OPTIMUM STRUCTURAL DESIGN

Concepts, Methods, and Applications

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PREFACE

This book was developed while teaching a graduate course in the Faculty of Civil Engineering, Technion—Israel Institute of Technology. The purpose of the book is to introduce the basic concepts, numerical methods, and possible applications of optimum structural design. Much work has been done and many studies have been published on this subject in recent years. The book is an attempt to collect together selected topics of this literature and to present them in a unified approach. It meets the need for a comprehensive textbook covering the various aspects associated with this area. This should prove useful to both the student and the practising structural engineer. The book does not require previous knowledge of optimization methods; however, the reader is expected to be familiar with matrix structural analysis and with basic concepts of structural design. Most of the material, including the problems at the end of each chapter, is suitable for mini-computer applications. It is believed that the growing use of such computers combined with familiarity with the topics presented in the book will encourage more structural engineers to employ optimization approaches in practical design.

The emphasis throughout is on problem formulation, relative merit of various methods, and possible difficulties in applications. It is shown how alternative formulations and methods can be combined to solve different design problems. To stress that certain methods are applicable in various problems and to show that more than one method can be used in the solution of a specific problem, the methods of solution are developed separately from the presentation of design applications. The methods are discussed in general terms and their application to structural design is illustrated by a variety of examples.

Part 1 (Chaps. 1 to 4) deals with the basic concepts of optimum structural design. The role of automated numerical optimization in the overall design process is outlined and the general problem is mathematically formulated. Terminology used throughout the text is defined and alternative approaches to structural design and optimization are discussed.

Chapter 4 contains the background material on structural analysis needed in the rest of the book. Analysis is a main part of any optimal design process and topics presented in this chapter include force and displacement methods for framed structures, the finite element approach for continuum structures, and kinematic and static formulations for plastic analysis of framed structures.

Part 2 (Chaps. 5 to 11) deals with numerical methods for optimal design. Many design problems can be solved by various procedures and there is no single best method. Each procedure has its own strengths, weaknesses, and typical characteristics. In order to choose the most effective procedure for a specific application, the user should be familiar with the different methods.

Chapter 5 develops reanalysis methods, intended to analyze efficiently new designs using information from previous ones. Direct, iterative, and approximate methods are discussed.

Chapter 6 introduces optimality criteria methods, which can be used to produce designs in simple and efficient ways. Fully stressed design algorithms and iterative procedures for problems with stress, displacement, and design constraints are presented.

Chapter 7 discusses the basic linear programming problem, where all the constraints and the cost function to be minimized are expressed in linear terms of the variables.

Chapter 8 is devoted to the numerical methods of nonlinear programming. Only those methods which are most commonly used in optimal design are developed and certain guidelines for choosing a method are established.

Chapter 9 presents formulation and solution of nonlinear programming problems by the geometric programming method.

Chapter 10 shows how large-scale nonlinear programming problems can be decomposed into a number of smaller subproblems, each of which yields the overall system optimum when solved independently.

Chapter 11 covers the dynamic programming approach for optimization of serial systems. Problems of initial value, final value, and boundary value are presented, and computational considerations are discussed.

Part 3 (Chaps. 12 to 17) shows how the concepts and the methods are used for solving different design problems. Typical examples of realistic and large-scale structures demonstrate a wide variety of applications.

Chapter 12 is devoted to optimization of simple elements of steel, reinforced concrete, and prestressed concrete. Alternative possible formulations and solution by different algorithms are illustrated.

Chapter 13 deals with the optimization of members' cross-sectional dimensions in truss systems with particular emphasis on various formulations of elastic and plastic optimal design.

Chapter 14 presents the optimization of cross sections in flexural systems. Elastic grillages are optimized by various nonlinear programming methods. Optimal prestressing of elastic systems and optimal plastic design of frames are formulated as linear programming problems.

Chapter 15 introduces some general topics related to optimization of structural systems. Approaches for calculating constraint derivatives and constraint explicit approximations are developed. The discussion on the relationship between mathematical programming and optimality criteria shows that the two approaches have many common ideas, and the positive features can be used to establish better solution procedures.

Chapter 16 discusses approaches to the optimal design of large and continuum systems. Techniques based on substructuring, decomposition, and approximation concepts are developed. Shape and cross section optimization of continuum systems are demonstrated, using mathematical programming and finite element analysis. Different optimization methods are combined to achieve efficient solution of complex systems such as aircraft, ship, and building structures.

Chapter 17 illustrates optimal design of geometry and topology of structures. Geometric variables, representing the coordinates of joints and location of supports, are optimized together with the cross-sectional dimensions. Procedures for optimization of the topology, based on elimination of uneconomical members from the structure, are demonstrated. The dynamic programming approach is used to optimize both geometric and topological variables.

Some sections in the book are necessary for continuity, while others are needed only for those interested in greater depth in a particular area. Many sections are independent and can be omitted, or their order can be changed. As a text the book can be used for a one-semester or two-semester course in departments of civil, aeronautical, or mechanical engineering. Chapters that can be omitted in a one-semester course are 3, 4, 5, 9, and 10. Chapters 1, 2, 7, and 8 are necessary for continuity, but it is also desired to include in a one-semester course Chaps. 6 and 11, and selections from Chaps. 12 to 17. In a two-semester course, it is possible to separate the concepts and methods (first semester, possibly for senior undergraduate students) from the applications (second semester). However, it might be useful to combine the methods and the applications. One possibility for such a two-semester course is as follows:

1st semester: Chaps. 1, 2, 3, 6, 7 plus applications (e.g., 12.4, 13.3, 13.5, 14.3, 14.4), and 11 plus applications (e.g., 17.4).

2nd semester: Chaps. 8 plus applications (e.g., 12.2, 13.2, 14.2, 17.2, 17.5), 9 plus applications (e.g., 13.4), and 10 plus applications (e.g., 16.1, 17.3).

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PART ONE

CONCEPTS

CHAPTER

ONE

INTRODUCTION

The design process may be divided into the following four stages:[1] †

a. *Formulation of functional requirements*, which is the first step in any design procedure. In some cases the functional requirements are not explicitly stated beforehand, and the designer has to investigate and take part in formulating these requirements. However, functional requirements are often established already before the structural engineer enters the design process. Examples of such situations include the required number of lanes on a bridge or the required space in an industrial building.

b. The *conceptual design* stage, characterized by ingenuity, creativity, and engineering judgment of the designer, is a critical part of the design process. It deals with the overall planning of a system to serve its functional purposes. At this stage, the designer experiences the greatest challenges as well as chances of success or failure. Selection of the overall topology, type of structure, and materials are some of the decisions made by the designer at the conceptual design stage. In general, this part of the design process cannot be performed by a computer. The designer may, however, use the computer to make systematic investigations into the relative merit of several competing concepts in search of the best one.

c. *Optimization*. Within a selected concept there may be many possible designs that satisfy the functional requirements, and a "trial-and-error" procedure may be employed to choose the optimal design. Selection of the best geometry of a truss or the cross sections of the members are examples of optimal design

† Superscript numerals refer to bibliographical items cited at the end of each chapter.

procedures. The computer is most suitable to carry out this part of the design, using methods of automated search for the optimal solutions. Thus, optimization in the present context is an automated design procedure giving the optimal values of certain design quantities, considering desired criteria and constraints.

d. Detailing. After completing the optimization stage, the results obtained must be checked and modified if necessary. In the final detailing stage, engineering judgment and experience are required, and it is again usually necessary for the designer to take part in the decision-making process.

Iterative procedures for the four stages are often required before the final solution is achieved. The portion of the structural design process that can be optimized automatically has been considerably increased in recent years. The significant progress in this field is a result of developments in structural analysis, digital computers, and optimization methods. In general, it is not practical to introduce one automated design program that solves the complete optimization problem without human interaction. Optimization procedures are usually used to solve specific subproblems and the field of automated design is strongly connected with computer-aided design. Computer-aided design involves user-machine interactions and it is characterized by the designer's decisions based on displayed information supplied by the computer. The use of graphical input-output devices facilitates crossing the user-machine interface. Automated procedures for optimal design, on the other hand, seek the optimum in a defined sense and are characterized by preprogrammed logical decisions based upon internally stored information. These procedures are aimed at keeping the quantifiable portion of the design in the computer and thus avoiding the crossing of this interface. The two approaches of automated optimal design and interactive computer-aided design are not mutually exclusive, but rather they complement one another. Both approaches are suitable for the effective use of large amounts of information associated with matrix analysis methods. As the techniques of interactive computer-aided design develop further, the needs to employ standard routines for automated design of structural subsystems will become increasingly apparent.

The available methods of optimization may conveniently be subdivided into two distinctly different categories as follows:[2,3]

a. Analytical methods, which are usually employing the mathematical theory of calculus, variational methods, etc., in studies of optimal layouts or geometrical forms of simple structural elements, such as beams, columns, and plates. Analytical methods are most suited for such fundamental studies of single structural components, but are not able to handle larger structural systems. In analytic optimization problems the structural design is represented by a number of unknown functions and the goal is to find the form of these functions. The optimal design is theoretically found exactly through the solution of a system of equations expressing the conditions for optimality. Analytical solutions, when they can be found, provide valuable insight and facilitate the comparison between forms. An example for this approach is the theory of layout, which seeks the arrangement of uniaxial structural members that produces a minimum-volume structure for specified loads and materials. The basic theorems of this approach were established by Maxwell[4] and Michell.[5] Since they are applied without meaningful constraints on the geometric form of the structure, such theorems often yield impractical solutions. Analytical methods are beyond the scope of this text and are not discussed here.

b. *Numerical methods*, which are usually employing a branch in the field of numerical mathematics called programming methods. The recent developments in this branch are closely related to the rapid growth in computing capacities affected by the development of computers. In the numerical methods, a near optimal design is automatically generated in an iterative manner. An initial guess is used as starting point for a systematic search for better designs. The search is terminated when certain criteria are satisfied, indicating that the current design is sufficiently close to the true optimum. Rapid developments in the programming methods as well as in the application of such methods in design facilitate the solution of realistically large practical design problems. Problems solved by numerical methods are called *finite optimization problems*. This is due to the fact that they can be formulated by a finite number of variables. Assignment of numerical values to these variables specifies a unique structure. Design optimization of practical structures is accomplished mainly by the use of finite formulations.

This text is only dealing with finite formulations and numerical methods of structural optimization.

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CHAPTER

TWO

GENERAL FORMULATION OF OPTIMAL DESIGN

2-1 DESIGN VARIABLES

A structural system can be described by a set of quantities, some of which are viewed as variables during the optimization process. Those quantities defining a structural system that are fixed during the automated design are called *preassigned parameters* and they are not varied by the optimization algorithm. Those quantities that are not preassigned are called *design variables*. The preassigned parameters, together with the design variables, will completely describe a design. Quantities are designated as preassigned parameters for a variety of reasons. It may be that the designer is not free to choose certain parameters, or it may be known from experience that a particular value of the parameter produces good results. Often, by considering some quantities fixed, i.e., invariant during the optimization process, the problem is greatly simplified. From a physical point of view, the design variables that are varied by the optimization procedure may represent the following properties of the structure:[1]

1. the mechanical or physical properties of the *material*
2. the *topology* of the structure, i.e., the pattern of connection of members or the number of elements in a structure
3. the *configuration* or *geometric layout* of the structure
4. the *cross-sectional dimensions* or the member sizes

From a mathematical point of view, it is important to distinguish between *continuous* and *discrete* design variables. In cases of discrete variables with a