Concepts of Modern Physics

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CONCEPTS OF MODERN PHYSICS

McGraw-Hill BOOK COMPANY, INC.

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Library of Congress Catalog Card Number: 63-13154

Cover photograph by Berenice Abbott
Text illustrations by Felix Cooper

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R	Activity
s	Spin quantum number
$s, p, d, f, g, \ldots$	Angular momentum states of $l = 0, 1, 2, 3, 4, \ldots$
S	Strangeness number
T	Kinetic energy; absolute temperature
$T_{1/2}$	Half life
u	Group velocity; particle energy
u_F	Fermi energy
U	Total energy of assembly of particles
v	Vibrational quantum number
v_d	Drift velocity
V	Potential energy
V_m	Magnetic potential energy
w	Atomic weight, wave (phase) velocity
W	Total probability of a distribution
Z	Atomic number
α	Polarizability
η	Free electron density
γ	Gamma ray
λ	Wavelength; decay constant
$\boldsymbol{\mu}$	Magnetic moment
μ_s	Spin magnetic moment
ν	Frequency
ω	Angular frequency
$\psi(x, y, z)$	Time-independent wave function
ψ^*	Complex conjugate of ψ
ψ_n	Wave function corresponding to quantum number n
$\Psi(x, y, z, t)$	Time-dependent wave function
Ψ^*	Complex conjugate of Ψ
ρ	Mass density
σ	Cross section
τ	Volume of cell in phase space
$\boldsymbol{\tau}$	Torque

PREFACE

This book is intended for use with one-semester courses in modern physics that have elementary classical physics and calculus as prerequisites. The first topics considered are relativity and quantum theory, which provide a framework for understanding the physics of atoms and nuclei. The theory of the atom is then developed with emphasis on elementary quantum-mechanical notions, and is followed by a discussion of the properties of aggregates of atoms. Finally, atomic nuclei and the related subject of elementary particles are discussed. The balance in the book leans more toward ideas than toward experimental methods and practical applications, because I believe that the beginning student is better served in his introduction to modern physics by a conceptual framework than by a mass of individual details. However, all physical theories live or die by the sword of experiment, and it seemed appropriate to present certain extended derivations, such as those of the Rutherford scattering formula, the quantum-mechanical treatment of the hydrogen atom, the various distribution laws of statistical mechanics, and the theory of alpha decay, in order to demonstrate exactly how an abstract concept can be related to experimental measurements. Further to this end, special descriptive passages have been inserted to accompany the regular text where appropriate.

Arthur Beiser

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BASIC CONCEPTS

THE SPECIAL THEORY OF RELATIVITY

Our study of modern physics begins with a consideration of the special theory of relativity. This is a logical starting point, since all of physics is ultimately concerned with measurement and relativity involves an analysis of how measurements depend upon the observer as well as upon what is observed. From relativity emerges a new mechanics in which there are intimate relationships between space and time, mass and energy. Without these relationships it would be impossible to understand the microscopic world within the atom whose elucidation is the central problem of modern physics.

1.1 The Michelson-Morley Experiment

The wave theory of light was devised and perfected several decades before the electromagnetic nature of the waves became known. It was reasonable for the pioneers in optics to regard light waves as undulations in an all-pervading elastic medium called the *ether*, and their successful explanation of diffraction and interference phenomena in terms of ether waves made the notion of the ether so familiar that its existence was accepted without question. Maxwell's development of the electromagnetic theory of light in 1864 and Hertz's experimental confirmation of it in 1887 deprived the ether of most of its properties, but nobody at the time seemed willing to discard the fundamental idea represented by the ether: that light propagates relative to some sort of universal frame of reference. Let us see just what this idea implies by considering a simple analogy.

Figure 1-1 is a sketch of a river of width D which flows with the speed v. Two boats start out from one bank of the river with the same speed V. Boat A crosses the river to a point on the other bank directly opposite the starting point and then returns, while boat B heads downstream for the distance D and then returns to the starting point. Let us calculate the time required for each round trip.

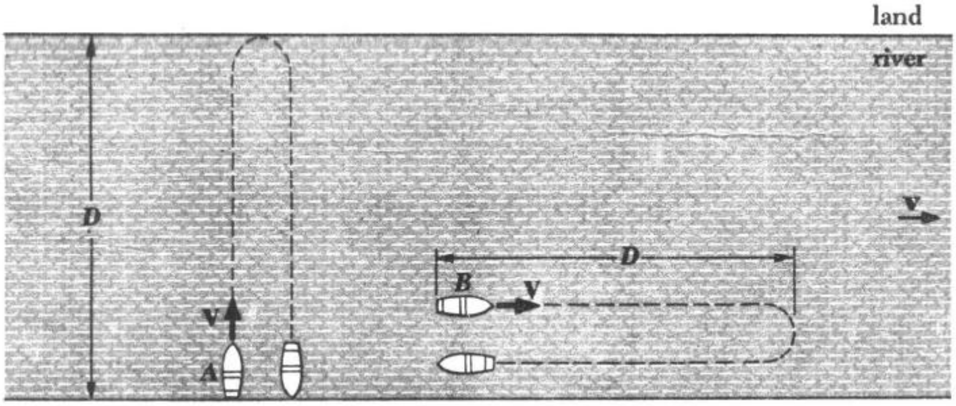
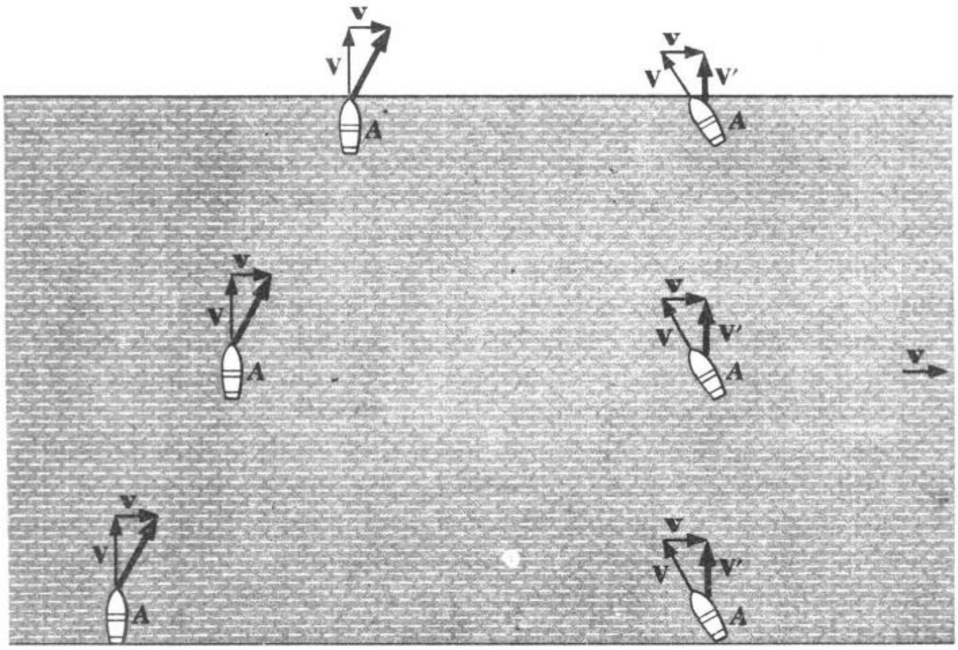


FIGURE 1-1 Boat A goes directly across the river and returns to its starting point, while boat B heads downstream for an identical distance and then returns.

We begin by considering boat A. If A heads perpendicularly across the river, the current will carry it downstream from its goal on the opposite bank (Fig. 1-2). It must therefore head somewhat upstream in order to compensate for the current. In order to accomplish this, its upstream component of velocity should be exactly $-\mathbf{v}$ in order to cancel out the river current $\mathbf{v}$,

FIGURE 1-2 Boat A must head upstream in order to compensate for the river current.



leaving the component V′ as its net speed across the river. From Fig. 1-2 we see that these speeds are related by the formula

$$V^2 = V'^2 + v^2$$

so that the actual speed with which boat *A* crosses the river is

$$\begin{aligned} V' &= \sqrt{V^2 - v^2} \\ &\doteq V\sqrt{1 - v^2/V^2} \end{aligned}$$

Hence the time for the initial crossing is the distance *D* divided by the speed V'. Since the reverse crossing involves exactly the same amount of time, the total round-trip time t_A is twice D/V', or

$$t_A = \frac{2D/V}{\sqrt{1 - v^2/V^2}} \tag{1.1}$$

The case of boat *B* is somewhat different. As it heads downstream, its speed relative to the shore is its own speed *V* *plus* the speed *v* of the river (Fig. 1-3), and it travels the distance *D* downstream in the time

$$\frac{D}{V + v}$$

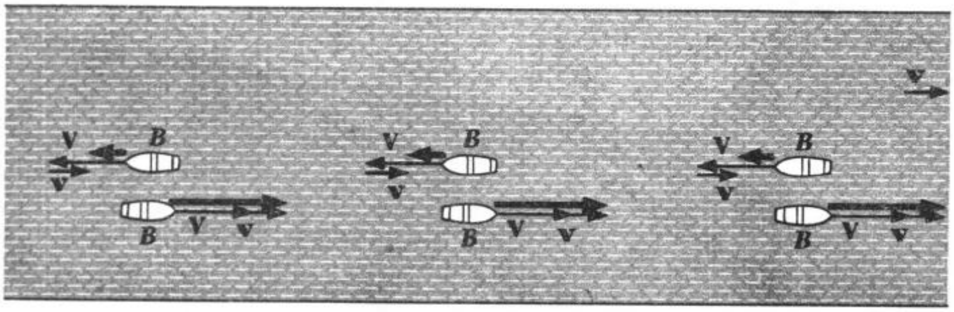
On its return trip, however, *B*'s speed relative to the shore is its own speed *V* *minus* the speed *v* of the river. It therefore requires the longer time

$$\frac{D}{V - v}$$

to travel upstream the distance *D* to its starting point. The total round-trip time t_B is the sum of these times, namely,

$$t_B = \frac{D}{V + v} + \frac{D}{V - v}$$

FIGURE 1-3 **The speed of boat *B* downstream relative to the shore is increased by the speed of the river current while its speed upstream is reduced by the same amount.**



Using the common denominator $(V + v)(V - v)$ for both terms,

$$t_B = \frac{D(V - v) + D(V + v)}{(V + v)(V - v)}$$

$$= \frac{2DV}{V^2 - v^2}$$

$$= \frac{2D/V}{1 - v^2/V^2} \tag{1.2}$$

which is greater than t_A, the corresponding round-trip time for the other boat.

The ratio between the times t_A and t_B is

$$\frac{t_A}{t_B} = \sqrt{1 - v^2/V^2} \tag{1.3}$$

If we know the common speed V of the two boats and measure the ratio t_A/t_B, we can determine the speed v of the river.

The reasoning used in this problem may be transferred to the analogous problem of the passage of light waves through the ether. If there is an ether pervading space, we move through it with at least the 3×10^4 m/sec (18.5 mi/sec) speed of the earth's orbital motion about the sun; if the sun is also in motion, our speed through the ether is even greater (Fig. 1-4). From the point of view of an observer on the earth, the ether is moving past the earth.

FIGURE 1-4 **Motions of the earth through a hypothetical ether.**

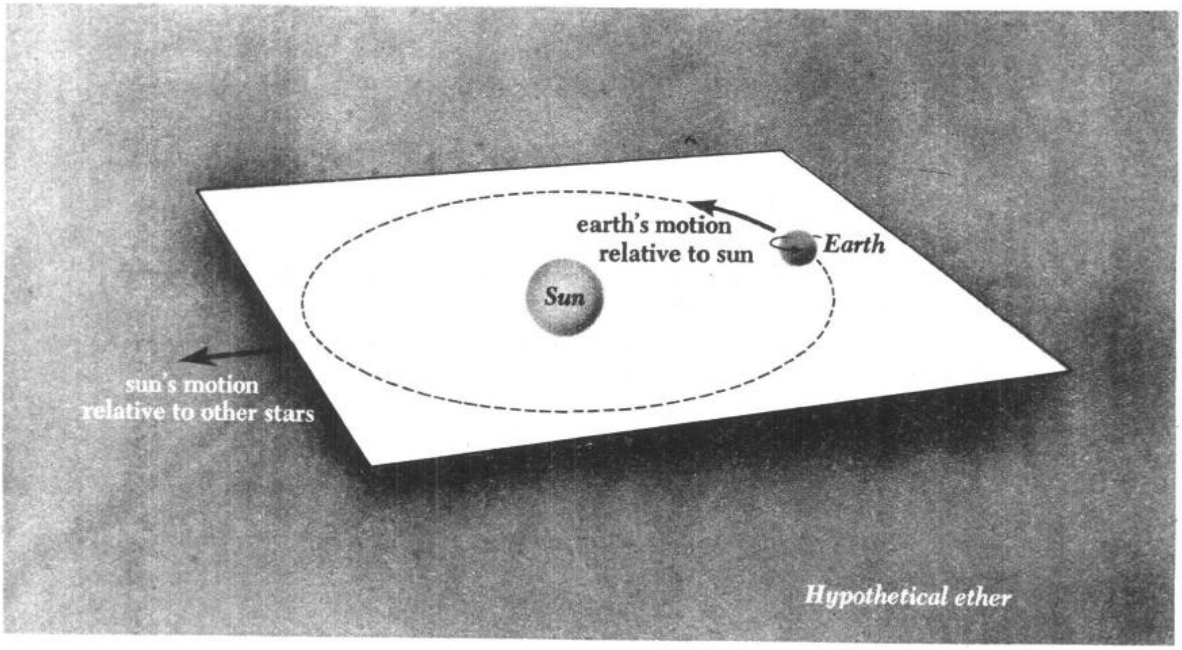
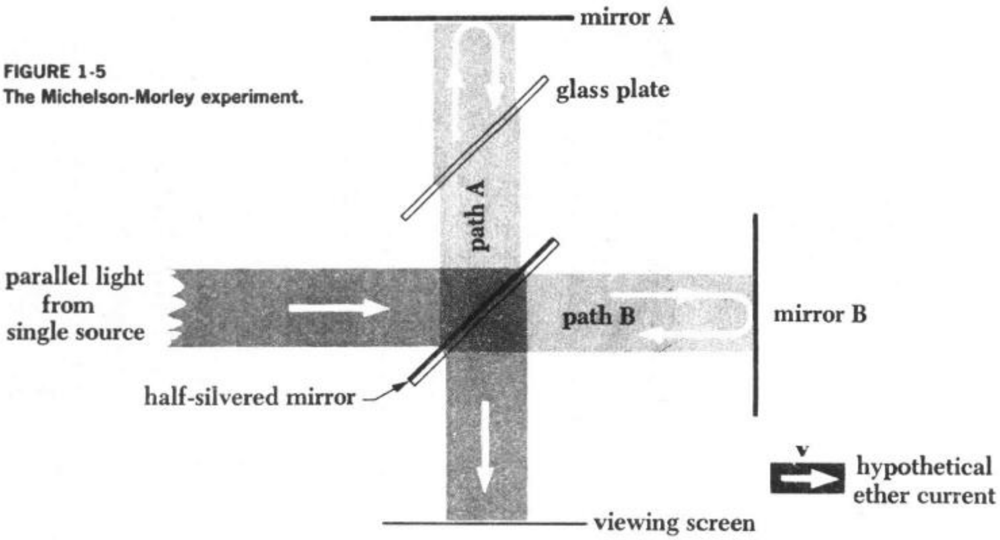


FIGURE 1-5
The Michelson-Morley experiment.



To detect this motion, we can use the pair of light beams formed by a half-silvered mirror instead of a pair of boats (Fig. 1-5). One of these light beams is directed to a mirror along a path perpendicular to the ether current, while the other goes to a mirror along a path parallel to the ether current. The optical arrangement is such that both beams return to the same viewing screen. The purpose of the clear glass plate is to ensure that both beams pass through the same thicknesses of air and glass.

If the path lengths of the two beams are *exactly* the same, they will arrive at the screen in phase and will interfere constructively to yield a bright field of view. The presence of an ether current in the direction shown, however, would cause the beams to have different transit times in going from the half-silvered mirror to the screen, so that they would no longer arrive at the screen in phase but would interfere destructively. In essence this is the famous experiment performed in 1887 by the American physicists Michelson and Morley.

In the actual experiment the two mirrors are not perfectly perpendicular, with the result that the viewing screen appears crossed with a series of bright and dark interference fringes due to differences in path length between adjacent light waves (Fig. 1-6). If either of the optical paths in the apparatus is varied in length, the fringes appear to move across the screen as reinforcement and cancellation of the waves succeed one another at each point. The stationary apparatus, then, can tell us nothing about any time difference between the two paths. When the apparatus is rotated by 90°, however, the two paths change their orientations relative to the hypothetical ether stream, so that the beam formerly requiring the time t_A for the round trip now

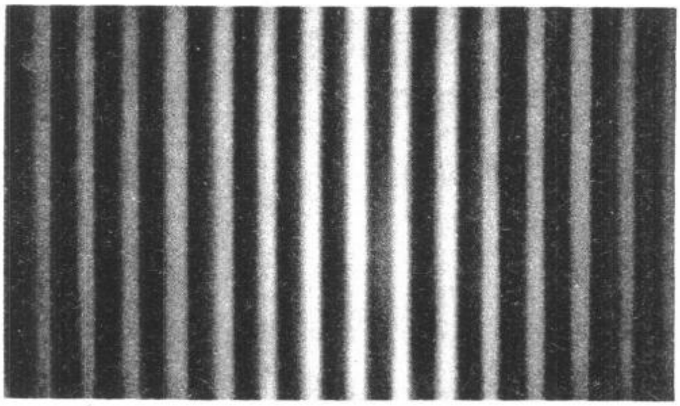


FIGURE 1-6 Fringe pattern observed in Michelson-Morley experiment.

requires t_B and vice versa. If these times are different, the fringes will move across the screen during the rotation.

Let us calculate the fringe shift expected on the basis of the ether theory. From Eqs. 1.1 and 1.2 the time difference between the two paths owing to the ether drift is

$$\begin{aligned}\Delta t &= t_B - t_A \\ &= \frac{2D/V}{1 - v^2/V^2} - \frac{2D/V}{\sqrt{1 - v^2/V^2}}\end{aligned}$$

Here v is the ether speed, which we shall take as the earth's orbital speed of 3×10^4 m/sec, and V is the speed of light c, where $c = 3 \times 10^8$ m/sec. Hence

$$\begin{aligned}\frac{v^2}{V^2} &= \frac{v^2}{c^2} \\ &= 10^{-8}\end{aligned}$$

which is much smaller than 1. According to the binomial theorem,

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!} \pm \frac{n(n-1)(n-2)x^2}{3!} + \dots$$

which is valid for $x^2 < 1$. When x is extremely small compared with 1,

$$(1 \pm x)^n \approx 1 \pm nx$$

We may therefore express Δt to a good approximation as

$$\begin{aligned}\Delta t &= \frac{2D}{c}\left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)\right] \\ &= \left(\frac{D}{c}\right)\left(\frac{v^2}{c^2}\right)\end{aligned}$$

Here D is the distance between the half-silvered mirror and each of the other mirrors. The path difference d corresponding to a time difference Δt is

$$d = c\,\Delta t$$

If d corresponds to the shifting of n fringes,

$$d = n\lambda$$

where λ is the wavelength of the light used. Equating these two formulas for d, we find that

$$\begin{aligned} n &= \frac{c\,\Delta t}{\lambda} \\ &= \frac{Dv^2}{\lambda c^2} \end{aligned}$$

In the actual experiment Michelson and Morley were able to make D about 10 m in effective length through the use of multiple reflections, and the wavelength of the light they used was about 5,000 A (1 A $= 10^{-10}$ m). The expected fringe shift in each path when the apparatus is rotated by 90° is therefore

$$\begin{aligned} n &= \frac{Dv^2}{\lambda c^2} \\ &= \frac{10\text{ m} \times (3 \times 10^4\text{ m/sec})^2}{5 \times 10^{-7}\text{ m} \times (3 \times 10^8\text{ m/sec})^2} \\ &= 0.2\text{ fringe} \end{aligned}$$

Since both paths experience this fringe shift, the total shift should amount to $2n$ or 0.4 fringe. A shift of this magnitude is readily observable, and therefore Michelson and Morley looked forward to establishing directly the existence of the ether.

To everybody's surprise, *no fringe shift whatever* was found. When the experiment was performed at different seasons of the year and in different locations, and when experiments of other kinds were tried for the same purpose, the conclusions were always identical: no motion through the ether was detected.

The negative result of the Michelson-Morley experiment had two consequences. First, it rendered untenable the hypothesis of the ether by demonstrating that the ether has no measurable properties—an ignominious end for what had once been a respected idea. Second, it suggested a new physical principle: the speed of light in free space is the same everywhere, regardless of any motion of source or observer.

1.2 The Special Theory of Relativity

We mentioned earlier the role of the ether as a universal frame of reference with respect to which light waves were supposed to propagate. Whenever we speak of "motion," of course, we really mean "motion relative to a frame of reference." The frame of reference may be a road, the earth's surface, the sun, the center of our galaxy; but in every case we must specify it. Stones dropped in Bermuda and in Perth, Australia, both fall "down," and yet the two move in exactly opposite directions relative to the earth's center. Which is the correct location of the frame of reference in this situation, the earth's surface or its center? The answer is that *all* frames of reference are equally correct, although one may be more convenient to use in a specific case. *If* there were an ether pervading all space, we could refer all motion to it, and the inhabitants of Bermuda and Perth would escape from their quandary. The absence of an ether, then, implies that there is no universal frame of reference, so that all motion exists solely relative to the person or instrument observing it. If we are in a free balloon above a uniform cloud bank and see another free balloon change its position relative to us, we have no way of knowing which balloon is "really" moving (Fig. 1-7). Should we be isolated in the universe, there would be no way in which we could determine whether we are in motion or not, because without a frame of reference the concept of motion has no meaning.

The theory of relativity resulted from an analysis of the physical consequences implied by the absence of a universal frame of reference. The special theory of relativity, developed by Albert Einstein in 1905, treats problems involving the motion of frames of reference at constant velocity (that is, both constant speed and constant direction) with respect to one another; the general theory of relativity, proposed by Einstein a decade later, treats problems involving frames of reference accelerated with respect to one another. The special theory has had a profound influence on all of physics, and we shall restrict ourselves to it.

The special theory of relativity is based upon two postulates. The first states that **the laws of physics may be expressed in equations having the same form in all frames of reference moving at constant velocity with respect to one another.** This postulate expresses the absence of a universal frame of reference. If the laws of physics had different forms for different observers in relative motion, it could be determined from these differences which objects are "stationary" in space and which are "moving." But because there is no universal frame of reference, this distinction does not exist in nature; hence the above postulate.

The second postulate of special relativity states that **the speed of light in free space has the same value for all observers, regardless of their state of**

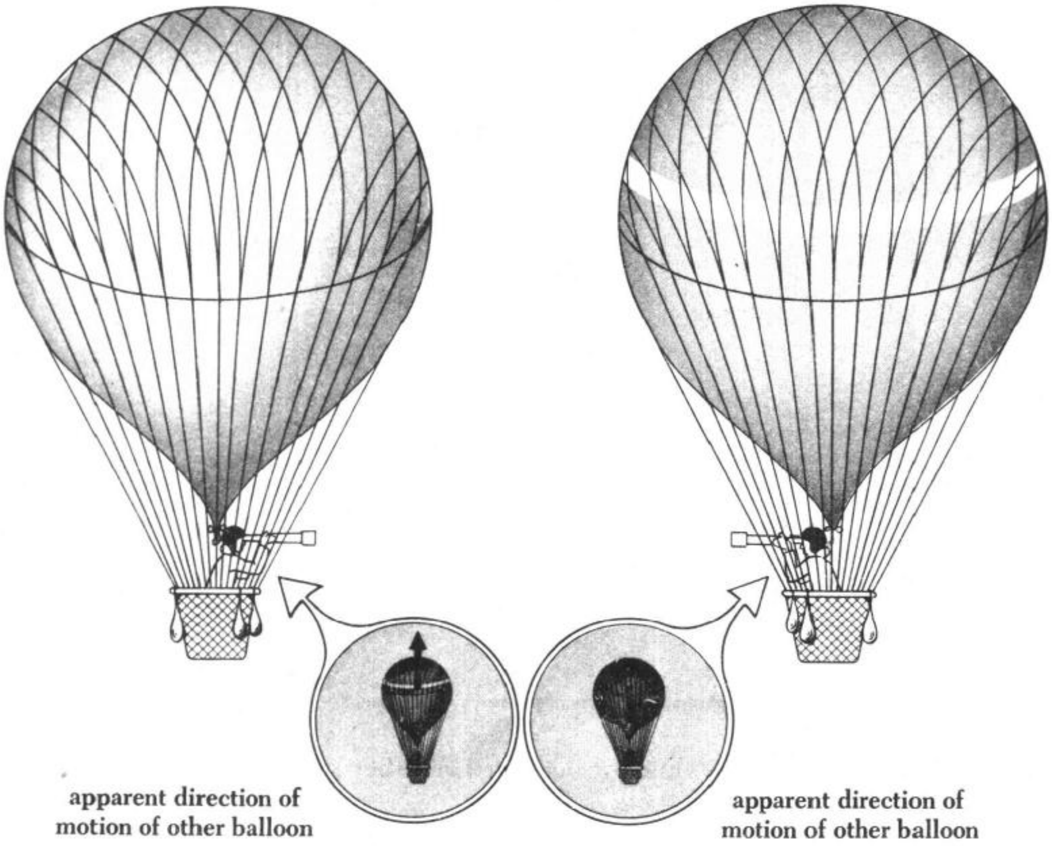


FIGURE 1-7 **All motion is relative to the observer.**

motion. This postulate follows directly from the result of the Michelson-Morley experiment.

At first sight these postulates hardly seem radical. Actually they subvert almost all of the intuitive concepts of time and space we form on the basis of our daily experience. A simple example will illustrate this statement. In Fig. 1-8 we have the two boats *A* and *B* once more, with boat *A* stationary in the water while boat *B* drifts at the constant velocity v. There is a dense fog present, and so on neither boat does the observer have any idea which is the moving one. At the instant that *B* is abreast of *A*, a flare is fired. The light from the flare travels uniformly in all directions, according to the second postulate of special relativity. An observer on either boat must see a sphere of light expanding with *himself* at its center, according to the first postulate of special relativity, even though one of them is changing his position with respect to the point where the flare went off. The observers cannot detect which of them is undergoing such a change in position since the fog eliminates any frame of reference other than each boat itself, and so, since the