Compressor Aerodynamics

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Preface

The idea for this book was given to me by Dr R C Dean early in 1973. His reason was that *Axial compressors* by J H Horlock was out of date. That was 15 years after the publication of *Axial compressors* and another 15 years have elapsed between then and now. I am sure that it is fortunate that I did not try to write a book at that time.

The title of this book may seem a little bald with so many books being entitled 'An introduction to' or 'Fundamentals of' but that is not a style which I like. I am however very aware that what I have written is only scratching the suface of a very big topic. It has also been a surprise to me how much I have had to leave out: in particular, discussion of ideas which are new (and may not be correct) but are currently interesting. Although I had set out to write a short book, I seem to have failed in this. In stressing that this is inevitably a brief account of a very big field, one occupying hundreds of people and vast expenditures of money, it must be realized that most of this work is being carried out by companies and the results are predominantly confidential. It is one of the main difficulties that much of what really matters is never released into the public domain, a trend that is more pronounced now than when *Axial compressors* was being written, for example. One of the consolations for an author is that those people inside the large companies who have access to this knowledge are not likely to write a book about it.

Writing a book about compressors thirty or so years ago, it probably seemed reasonable to give a guide to design and this is evident in, for example, Eckert and Schnell's book *Axial- und Radialkompressoren* as well as in Horlock's *Axial compressors*. Writing today this does not seem so appropriate. This is partly because of the greater concentration of knowledge and expertise in confidential form, referred to above, but also the greater use of numerical methods. The computer has come to dominate the design and assessment of compressors, so that it is not very likely that a major design would be carried out nowadays without the extensive use of numerical methods. Such methods are not necessarily described at all well in design manual form. In short this book does not set out to be a design guide but to provide ideas and clarification which will in turn lead to improvements in design. It is my hope that compressor aerodynamicists and designers will find it helpful in this way and that it will have a use in introducing newcomers to the topic. Some of the sections are relatively specialized, others are quite general and have application to all turbomachines and to wide areas of fluid dynamics.

On the whole I have not included much mathematics; someone wanting the details will be able to find them in one of the referenced works. Again the shift to numerical methods has somewhat diminished the attraction of the complete mathematical coverage of a topic and mathematical analysis has become relegated to certain specialist fields, mainly those such as stall or vibration where linear analysis is valid. As a rule equations are given when they are needed to follow the argument. In so far as I am able I have held my argument together by using results in graphical form and as a result there are a lot of figures in this book.

I would be giving a wrong impression if I suggested that to be modern is to relegate what was done in the past to obscurity. There is, I believe, a lot to be learned from the work which was done in the 1940s and 50s and I have included some of this. Naturally the areas where this is most apparent are those in which the early workers were most active, so in the chapters on blade-to-blade flows in axial machines and on centrifugal impellers there is quite extensive reference to much of the excellent early work.

This does not, however, set out to be an historical account attempting to put the record straight about who did what. Where it is appropriate I have referred to the originators of ideas, but this is not the primary purpose of the book or of the reference list. I have, of course, included any reference from which I have taken a diagram as well as those which I think will be useful, either because they are themselves interesting or because they in turn provide a very complete bibliography. Inevitably many first-class pieces of work are not referred to, though the person wishing to dig deeper will find them from the reference lists in other papers. In general I have tried to give references which are easily obtained and this necessarily distorts history to some extent. Thus, for example, I have referenced Day and Cumpsty (1978), which is easily available in libraries around the world, whereas the work is first described by Day in a doctoral dissertation held in the library of Cambridge University. To the majority of readers this distortion will probably seem a price worth paying for convenience.

As the author I feel more comfortable with some topics than others. In writing about blade vibration I was very conscious of having a fairly superficial grasp of the topic. I am very grateful to Dr D S Whitehead for his help with this and on the strength of this feel confident to see what I have written published. Similarly the computation of flows in turbomachines is a specialized field with which I felt unfamiliar. In Chapter 11 there is a relatively short section on numerical methods which is based on a lecture by Dr J D Denton who was kind enough to review what I wrote; again with this support I am prepared to see what I have written published, believing it to be free from the worst errors and a helpful introduction for the non-specialist.

A sense of realism tells me that there will be mistakes which slip through into the published text. In the hope that there will ultimately be a second edition I should be very pleased to have readers send me corrections, most particularly corrections of fact. If they have illustrations which they think would help a second edition to be clearer, perhaps even replacing existing examples, I should be very pleased to receive them.

Acknowledgements

I feel that in writing this book in the Whittle Laboratory I have been very fortunate; the combination there of resources, ability and a willingness to be helpful is all that one could ask for.

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In academia no one could have done more to help me than Professor E M Greitzer of MIT, both by encouragement and by argument. Dr T P Hynes, with an office next to mine in the Whittle Laboratory, has been an invaluable colleague to test my ideas on and I owe him a great deal, both for his exceptional forbearance as well as his great abilities. (He also took a very active role in the section on matching of multistage compressors described in Chapter 2.) My debt to Drs Denton and Whitehead I have referred to in the preface, but to this must be added the use of computer programs they have written. Dr Dawes has helped me with calculations and advice. Others in the Whittle Laboratory have helped me substantially, in particular my students and former students who had the misfortune to be here when the book was being prepared; Drs S G Gallimore, Y Dong, and N McDougall and Mr Y S Li have helped both by their influence on my ideas and by carrying out calculations for me.

There is a special note of thanks I wish to record to those who have helped me with the book in its later stages. Foremost of these is Professor Greitzer who has done his best with savage injunctions to improve the book and has read most of the chapters for me. Dr Casey read three chapters and made invaluable contributions. Mr Freeman and Dr Gallimore took great pains over Chapter 3 when I was having a lot of trouble with it. Mr M Howard checked Chapters 4 and 5 for me. Dr McDougall has helped me in many ways, but I particularly appreciate his assistance with the later stages when he himself was under pressure. The contribution of the following in reading and checking the proofs is gratefully acknowledged: Dr D Andrews, Mr L He, Dr D Lambie, Mr Y S Li, Dr J Longley, Mr J A Storer and Dr M Zangeneh-Kazemi.

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Notation

There are very many notations in use for the consideration of turbomachines and it is just about impossible to evolve a system which has no duplication of symbols without recourse to excessive use of subscripts. It is hoped that the system adopted will represent a reasonable compromise which is fairly transparent and agrees with that generally used for the topic being discussed. The overlap that does exist here (for example m denotes mass flow rate and meridional distance) should not confuse the reader too much. There are other inconsistencies (as Emerson wrote, a foolish consistency is the hobgoblin of little minds) but this should not be too irritating. The list given is not an exhaustive one and various additional symbols are introduced throughout the book.

General points

Throughout the book all angles are measured from the meridional flow direction, which reverts to the axial direction for the blade-to-blade flow in axial machines and the radial direction towards the outlet of centrifugal compressors. (The terms radial or centrifugal compressor are used as alternatives without any implied difference; both are in common use).

The velocity magnitude and direction in the relative or rotating frame of reference are denoted by W and β whilst in the absolute or stationary frame of reference they are denoted by V and α.

As is common in British and American practice for compressors, a convention of positive and negative signs for flow or blade angles is *not* used; angles are taken as positive and the appropriate sense adopted.

The word stagnation is normally used, as for stagnation enthalpy $h_0 = h + V^2/2$, and not the word total. The usage total-to-static, as in total-to-static efficiency, is so widespread and the corresponding term based on stagnation so much harder to say that this is retained.

The outer diameter of axial machines is sometimes called the tip. This may be ambiguous, for the tip of the stators is at the hub. The word casing is therefore preferred for the outer diameter. The phrase hub–tip ratio is so common that this is occasionally used in place of hub–casing ratio for the ratio of the hub diameter divided by the casing diameter.

Variables commonly used

Geometric variables

b	passage width in spanwise direction, used for centrifugal compressors
c	blade chord
d, D	diameter
g	staggered gap, pitch resolved normal to the flow direction
h	blade height, used mainly for axial compressors
m	distance in meridional direction $dm = \sqrt{(dx^2 + dr^2)}$, $dx/V_x = dr/V_R$
r, R	distance in the radial direction
s	blade pitch
s	distance along streamline $ds = \sqrt{(dx^2 + dr^2 + r^2 d\theta^2)}$, $dx/V_x = dr/V_R = r d\theta/V_\theta$
t	blade thickness
t	tip clearance
x	distance in axial direction
y	distance in the pitchwise direction
z	distance normal to x and y
σ	solidity c/s

Angles Relating to Blading (see Fig. 4.1)

ϵ	angle between a blade filament and the radial direction in axial view (blade lean)
ξ	stagger (angle of chord line measured from the axial† direction)
θ	camber
θ	angle in the circumferential direction
χ_1	blade inlet angle (measured from the axial† direction)
χ_2	blade outlet angle (measured from the axial† direction)
λ	blade lean in radial compressors

Flow variables

Stationary frame of reference

α_1	flow inlet angle (measured from the axial† direction)
α_2	flow outlet angle (measured from the axial† direction)
V_1	inlet flow velocity
V_2	outlet flow velocity

Rotating frame of reference

β_1	flow inlet angle (measured from the axial† direction)
β_2	flow outlet angle (measured from the axial† direction)

† For radial and mixed flow machines, angles are measured from the meridional direction rather than the axial direction. For axial machines when the meridional streamlines are inclined at a substantial angle to the axial direction, the angles are also sometimes referred to the meridional.

W_1	inlet flow velocity
W_2	outlet flow velocity

Subscripted velocities

$V_{\theta 1}$	tangential component of velocity into blade row
V_{R1}	radial component of velocity into blade row
V_{x1}	axial component of velocity into blade row
	... likewise for other velocities, V_2, W etc
V_m	meridional component velocity, $V_m = \sqrt{(V_x^2 + V_R^2)}$

Special angles

i	incidence (angle between inlet flow direction and blade inlet direction, $i = \alpha_1 - \chi_1$ or $i = \beta_1 - \chi_1$ for stator or rotor respectively)
A	angle of attack (angle between inlet flow direction and the chord line, $A = \alpha_1 - \xi$ or $A = \beta_1 - \xi$)
δ	deviation (angle between outlet flow angle and blade outlet angle, $\delta = \alpha_2 - \chi_2$ or $\delta = \beta_2 - \chi_2$)
ϕ	inclination of meridional streamline to axial direction
γ	inclination of meridional streamline to axial direction (used for radial machines)

General variables

A	streamtube cross-sectional area
a	velocity of sound
a^*	velocity of sound at condition when flow sonic (similarly p^*, ρ^* etc.)
$AVDR$	axial velocity–density ratio $\rho_2 V_{x2} / \rho_1 V_{x1}$
b	streamtube depth measured normal to two-dimensional surface
B	blockage, 1-(mass flow ÷ mass flow across same section in ideal flow)
C	velocity of sound (Chapter 10)
C_D	dissipation coefficient or integral
c_f	skin friction coefficient, $\tau_w / (\frac{1}{2} \rho U^2)$
c_p	specific heat capacity at constant pressure
c_p	static pressure rise coefficient, $(p - p_1)/(p_{01} - p_1)$
DF	Lieblein's diffusion factor
F	flow function, $\dot{m}(c_p T_0)^{1/2} / A p_0$
h	specific enthalpy
h_0	specific stagnation enthalpy, $h + V^2/2$
I	specific rothalpy, $h + W^2/2 - U^2/2$
k	acoustic wavenumber
m	mass flow rate
M	Mach number

N	number of blades
N	angular velocity rev/min
p	static pressure
p_0	stagnation pressure, sometimes termed total pressure
Q	volume flow rate
R	gas constant
R	degree of reaction
s	specific entropy
T	static temperature
T_0	stagnation temperature, sometimes termed total temperature
U	blade speed
δ	boundary layer thickness
δ^*	boundary layer displacement thickness
δ_F	force deficit thickness
η	efficiency
θ	boundary layer momentum thickness
γ	ratio of specific heat capacities c_p/c_v
λ	swirl parameter V_θ/V_R, used for radial compressors
λ	acoustic wavelength
μ	dynamic viscosity
ν	kinematic viscosity, $\nu=\mu/\rho$
ρ	density
σ	slip factor, (absolute whirl velocity $\div$ ideal absolute whirl velocity)
τ	shear stress
ϕ	flow coefficient; V_x/U for axial, different definitions for radial compressors
ϕ	velocity potential
ψ	stream function
ψ	loading, $\Delta h_0/U^2$
ω	loss coefficient, $\Delta p_0/(p_{01}-p_1)$
ω	angular velocity
Ω	vorticity

Contents

1 Useful basic ideas

1.1 Introduction

It is assumed in this book that the reader is familiar with the concepts and methods of fluid mechanics and engineering thermodynamics and no attempt will be made here to survey these. This chapter will look at topics which are of particular relevance to this book, either because they will be used or because they are basic ideas which are useful and have not been very satisfactorily treated elsewhere. Some of the sections are very basic and may serve some readers merely to establish the notation, others are much more challenging and presuppose a fairly thorough understanding of fluid mechanics and turbomachinery.

1.2 Blades and flow

The essential elements of turbomachines are the blades or vanes because it is these which impart the force and, more relevantly, the moment to the flow. Sometimes it is more convenient to think of the blades turning the flow and this is particularly the case with axial machines where the flow is very often at nearly constant radius. On other occasions it is the blade force per unit area (i.e. the pressure difference) which is more helpful.

Blades in axial compressors have some features in common with aircraft wings but the situation is more complicated. Thus attempts to take over successful concepts from aeronautics are by and large not very useful. The flow varies very strongly along the span, both because of endwall boundary layer effects and because the radius and the blade speed change markedly. Also a compressor blade is just one element surrounded by many other blades and blade rows; the whole flow is the result of all the blade rows and any one blade can itself have little effect. The design of blades or blade rows in multistage applications should be thought of as choosing a configuration which is compatible with a desired flow in which it is immersed. Fortunately the blades are, like wings, able to tolerate a range of inlet flow angles (i.e. a *range of incidence*). In addition to a first approximation the outlet flow angle remains constant because of the constraint of the bladed assembly.

Although the outlet flow angle is nearly constant it is not the same as the outlet flow direction of the blades themselves. For an axial turbomachine the difference between the outlet flow angle α_2 and the blade outlet angle χ_2 is referred to as the deviation defined by $\delta=\alpha_2-\chi_2$. In the case of the radial impeller it is normal to define a slip factor σ by the ratio of the measured average absolute tangential velocity out of the impeller to the absolute tangential velocity if the outlet flow were uniform and in the direction of the blades at outlet. Both the deviation and slip factor are predominantly inviscid effects to which the boundary layer fluid makes only a small additional contribution, a point taken further in Chapters 4 and 6. In the idealized case with very thin boundary layers the streamline leaving the trailing edge would do so in the blade direction, a property associated with the Kutta–Joukowsky condition that the pressure difference should go to zero there. Out across the passage the flow is inclined to this direction, the sense of the inclination being that which reduces the force on the blades. Methods for estimating deviation and slip will be discussed in later chapters, but it is worthwhile in this introduction giving broad estimates: for a radial impeller with a typical number of vanes the slip factor is approximately equal to 0.9; for axial blades the deviation is given approximately by $\delta=0.3\theta\sqrt{(s/c)}$ where θ is the camber, s is the blade pitch and c is the chord. The expression for deviation allows some simple generalization for axial blading. The outlet flow direction is given by

$$\alpha_2 = \xi - \theta/2 + \delta$$

where ξ is the blade stagger, the inclination to the axial direction of the chord, the line joining the leading and trailing edges. (This is valid only for circular arc camber lines but this is a very common choice.) Introducing the estimate for the deviation for a solidity $c/s \approx 1.0$ gives

$$\alpha_2 \approx \xi - \theta/2 + 0.3\theta$$

so that $\qquad \alpha_2 \approx \xi - 0.2\theta$

In other words the outlet flow direction depends to only a fairly small extent on the camber whereas it is the stagger angle ξ which really has a big effect. At low inlet Mach numbers most axial blades are able to tolerate quite a large incidence range, so again the camber is of secondary importance. At high inlet Mach numbers, even high subsonic ones, the blade performance is strongly affected by incidence and the overriding dependence on stagger to the relative exclusion of camber is no longer so true. It is nevertheless the principle behind the use of variable stagger stators in high-speed compressors. Stagger remains a very important variable at high Mach numbers because it has such a large effect on the blade passage areas and therefore on the mass flow capacity.

The flow in most turbomachinery blade passages is extraordinarily complicated and a major reason for this is the unsteadiness of the flow. It is essential that there should be stationary and rotating components and movement

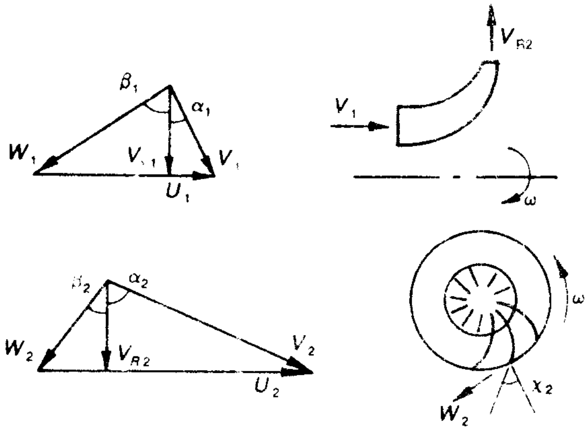

Fig. 1.1 The velocity triangles for flow entering and leaving a moving component, in this example an impeller with backsweep χ_2

of one past the other inevitably creates unsteadiness. For most purposes it is possible to ignore the unsteadiness by working in a frame of reference fixed to the component under consideration: for stator blades a coordinate system is used which is stationary (sometimes called the absolute frame) and for rotor blades or centrifugal impellers the frame of reference moves at the local blade speed (this is often referred to as the relative frame).

As a convenience in changing the frame of reference it is usual to describe the flow with vector triangles and Fig. 1.1 illustrates this for a backswept centrifugal impeller in which the flow at inlet has components in the axial and tangential directions only, while at outlet it has components only in the radial and tangential directions. Throughout this book the velocities in the stationary frame will be denoted by V and in the relative or moving frame by $\boldsymbol{W}$. The corresponding flow angles will be denoted at inlet by α and β for the absolute and relative flows respectively.

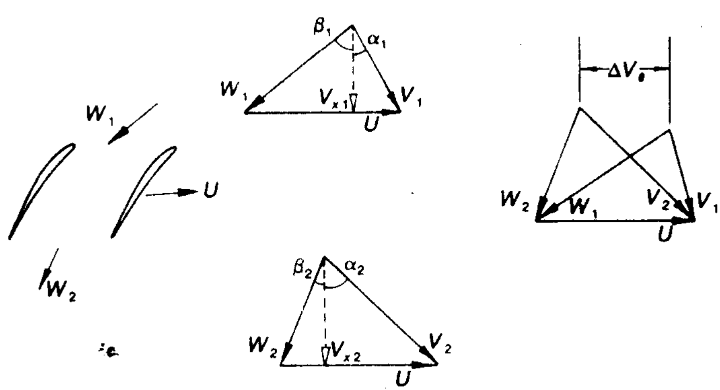

Fig. 1.2 The velocity triangles into and out of an axial rotor row