$$

If V is the resultant velocity, it has already been seen that

$$V = \frac{AF}{dt},$$

so that
$$AF : AE = Vdt : v_2dt$$
$$= V : v_2,$$
and hence
$$Ar : Ap = V : v_2.$$

Thus Ar represents the magnitude of the velocity V on the same scale as that on which Ap represents the velocity v_2. Also since Ar is in the direction of AF, the resultant motion, we see that Ar represents the velocity V both in magnitude and direction. We have accordingly proved the following theorem:

THEOREM. *If two velocities are represented in magnitude and direction by the two sides of a parallelogram which start from any point A, then their resultant is represented in magnitude and direction on the same scale by the diagonal of the parallelogram which starts from A.*

This theorem is known as the *parallelogram of velocities.* We may illustrate its meaning by two simple examples.

A. Suppose that a carriage is moving on a level road with velocity V. As a first frame of reference let us take the body of the carriage; as a second frame take the road itself. The velocity of frame 1 relative to frame 2 is then V. Relatively to frame 1, the center of any wheel P is fixed, so that any point on the rim describes a circle about P. Relatively to frame 1 the road is moving backward with velocity V, so that if there is to be no slipping between the rim and the road, the velocity of any point on the rim, relative to the first frame (the carriage), must be V. Thus the velocity of any point Q on the rim relative to frame 1 will be a velocity V along the tangent QT. Representing this by the line QT, the velocity of the carriage relative to the road is represented by an equal line QH parallel to the road. Thus the resultant velocity of the point Q is represented by the diagonal QS of the parallelogram $QHST$. Clearly its direction bisects the angle HQT. Let L be the

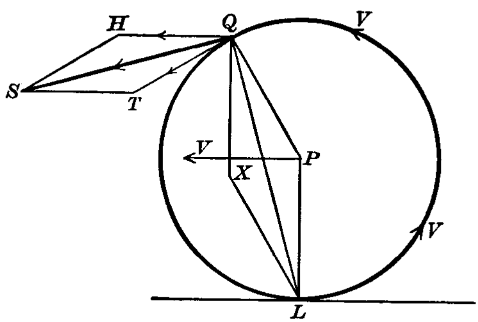

FIG. 4

lowest point of the wheel, and let X complete the parallelogram $QPLX$. Obviously this parallelogram is similar to the parallelogram $QTSH$, corresponding lines in the two parallelograms being at right angles. Thus

$$QS : QT = QL : QP.$$

So that on a scale in which the velocity of the carriage is represented in magnitude by QP, the radius of the wheel, the velocity of the point Q will be represented by QL. Thus the velocities of the different points on the rim are proportional to their distances from L, their directions being in each case perpendicular to the line joining the point to L.

2. A battle ship is steaming at 18 knots, and its guns can fire projectiles with velocities of 2000 feet per second relative to the ship. How must the guns be pointed to hit an object the direction of which from the ship is perpendicular to that of the ship's motion?

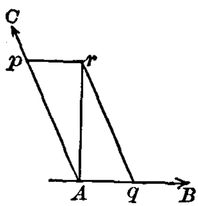

FIG. 5

Let AB be the direction of the ship's motion, and let us suppose the gun pointed in a direction AC. Then the velocity of the shot relative to the ship can be represented by a line Ap along AC, while that of the ship relative to the sea can be represented by a line Aq along AB. Completing the parallelogram $Aprq$, we find that the diagonal Ar will represent the velocity of the shot relative to the sea in magnitude and direction. Hence Ar must, from the data of the question, be at right angles to AB. If θ is the angle pAr through which the gun must be turned after sighting the object to be hit, we have

$$\sin\theta = \frac{pr}{Ap} = \frac{\text{velocity of ship}}{\text{velocity of firing of shot}}.$$

The velocity of the ship is 18 knots, or 18 nautical miles per hour. Now 1 nautical mile $= 1.1515$ ordinary miles $= 6080$ feet, so that a velocity of 18 knots is equal to 109,440 feet per hour, or 30.4 feet per second. Thus $\sin\theta = \frac{30.4}{2000} = .0152$, whence we find that $\theta = 0° 52' 16''$.

Triangle of Velocities

10. We can also compound velocities by a rule known as the *triangle of velocities*. In fig. 3 the two velocities were represented by Ap, Aq, and their resultant by Ar. The two velocities, however, might equally well have been represented by Ap, pr, and their resultant by Ar, from which we obtain the following rule:

If two velocities are represented by the two sides of a triangle taken in order, their resultant will be represented by the third side, taken in the direction from the first side to the second side.

For example, let OP_1, OP_2 be two lines drawn through O to represent, on any scale, the velocities of a moving point at instants t_1, t_2. Then P_1P_2 will, on the same scale, represent the additional velocity acquired by the point in this interval.

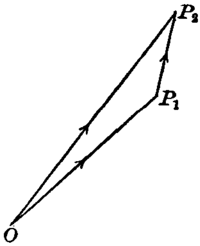

FIG. 6

For we can imagine a frame moving with the uniform velocity OP_1 of the particle at instant t_1. The velocity OP_2 at instant t_2 may be supposed compounded of the velocity OP_1 of the frame and a velocity P_1P_2 relative to the frame. Obviously this latter is the increase of velocity.

EXAMPLES

1. A car is running at 14 miles an hour, and a man jumps from it with a velocity of 8 feet per second in a direction making an angle of 30° with the direction of the car's motion. What is his velocity relative to the ground?

2. A railway train, moving at the rate of 60 miles an hour, is struck by a bullet, which is fired horizontally and at right angles to the train with a velocity of 440 feet a second. Find the magnitude and direction of the velocity with which the bullet appears to meet the train to a person inside.

3. A ship whose head points northeast is steaming at the rate of 12 knots in a current which flows southeast at the rate of 5 knots. How far will the ship have gone in $2\frac{1}{2}$ hours?

4. A train is traveling at the rate of 30 miles an hour, and rain falls with a velocity of 22 feet per second at an angle of 30° with the vertical in the same direction as the motion of the train. Find the direction of the splashes made on the windows by the raindrops.

5. A steamer's course is due south, and its speed is 20 knots; the wind is from the west, but the line of smoke from the steamer is observed to point in a direction 30° east of north. What is the velocity of the wind?

6. A man rows across a stream a mile wide, pointing his boat upstream at an angle of 30° with the bank. How long does he take to cross, if he rows with a velocity of 4 miles an hour and if the current has an equal velocity?

7. A stream has a current velocity a, and a man can row his boat with a velocity b. In what direction must he row, if he is to land at a point exactly opposite his starting point? And in what direction must he row so as to cross in the shortest time?

8. A ship whose head is pointing due south is steaming across a current running due west; at the end of two hours it is found that the ship has gone 36 miles in the direction 15° west of south. Find the velocities of the ship and current.

9. A person traveling eastward at the rate of 3 miles an hour finds that the wind seems to blow directly from the north; on doubling his speed it appears to come from the northeast. Find the direction of the wind and its velocity.

ACCELERATION

11. Acceleration is rate of increase of velocity. If we find that the velocity of a moving point increases by an amount f in a second, no matter which second is selected, we say that the motion of the point has a uniform acceleration f per second. For instance, a stone or other body falling under gravity is found to increase its velocity by a certain constant velocity f per second, where f denotes a velocity of about 32 feet per second. Thus we say that a falling stone has a uniform acceleration of f per second, or of about 32 feet per second per second.

Generally, however, an acceleration will not be uniform; the rate of increase of velocity will be different at different stages of the journey. To find the acceleration at any instant, we observe the change in velocity during an infinitesimal interval dt of time. If dv is the increase of velocity, we say that $\frac{dv}{dt}$ is the acceleration at the instant at which dt is taken. An acceleration will of course have sign as well as magnitude, for the velocity may be either increasing or decreasing. When the velocity is decreasing, the acceleration is reckoned with a negative sign. A negative acceleration is spoken of as a *retardation*. Thus a retardation f means that the velocity is diminished by an amount f per unit of time.

EXAMPLES

1. A workman fell from the top of a building and struck the ground in 4 seconds. With what velocity did he strike the ground, the acceleration due to gravity being 32 feet per second per second?

2. A train has at a given instant a velocity of 30 miles an hour, and moves with an acceleration of 1 foot per second per second. Find its velocity after 20 seconds.

3. A train comes to rest after the brakes have been applied for ten seconds. If the retardation was 8 feet per second per second, what was the velocity of the train when the brakes were first drawn?

4. How long does it take a body starting with a velocity of 22 feet per second and moving with an acceleration of 6 feet per second per second, to acquire a velocity of 60 miles an hour?

5. Two bodies start at the same instant with velocities u and v respectively; the motion of the first undergoes a retardation of f feet per second per second, while that of the second is uniform. How far will the second have gone by the time that the first comes to rest?

6. A body starting from rest moves for 4 seconds with a uniform acceleration of 8 feet per second per second. If the acceleration then ceases, how far will the body move in the next 5 seconds?

7. A train has its speed reduced from 40 miles an hour to 30 miles an hour in 5 seconds. If the retardation be uniform, for how much longer will it travel before coming to rest?

8. A body falling under gravity has an acceleration of 32.2 feet per second per second. Express this acceleration when the units are (*a*) centimeter, second; (*b*) mile, hour.

12. Parallelogram of accelerations. THEOREM. *Let the velocity of a point be compounded of two velocities v_1, v_2 along given directions, and let these velocities be variable, their accelerations being f_1, f_2. Then if two lines be drawn in the direction of the velocities, to represent f_1, f_2 on any scale, the resultant acceleration will be represented on the same scale by the diagonal of the parallelogram of which these lines are edges.*

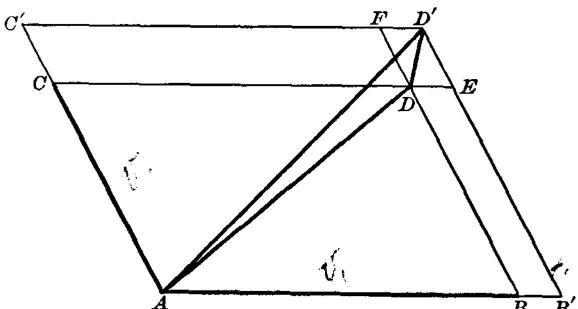

FIG. 7

To prove the theorem, we consider the motion during any small interval dt at which the component accelerations are f_1, f_2. In fig. 7 let AB, AC represent the two velocities v_1, v_2 at the beginning of this interval. Let BB', CC' represent, on the same scale, the infinitesimal increments in velocity in the interval dt, namely f_1dt, f_2dt. Then AB', AC' will represent the velocities at the end of the interval dt.

In the figure the lines BDF, $B'ED'$, CDE, $C'FD'$ are drawn parallel to AB and AC. Thus AD represents the resultant velocity at the beginning of the interval dt, and AD' that at the end of the interval. The velocity AD' can be regarded as compounded of the two velocities AD, DD', and, as in § 10, DD' represents the increment in velocity in time dt. Thus, if F is the resultant acceleration, the line DD' will represent a velocity Fdt. On the same scale DE, DF represent velocities f_1dt, f_2dt, and $DED'F$ is a parallelogram.

If OF_1, OF_2 (fig. 8) represent the accelerations f_1, f_2 on any scale, and if OG is the diagonal of the completed parallelogram, we clearly have $OF_1 : OF_2 = f_1 : f_2 = DE : DF$, so that the parallelograms OF_1GF_2 (in fig. 8) and $DED'F$ (in fig. 7) will be similar and similarly situated. Thus

$$OG : OF_1 = DD' : DE = Fdt : f_1dt = F : f_1,$$

so that OG represents the acceleration F on the same scale as that on which OF_1, OF_2 represent f_1, f_2; and OG, being parallel to DD', will also represent the direction of F, proving the theorem.

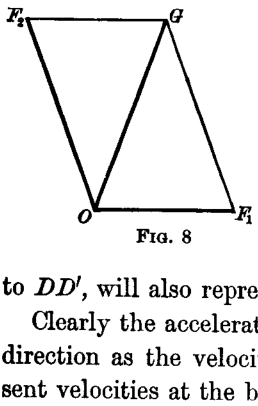

Fig. 8

Clearly the acceleration at any instant need not be in the same direction as the velocity. In fig. 7 the directions AD, AD' represent velocities at the beginning and end of the interval dt. When in the limit we take $dt = 0$, these lines coincide, and the direction of the velocity at the instant at which dt is taken is that of AD. The direction of the acceleration at this instant is, however, DD'.

As an illustration of this, let us consider the motion of a particle moving uniformly in a circle; e.g. a point on the rim of a wheel, turning with uniform velocity V about its center.

Let A, B (fig. 9) be the positions of the point at two instants, let the tangents at A, B meet in C, and let D complete the parallelogram $ACBD$.

The velocity at the first instant is a velocity V along AC. Let us agree to represent this by the line AC itself. At the second instant the velocity is a velocity V along CB; this may, on the same scale, be represented by the line CB, or more conveniently by AD. Since AC, AD represent the velocities at the two instants, the line CD will represent the change in velocity between these two instants.

Now let the two instants differ only by an infinitesimal interval dt, so that the points A, B coincide except for an infinitesimal arc Vdt. In the figure, CD passes through P wherever A, B are on the circle, so that when B is made to coincide with A, CD coincides with the radius through A. But if F is the acceleration of the moving point, the change in velocity produced in time dt must be Fdt. Thus CD represents the change of velocity Fdt in direction and magnitude, so that the change of velocity, and hence the acceleration at A, is along the radius at A.

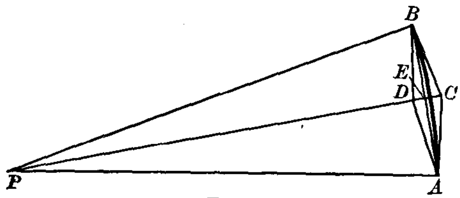

FIG. 9

Here, then, we have a case in which the acceleration is at right angles to the velocity.

To find the magnitude of the acceleration, we notice that $CD = 2\,CE$, and that, by similar triangles,

$$EC : CB = BE : BP.$$

Now EC, or $\frac{1}{2}\,CD$, represents the velocity $\frac{1}{2}\,Fdt$, while CB on the same scale represents the velocity V.

Thus $$\tfrac{1}{2}\,Fdt : V = BE : BP.$$

In the limit when BA is very small, BE, or $\frac{1}{2}\,BA$, becomes identical with half of the arc BA of the circle, and therefore with $\frac{1}{2}\,Vdt$. Thus, if a is the radius of the circle,

$$\tfrac{1}{2}\,Fdt : V = \tfrac{1}{2}\,Vdt : a,$$

giving $F = \dfrac{V^2}{a}$ as the amount of the acceleration.

EXAMPLES

1. A windmill has sails 20 feet in length, and turns once in ten seconds. Find the acceleration of a point at the end of a sail.

2. A wheel of radius 3 feet spins at the rate of 10 revolutions a second and is at the same time falling freely with an acceleration of 32 feet per second per second due to gravity. Find the resultant accelerations of the different points on the rim of the wheel.

3. Taking the earth to have an equatorial diameter of 7927 miles, find the acceleration towards the earth's center of (a) a point at rest, relative to the earth's surface, on the equator; (b) a body falling under gravity at the equator,

with an acceleration, relative to the earth's surface, of 32.09 feet per second per second.

4. Supposing that the moon describes a circle of radius 240,000 miles round the earth in $29\frac{1}{2}$ days, find its acceleration towards the earth.

5. Assuming that the planets describe circles round the sun with different periodic times, such that the squares of the periodic times are proportional to the cubes of the radii of the circles, show that the accelerations of the planets are inversely proportional to the squares of their distances from the sun.

Vectors

13. We have found three kinds of quantities, — motion, velocity, and acceleration, — all of which can be compounded according to the parallelogram law.

Quantities which can be compounded according to the parallelogram law are called *vectors.* A vector must have magnitude and direction, and hence must be capable of representation, on an assigned scale, by a straight line. We have seen that motion, velocity, and acceleration are all vectors.

Composition and Resolution of Vectors in a Plane

14. By definition of a vector, two vectors can be compounded into one, by application of the parallelogram law. It also follows from the definition that any one vector may be regarded as equivalent to two, these two being represented by the edges of a parallelogram constructed so as to have the original vector represented by the diagonal; or, as we shall say, any vector can be *resolved* into two others.

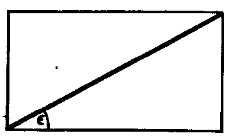

FIG. 10

In particular, if we construct a rectangular parallelogram so as to have a line which represents a vector R as its diagonal, we find that the vector R can be resolved into two vectors $R\cos\epsilon$ and $R\sin\epsilon$, at right angles to one another, and in directions such that R makes angles ϵ, $\frac{\pi}{2}-\epsilon$ with them.

If we take two fixed rectangular axes Ox, Oy in a plane, we see that any vector R can be resolved into two components $R\cos\epsilon$,